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Uncertainty management for coastal defence systems.

Uncertainty Management for Coastal Defence Systems

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Abstract

Coastal engineers are challenged by intensifying demands for efficiency and sustainability in their efforts to manage the hazards of flooding and coastal erosion. The aim of the research described in this thesis was to improve decision-making in coastal engineering by addressing aspects of uncertainty that have tended to be neglected in the past. Interviews with coastal engineering practitioners demonstrated how uncertainty has its origins in issues of values, communication, and organisational constraints, as well as in the models of how coastal defences may perform in future. Decision-makers should therefore be as concerned with the decision-making *process* as they are with uncertain model predictions. The process models constructed in this thesis identified hundreds of data collection, analysis and design processes that contribute evidence to key strategic or project appraisal decisions. These processes are themselves integrated in a wider socio-technical system of ongoing coastal management.

Mathematical treatment of uncertainty in coastal engineering has to date focussed on probabilistic methods. Yet uncertainty is too rich and complex a phenomenon to be expressible solely in the language of probability. Alternative uncertainty representations, mostly generalisations of probability theory, are explored. A new approach to using Interval Probability Theory, which is one such generalisation, for uncertain inference in multi-dimensional problems, is introduced. These developments have been implemented in Visual C++ and integrated with a hierarchical process modelling software package developed by co-researchers. It is demonstrated how different types of uncertain information can be combined to choose a preferred coastal defence option.

The new concepts and techniques have been applied in practice to project appraisal decisions for three Environment Agency sea defence projects. These case studies demonstrated how hierarchical process models, with uncertainty representation using Interval Probability Theory, provide an overview of the sources of uncertainty in the decision-making process. Communication of uncertainty is supported by externalising and structuring expert judgements. These developments enable decision-makers to take better account of uncertainty and adopt resilient coastal defence strategies.

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Finally, I am deeply grateful, as always, to Laura who, despite my sometimes-obsessional preoccupation with this thesis (not to mention mountaineering), nonetheless agreed to marry me.

to Laura

Declaration

I declare that the work in this dissertation was carried out in accordance with the Regulations of the University of Bristol. The work is original except where indicated by special reference in the text and no part of the dissertation has been submitted for any other degree.

Any views expressed in the dissertation are those of the author and in no way represent those of the University of Bristol.

The dissertation has not been presented to any other university for examination either in the United Kingdom or overseas.

Signed: JW Hall

Date: 22/12/99

The papers:

HALL, J.W., BLOCKLEY, D.I. and DAVIS, J.P. Uncertain inference using interval probability theory. *International Journal of Approximate Reasoning*, Vol.19, No.3-4 (October 1998) pp. 247-264.

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are based on the work described in this thesis.

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The complexity of the universe is beyond expression in any possible notation.

Lift up your eyes. Not even what you see before you can ever be fully expressed.

Close your eyes. Not even what you see now.

(Michael Frayn, 1974, Constructions)

It is increasingly apparent that the engineering sciences face a historic turning point: they can continue to think and work in the worn-out ways of the nineteenth century. Then they will confuse the problems of the risk society with those of early industrial society. Or they can face the challenges of a genuine, preventative management of risks. Then they must rethink and change their own conceptions of rationality, knowledge and practice, as well as the institutional structures in which they are put to work.

(Ulrick Beck, 1986, Risk Society: Towards a New Modernity)

CHAPTER 1

The problem domain: coastal defence in the UK

1.1 Objectives of Chapter 1

- to state the objectives of this thesis and outline how they are to be achieved;
- to introduce the UK coastal defence system;
- to explain the increasing pressure for improved decision-making in UK coastal engineering;
- to demonstrate the need to study uncertainty in decision-making for coastal defence systems.

1.2 Background to coastal defence and coastal management

In the UK the aim of the Ministry of Agriculture Fisheries and Food's (MAFF, 1993*a*) flood and coastal defence policy is

> *To reduce risks to people and the developed and natural environment from flooding and coastal erosion by encouraging the provision of technically, environmentally and economically sound and sustainable defence measures.*

Risk reduction therefore lies at the heart of coastal defence activities, and inevitably linked to reduction or management of risk are issues of uncertainty. It is also clear from MAFF's policy that the objectives of risk reduction are multiple. The objectives, at least, encompass

- economic efficiency,
- safety of people, and
- environmental sustainability.

Coastal defence is, therefore, a multi-objective activity concerned with risk reduction. It is conducted on the interface between human and natural systems. Flooding may in biblical terms have been regarded as an act of God, in other words a phenomenon in the meta-system over which humans have no control. However, in modern times the coastal system is now strongly influenced by engineered works on the coast, be they coastal defence works or other constructions, for example associated with ports and harbours. On the developed stretches of mobile (*i.e.* not rocky) coast which are of the most concern to coastal engineers it becomes practically meaningless to disentangle natural and human-induced changes. In other words the coastal system has become co-evolutionary (Turner *et al.*, 1998) with complex dynamic feedbacks between human and natural influences. These feedbacks lead to the idea of 'joint determinism' in which human and natural processes are now meshed. At many coastal sites the consequence of human intervention in such an energetic natural environment has been to induce an over-engineered state of disequilibrium (Klein *et al.*, 1998, Helmer *et al.*, 1996), which may make coastal responses to change (for example climate change) increasingly unstable.

Equally if not more complex than the physical processes at work on the coast are the human systems, both those directly associated with coastal defence and the wider set of human activities that have an influence on the coast. Experience demonstrates that engineered systems are inevitably fallible. Coastal engineering is no exception. If, as Blockley (1980) and others (Turner and Pigeon, 1997) suggest, engineering failures are a result of the unforeseen consequences of human actions, then the human side of the coastal management system should be the subject of more scrutiny that has been the case in the past. The institutional arrangements for coastal management in the UK will be outlined shortly, but it should already be clear from the multiple objectives introduced above that management of coastal defence is a complex problem. It is an example of what Donald Schön (1983) referred to as "a mess" or what Conklin and Weil (1999) referred to as a "wicked problem".

1.2.1 A complex natural environment

Analysis of coastal engineering systems often begins with the loading regime and it is here that complexity and uncertainty first become apparent. Loads (waves heights and water levels) are traditionally described in statistical terms. However, historic data sets at many sites are rather short for extreme value analysis. Analysis of longer data sets can demonstrate that loading conditions may be far from stationary in statistical terms. The Intergovernmental Panel on Climate Change (IPCC) recognised in 1996 that "evidence suggests that there is a discernible human influence on climate" (quoted in Turner *et al.*, 1998). One effect is that the long-term rise in sea level since the end of the last Ice Age is believed to be accelerating, though apparently that acceleration, the subject of much debate in climate circles, began during the second half of the last century (Woodworth, 1999). In the UK the situation is most severe in the south and east of the country due to isostatic adjustment. By 2050 Scotland is likely to witness a 16-20cm rise in relative sea levels, whilst by the same time relative sea levels in East Anglia are likely to be 30cm higher. One of the most significant impacts from rising sea levels will be the change in frequency of high water levels. Though predictions are highly uncertain, best estimates suggest that by 2050 a storm surge which currently has a 100 year return period will occur with an estimated return period of once a decade. There have also been suggestions that storminess and prevailing wave directions may be changing, with consequent long-term impacts on littoral processes, but evidence to support this assertion is scarce (Hadley Centre for Climate Prediction and Research, 1998).

Yet more complex than the loading regime on the coast are the mechanisms by which coastal sediments and structures respond to those loads. These are topics that have been the subject of intense research activity over the last fifty years or more, but it now seems that there may be fundamental limits to the predictability of some coastal phenomena. Southgate (1995) has demonstrated the influence on non-linear effects on the prediction of beach evolution. Reeve and Fleming (1999) have detected chaotic effects in historic records of beach profiles. In an important pair of papers on coastal modelling de Vriend (1991*a*, 1991*b*) describes the coast as "a multi-scale non-linear dynamic system" in which long term effects are "a weak residual of a very noisy signal", with consequent limits to predictability in the long term. The coast can be addressed at different levels of definition (Komar, 1999)

- *oceanic* e.g. the North Sea and Mediterranean;
- *coastal cell*: largely self contained stretches of coastline with respect movement of larger grained sediments;
- *behavioural units*: local sections of coast which interact with neighbouring sections but can be distinctly characterised by hydraulic/morphological regime *e.g.* individual beaches (CIRIA, 1996*a*), cliff behavioural units (Lee, 1999) or estuaries;
- *local responses*: distinct responses to individual features or human interventions; morphological features such as sand bars;
- *individual sediment grains.*

Following de Vriend's arguments it is not surprising that compromising so-called 'top down' models of long term large scale coastal change with 'bottom up' models based on integrating small scale models of over short time steps has proved to be such a difficult task (HR Wallingford, 1997).

1.2.2 Conflicting human demands

The coast is valued by society for diverse reasons. The total UK annual turnover by the marine related sector of the economy has been estimated at £51.2 billion (1994-95 prices) and the total marine related value added has been put at £27.8 billion. The most important marine-related sector of the economy in financial terms (contributing 40% of the total sector in value added) is in fact the oil and gas, in which most activity is offshore rather than on the coast. However, marine related leisure and recreation, which is concentrated on the coast, is a growing market sub-sector (21% of the value added) accounting for some £1.7 billion of overseas earnings. Other components of the marine related economy such as shipping, ship building, ports, fishing *etc.* contribute smaller individual value added totals (between 1% and 8% of the total each) (Office of Science and Technology, 1997).

Ports and harbours

The UK, by its island nature, is dependent on ports for a great proportion of its international trade. Growing trade with Europe has increased the importance of ports, especially in the South and Southeast of England. Several UK ports have recently undertaken expansion works (including major dredging programmes) in response to the international trend of increasing vessel sizes.

Dredging of marine aggregates

The dredging of marine aggregates accounts for around 10% of UK aggregate production the main uses being the concrete industry and beach nourishment (CIRIA, 1996*b*). Government approval of extraction licenses is contingent on it being shown that the extraction will not affect coastal processes. Navigation dredging, which is nearer to the shore and removes material from active regions of the littoral zone, can be expected to have much greater impacts on the coast than marine aggregate extraction. The impacts on fisheries of aggregates dredging are more controversial, bringing promoters of beach nourishment schemes, which rely on marine dredging, into conflict with fisheries interests.

Coastal flooding and coastal erosion

Over 5% of the UK population live in the 2,200 km^2 of land most at risk from flooding by the sea. 57% of Britain's most productive agricultural land lies below 5mOD. Smaller, but also of economic and political significance is the proportion of the population living on or near the coast on land above 5mOD that may nonetheless be at risk from coastal erosion. The current annual average land loss in England of around 25 ha (Lee, 1999). Archives show that Norfolk has lost 21 coastal towns and villages since the eleventh century. Some 11.5% of the British coastline and 32% of the English coast are protected by artificial structures (Turner *et al.*, 1998). Annually more than £100 million is invested in capital works of coastal engineering (Simm and Cruickshank, 1998). Nationally if there were no form of defence MAFF estimates that the annual average damage from flooding and coastal erosion in England and Wales would be of the order of £2.1 billion (House of Commons, 1998).

Coastal housing

Populations are drawn to the coast for economic reasons, but also because the coast is in itself attractive. Continued efforts from agencies/authorities responsible for flood and coastal defence to restrict development in zones at risk from flooding and coastal erosion (Environment Agency, 1998*a*) come into conflict with the demographic demands for more houses, particularly near the coast.

Assimilation of waste

Coastal and deeper waters have traditionally been used as a major repository for waste. Wastes enter coastal waters as a consequence of land and sea-based human activities. Physical, chemical and biological processes determine the capacity of the coast to assimilate waste without undesirable changes to ambient water quality, which can affect, for example, beach-related recreation or commercial fishing. Following the 1994 EC Bathing Water Directive there has been major investment to remedy the pollution problems at the 17.5% of Britain's beaches that failed to meet European standards. Over the period 1995-2000, an investment of £900 million per annum is projected for construction of treatment plants and other facilities, mostly as a response of the Urban Wastewater Treatment Directive and the Bathing Water Directive. Operating costs have been assessed at £300 million (Turner *et al.*, 1998).

Coastal recreation

The coast is highly valued as an amenity resource by coastal inhabitants and visitors alike. In 1997, the British took 26.5 million seaside holidays in the UK, spending £4.7 billion. In addition , there were in excess of 190 million day visits made to the UK's coast, generating a further £2 billion spend (British Resorts Association, 1999). The beaches between Mablethorpe and Skegness, which are studied in detail in Chapter 7, were visited by 3,600 visitors daily on average during the peak summer season (Posford Duvivier, 1991). Research reported by Tunstall and Penning-Rowsell (1998) demonstrates the diversity of attractions which bring visitors to the coast, from traditional 'bucket and spade' recreation to bird watching and jet skiing, recreation activities which can bring different groups of coastal users into conflict.

Coastal habitats

The coast is an extremely important habitat. Preservation of saltmarshes, reedbeds and lowland wet grasslands features in the UK's commitments to biodiversity, set out in *Biodiversity: The UK Action Plan* (Department of the Environment, 1994). European environmental legislation is of direct relevance to the coast, significant parts of which are now designated as Special Protection Areas under the Birds Directive and/or Special Areas of Conservation under the Habitats Directive. The UK is also a signatory to the Ramsar Convention and many Ramsar sites, which are wetlands of international importance, are located on the coast. Saline habitats (primarily mudflats) are under threat from coastal squeeze between fixed defence lines and rising sea levels, resulting in an estimated 11,459ha loss of inter-tidal habitats over the next 50 years (4% of current resource) (Sharpe, 1999). Meanwhile freshwater wetland habitats can be threatened by the erosion of the natural or man-made defences that protect them from inundation by the sea. For example the eroding shingle bank at Cley on the North Norfolk Coast protects a freshwater wetland which is an important habitat for birds. The Environment Agency, which is responsible for the bank, is under pressure to intervene in the natural system in order to prevent the freshwater habitat being inundated with salt water (Klein, 1997).

Shifting priorities

The priorities and objectives associated with the coast are themselves dynamic. The demand for agricultural production during and immediately after World War II has now subsided to such an extent that large areas of low grade agricultural land reclaimed in Britain's estuaries are now set aside from production. Meanwhile, issues of sustainability have attracted increased political attention, which is reflected, for example, by the UK's international commitments on biodiversity.

1.2.3 The coastal management system in the UK

In the England overall policy responsibility for flood and coastal defence lies with the Ministry of Agriculture Fisheries and Food (MAFF) and in Wales with the National Assembly for Wales. MAFF develops coastal defence policy and, through the system of Grant Aid, funds capital works of flood and coastal defence. Through the administration of the Grant Aid system MAFF is able to exercise considerable control over the construction of flood and coastal defences. For a proposed coastal defence project to be eligible for Grant Aid, it has to be expected to provide an economic benefit (in terms of expected reduction in risk of flooding and/or coastal erosion) which exceeds the estimated cost of the scheme. It must also be sound in engineering terms and acceptable from an environmental point of view (MAFF, 1993*b*). The process of Project Appraisal, by which the eligibility of a prospective project for Grant Aid is assessed, is a key stage in the promotion of coastal defence projects in the UK.

Implementation and maintenance of coastal defences is the responsibility of the Environment Agency (EA) and local authorities on the coast (known as Maritime District Councils (MDCs)). In 1995 the EA inherited, from the National Rivers Authority (NRA), responsibility for sections of the coast at risk from flooding. The MDCs are responsible for implementation of coastal defences in areas that are not at risk from flooding, so are concerned with reducing the risk of coastal erosion by provision of engineering works which are usually referred to as 'coast protection'.

To avoid piecemeal responses to coastal problems, in recent years MAFF has promoted a strategic approach to coastal defence (MAFF, 1993*a*, Purnell, 1995). Research by HR Wallingford (Brampton and Motyka, 1993) subdivided the UK coastline into a series of coastal cells, which were considered to be reasonably self-contained in sedimentary terms. Authorities with responsibility for implementation of coastal defences within each coastal cell were encouraged to join together in Coastal Groups. Shoreline Management Plans (SMPs) were developed in order to set the general policy within each sub-cell. Four policy options are considered at SMP stage:

- do nothing,
- hold the line,
- retreat,
- advance

(MAFF, 1995). The SMP process is now largely complete and has brought together a wide range of bodies with an interest in the coast. The focus of SMP process is coastal defence. SMPs are therefore rather more specific that Coastal Zone Management (CZM) plans (European Commission, 1999), which address all of the planning issues associated the coast.

More specific design options and implementation plans are considered during Strategy Plans, which include costed plans for specific sections of coast. Because Strategy Plans involve assessment and outline costing of specific options they are more closely related to the Project Appraisal process than SMPs. Strategy Plans are usually developed by an individual MDC or office of the EA, though in some cases two authorities may choose to work together in the development of a Strategy Plan (Figure 1.1).

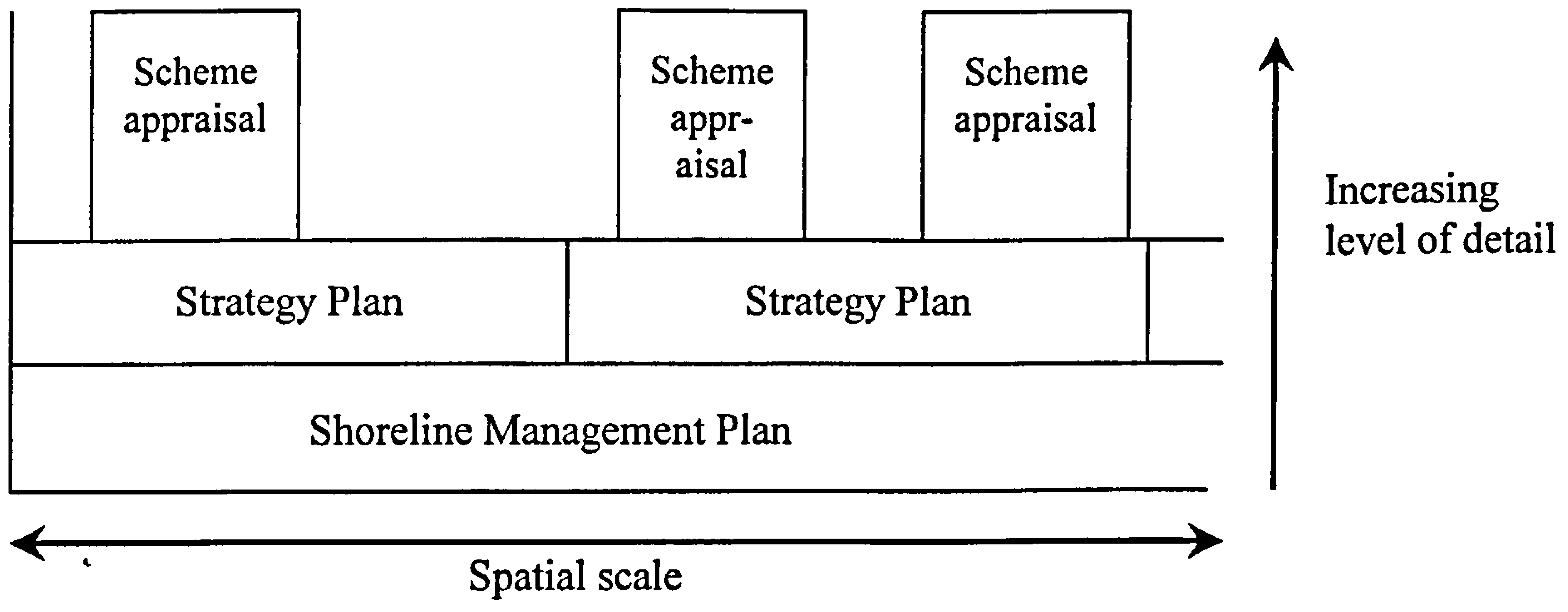

Figure 1.1 Interaction of planning and design in coastal management (after MAFF, 1997)

The move towards Strategy Planning represents an acknowledgement of the dynamic and continuous nature of coastal management. Coastal management is no longer seen as a process of constructing monolithic works on the coast followed by a minimum of intervention ('hard' coastal engineering). Many attempts to protect against flooding and coastal erosion using hard capital works are now seen as inappropriate investments of large sums of public money. Consequently there is a shift away from hard one-off capital works towards ongoing operational coastal management systems. Coastal management is now seen as the dynamic control of a continuously changing system. It involves intelligent monitoring and effective strategies for responding to monitoring. The term 'soft engineering' is used to refer to coastal engineering that endeavours to work with natural systems rather than against them, for example through beach nourishment or mudflat generation. Implicit in the idea of soft engineering is recognition of the dynamics of the coastal system.

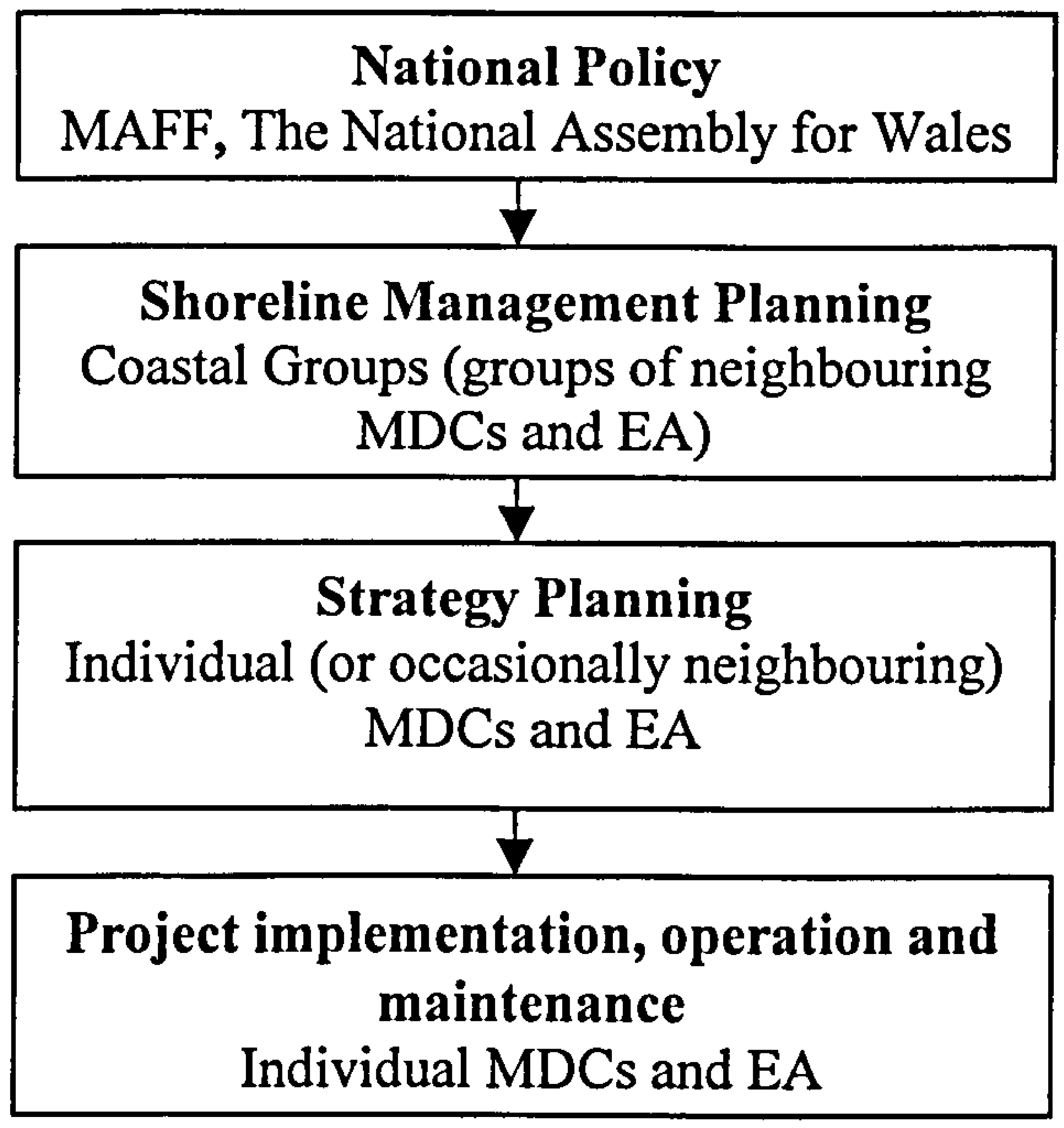

Figure 1.2 Levels of decision-making in flood and coast defence in the UK

The hierarchy of responsibility and decision-making in the UK coastal management system is illustrated in Figure 1.2. Policy decisions by MAFF are interpreted in regional terms in SMPs and more locally in Strategy Plans. Implementation and maintenance is the responsibility of the MDCs and EA who usually employ consultants and contractors to help them with their implementation role. Since the formation of the NRA and subsequently the EA there has been a tendency to contract out engineering services, a trend that has been mirrored in MDCs. The amount of in-house coastal engineering expertise within the EA and MDCs has reduced.

This hierarchical model is relatively recent and institutional arrangements are subject to ongoing change. MAFF's Project Appraisal Guidance Notes (PAGN) (MAFF, 1993*b*), which have been a major influence on the development of coastal defences in the UK, are to be replaced by an integrated set of six project appraisal guidance documents. These new guidance notes will include a document dedicated specifically to risk and one dedicated to environmental issues in project appraisal. The EA has implement significant changes to its management of flood defences in the wake of the Easter 1998 floods (Bye and Horner, 1998, Environment Agency, 1998*b*). It remains to be seen what influence the report of the House of Commons Agriculture Committee (House of

Commons, 1998) will have on administrative arrangements. In common, therefore, with the physical and human systems associated with the coast, the coastal management system is itself dynamic and subject to varied and complex influences.

1.3 The drivers for improved decision-making

From the brief introduction to contemporary coastal engineering and management in the UK some drivers for improved decision-making begin to emerge. The need for improved decision-making is driven by increasing demands for efficiency, safety and sustainability. Pressure for efficiency has resulted in a reduction in the number of individuals in the institutions responsible for implementation of coastal defences at a time when the move towards soft coastal engineering is increasing the information processing demands placed on decision-makers. Institutional change will be ongoing.

Soft coastal engineering places increasing information processing demands on decision-makers because it tends to rely on a deeper understanding of coastal processes than hard approaches, which were characterised by solid defences against natural processes. Modern coastal management makes more use of monitoring information and often elaborate models, information which the decision-maker is required to make sense of. Ongoing operational beach, cliff or estuarial management systems present the coastal manager with a bewildering array of options for planning, data collection, analysis, design and implementation. Temporally linear project-based management is being replaced by a spatially diffuse cycle of data collection, decision-making and actions. This emphasis on monitoring and control does relieve some of the unjustifiably high expectations which have in the past tended to be placed on predictive models but relies on engineers being able to design a high degree of flexibility in coastal defence works.

The move towards a more strategic and holistic view of coastal defence is welcome, but widens the scope of issues to be addressed by decision-makers. A strategic perspective requires analysis on a longer term and broader scale than was typically the case in the past. Holistic approaches require consideration of social and environmental considerations, which coastal engineers in the past would not have recognised as being within their remit.

The pressure placed on decision-makers by dynamic data-intensive engineering systems is well illustrated by an example from an analogous field of engineering, the well known failure of the Heathrow Express tunnel. The tunnel was being constructed using the so-called New Austrian Tunnelling Method (NATM). NATM relies on intensive monitoring of ground movements, with design and construction of the tunnel lining in response to feedback from the monitoring information. It is therefore analogous to dynamic systems of coastal management, which rely on effective response to monitoring information. In the enquiry into the Heathrow Tunnel collapse it became clear that whilst data was being collected it was not being used as intended. Signs of impending failure were ignored (*New Civil Engineer*, February 18, 1999, pp.3-5,10-14). The collapse reflects the importance of having effective decision-making systems in place, and the difficulty of achieving this in dynamic data-intensive situations. Pascoe and Pidgeon (1995) argue that in dynamic contexts the role of the individual decision-maker deserves special attention since his or her actions have more influence over the progress and the outcome of the task than is the case in most static contexts. High complexity tasks require not only dependable decisions, but more importantly appropriate timing, determined by the dynamic system rather than the decision-maker. As coastal defence becomes more dependent on vigilant monitoring and frequent intervention it may be becoming less robust with respect to failure of the human management system.

There is therefore an unfortunate paradox that on the one hand the coastal engineer's understanding and ability to model coastal processes has increased markedly over the years. However, on the other hand that increased understanding has arguably intensified the appreciation of the complexity of the coastal system and has certainty increased the complexity of the information available to the decision-maker.

Pressure for efficient and environmentally sound and sustainable solutions demands consideration of a wide range of potential impacts and interactions. The UK Government's signature of the Rio Treaty represents a commitment to sustainable development and biodiversity. The coastal engineer is under increasing pressure to pay attention to remote impacts of engineering on the coast and to prevent or mitigate potentially adverse impacts. Once again the coastal engineer's capacity to address these issues is increasing, but at a cost of increasing complexity.

Social tolerance of flooding and coastal erosion is not easy to monitor in sociological terms (see Chapter 2). However, evidence from the 1998 Easter floods (Bye and Horner, 1998) and ongoing media attention to the impacts of coastal erosion suggests that public expectations with regard to coastal defence are high and may be intensifying. Meanwhile there is a growing expectation for transparency and accountability in public decision-making. Enquiries by Peter Bye (Bye and Horner, 1988) and the House of Commons Agriculture Committee (House of Commons, 1998) reflect this process. These enquiries identified some shortcomings in the current arrangements and have intensified pressure for improved decision-making. The prospect of future climate change and the implications for sea level rise mean that the hazards of flooding and coastal erosion may well become more severe in coming years.

Increased interest in coastal risk and coastal infrastructure also comes from commercial directions. In recent years insurers have invested considerable sums in analysis of coastal flood risk (Madrell, 1995, Madrell *et al.*, 1996). Coastal flooding represents one of the largest risks to which UK insurers are potentially exposed. Unlike in the Netherlands and the USA, in the UK private flood insurance is available throughout the country. However, the insurance industry has made it clear that the universal availability of private flood insurance is under ongoing review (Whiting, 1995).

The private sector is also becoming involved in the provision of coastal defence. The first privately financed sea defence project at Pevensey Bay in the UK has recently been let to a consortium of contractors, consultants and financiers (*New Civil Engineer*, June 10, 1999, p.7). These innovative procurement arrangements are increasing the scope for flexible strategic coastal solutions but are also bringing a new range of options and considerations to the fore for coastal managers.

Risk issues have attracted increasing interest from the UK coastal engineering community. At first sight it may seem that risk management in coastal engineering is being addressed from rather different points of view. On the one hand there are management activities intended to improve the efficiency of delivery of coastal projects, in particular project risk management. On the other hand there are activities aimed at natural hazard management ("To reduce risks to people and the developed and natural environment from flooding and coastal erosion...", MAFF, 1993). Another perspective comes from the point of view of construction health and safety, for which coastal construction has a rather poor record (Simm and Cruickshank, 1998). The perspectives are convergent and all contribute to the high level objective of efficient natural hazard management. To efficiently reduce the risk of flooding and coastal erosion requires efficient and safe coastal engineering projects, as well as assessment and management of the natural hazard itself.

1.3.1 Towards uncertainty management

The drivers for improved decision-making may therefore be summarised as

- increasing pressure for economic efficiency of public investment in coastal defences
- reduced in-house capacity of authorities responsible for coastal defence
- increasing information processing and decision-making demands of soft coastal engineering
- increasing pressure to identify and mitigate adverse impacts
- new national and international commitments to biodiversity and coastal habitats preservation
- low social tolerance of flooding and intense public interest in issues of coastal erosion
- potential changes in industry attitude to insurance of coastal hazards
- innovation in procurement of coastal projects
- new emphasis on project risk management and health and safety management.

The general theme running through all of the drivers for decision support is one of uncertainty. In the past, uncertainty has tended to be coped with by using engineering judgement, factors of safety, semi-probabilistic methods, and sensitivity testing (which are examined in more detail in Chapter 2). These methods will continue to have their place. In particular, engineering judgement founded on experience will always be an indispensable aspect of responsible decision-making. However, it cannot be assumed that traditional methods of dealing with uncertainty are as efficient as they could be or that they will continue to be effective in increasingly complex coastal management systems in which more interactions and sensitivities are taken into account. Under these circumstances the need for improved management of uncertainty is clear. The term 'uncertainty management' is used to emphasise that uncertainty is an inevitable characteristic of coastal defence systems. Uncertainty is something to recognise and respond to, it cannot be removed and should not be ignored. It has to be managed.

1.4 Research aim and objectives

The aim of the research described in this thesis is to develop a framework for uncertainty management in coastal engineering and tools and techniques to support decision-making under conditions of uncertainty.

The objectives of the research are:

1. *to provide a critical analysis of the sources of uncertainty for coastal engineers and managers in the UK and review current methods for coping with uncertainty in decision-making*;
2. *to define uncertainty and to characterise the sources of uncertainty in the evidence assembled in the lead-up to a coastal engineering decision*;
3. *to identify the requirements for uncertainty management in coastal defence systems*;
4. *to review approaches to representing uncertainty with numerical structures and identify an appropriate syntax for representing the uncertainty in the evidence assembled in the lead-up to a coastal engineering decision*;
5. *to implement theoretical developments in uncertainty representation in a decision support tool*;
6. *to demonstrate how different types of uncertain information can be used when making a choice*;
7. *to demonstrate the new developments by application to case studies of UK coastal engineering projects.*

The starting point for this research was a situation of increasing concern about issues of uncertainty in coastal engineering and some isolated areas of research attention, notably in statistical treatment of loads and reliability analysis of coastal structures, but very little in the way of comprehensive treatment of uncertainty. A first objective, therefore, was to conduct empirical research to establish the scope of uncertainty issues and current responses to uncertainty. This then led to a developing view of uncertainty management in coastal engineering. The empirical work, together with theoretical examination of issues of uncertainty and decision-making, also helped to identify more specific issues to do with the process of assembling evidence in the lead-up to a decision, which demanded special research attention. These problems were the focus of the theoretical developments, tool development and demonstration. In parallel, the general view of the process of managing uncertainty in coastal engineering has been enriched, and is reflected in the case study work.

1.5 Outline of this thesis

In order to achieve the objectives outlined above, this thesis is structured as follows (Figure 1.3). The following chapter describes new empirical findings revealed in an analysis of the sources and implications of uncertainty for coastal managers in the UK is described. Current approaches to decision-making under uncertainty are discussed critically. Using the investigative method of Grounded Theory, developed for research in the social sciences, the study described in Chapter 2 develops an empirical justification for the subsequent treatment of uncertainty.

Key systems concepts are introduced in Chapter 3. The weakness of reductionist approaches to representing physical and human systems is demonstrated and the integrative ideas promoted by the systems movement are proposed as an alternative. Closely linked to systems ideas are the concepts of process. Process modelling provides a unified vocabulary that can be used to address human and physical systems, so is of great relevance to the problems being addressed in this thesis. Process modelling techniques, which are introduced in Chapter 3 are developed through Chapters 4 and 5 to model the uncertainty in the process of assembling evidence in the lead-up to a decision.

Chapter 4 develops much of the conceptual framework which the thesis is based upon. A philosophical understanding of the notion of uncertainty is developed. The types of uncertainty in information available to decision-makers at the moment of choice are described. Of particular importance is the concept of 'dependability' of information. The uncertainty management methods proposed in Chapters 4 and 5 and demonstrated in Chapter 7 are aimed at providing decision-makers with information about the dependability of the different items of evidence upon which a decision is based.

Chapter 1: ***The problem domain: coastal defence in the UK***
Introduction to the problem domain.

↓

Chapter 2: ***Decision-making for coastal defence: a study of the current UK situation***
Empirical study of the sources of uncertainty and needs for decision support.

↓

Chapter 3: ***Systems and processes***
Introduction to systems concepts and process modelling.

↓

Chapter 4: ***Uncertainty, information and decision-making***
Development of the philosophical basis and key concepts of uncertainty, information and decision-making.

↓

Chapter 5: ***Developments in the representation of uncertain knowledge using numerical structures***
Review of approaches to uncertainty representation using numerical structures. New theoretical developments in Interval Probability Theory. Implementation of Interval Probability Theory for process modelling and uncertainty representation.

↓

Chapter 6: ***A contingency approach to choice***
Development of a general framework for choice based on uncertain information.

↓

Chapter 7: ***Uncertainty management for coastal defences on the East Coast of the UK***
Demonstration and testing of new developments by application to case studies

↓

Chapter 8: ***Conclusions***
Review of main contributions in the thesis and directions for future development

Figure 1.3 Outline of this thesis

Chapter 5 is a mathematical treatment of uncertainty. Established and more recent theories of uncertainty representation with numerical structures are introduced and assessed for the first time in the context of coastal engineering decision-making. The links between various uncertainty representations are highlighted. Interval Probability Theory is identified as an appropriate mathematical vocabulary for modelling the uncertain dependability of evidence. New theoretical developments in uncertain inference using Interval Probability Theory are introduced and explained. An appropriate structure for modelling uncertainty using Interval Probability Theory is developed.

Chapter 6 addresses the issue of how different types of information are brought together in a decision-making process at the moment of choice. It is argued that the choice mechanism should be appropriate to the nature of the available information and meta-objectives of the decision-maker.

In Chapter 7 the new developments in uncertainty representation are demonstrated in case studies. The project appraisal of two sea defence projects on the East Coast of the UK are analysed retrospectively to identify the sources and sensitivities to uncertainty. A third study addresses a decision that was live at the time of analysis.

Finally, conclusions are drawn in Chapter 8 and potential directions for further research are identified.

1.6 Conclusions

1. The aim of coastal defence in the UK is to reduce risk to the developed and natural environment from flooding and coastal erosion. Risk, and by implication uncertainty, are therefore at the heart of flood and coastal defence policy and practice in the UK.

2. The coast is valued for economic, environmental and recreational reasons. Coastal defence is an activity with multiple objectives, including economic efficiency, safety of people and environmental sustainability. It is a 'wicked problem'.

3. The physical processes at work on the coast are extremely complex and modern research is revealing the limits to their predictability. Climate change, with consequent impacts of rising sea levels, increasingly severe surge water levels and possibly changes to wave climates, is increasing the stress on the coast. These stresses have been exacerbated by a legacy of misguided engineering works and coastal planning policies, which have reduced the flexibility of what was once a dynamic natural environment.

4. In England overall responsibility for flood and coastal defence lies with MAFF but coastal defence works are implemented by the EA and MDCs with the help of consultants and contractors. Decision-making in coastal engineering is a hierarchical process cascading from national policy to Shoreline Management Plans to strategy plans and individual projects. In recent years there has been a move towards more strategic planning, together with increasing emphasis on monitoring and flexible response. This is to be welcomed but, to be effective, relies on timely, dependable decision-making and action.

5. Coastal engineers are now confronted with increasing pressure for economic efficiency in public investment in coastal defences at a time of reduced in-house engineering capacity. The move towards soft engineering is intensifying the information processing demands placed on decision-makers, at a time of increased public interest in coastal issues and the impacts of

flooding. It cannot be assumed that traditional methods of dealing with uncertainty are as efficient as they could be or that they will continue to be effective in increasingly complex coastal management systems in which more interactions and sensitivities are taken into account.

6. The aim of this thesis is to improve decision-making in coastal engineering by addressing aspects of uncertainty that have tended to be neglected in the past. Uncertainty management involves giving an overview of where uncertainty lies in a decision. It will help the coastal manager by identifying the most significant sensitivities and sources of uncertainty in a decision.

7. This thesis develops an approach to uncertainty management through a combination of empirical analysis of sources of uncertainty and current decision-making practice, theoretical developments in uncertainty representation and process modelling, decision support tool development and demonstration on two of the EA's sea defence projects. The objectives of this thesis have been listed and are revisited in Chapter 8 to demonstrate how they have been achieved.

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CHAPTER 2

Uncertainty and decision-making for coastal defence: a study of the current UK situation

2.1 Objectives of Chapter 2

- to describe an empirical study of uncertainty and decision-making practice;
- to identify sources of uncertainty in coastal engineering;
- to identify common coastal engineering responses to uncertainty;
- to establish principles which will guide the development of uncertainty management concepts and tools.

2.2 Introduction

This chapter describes an empirical study of current decision-making practice in coastal engineering and its inherent uncertainties. Such an analysis must include the social and organisational issues as well as purely technical ones if it is to form a basis for feasible and desirable changes to decision-making practice. Descriptive analysis of complex processes on the socio-technical interface is not straightforward. Many of the issues taken into account in decision-making are unquantifiable and may not even be made explicit. An investigative technique called Grounded Theory developed by Glaser and Strauss (1967) for field work in the social sciences was, for the first time, used to analyse the problems which coastal engineers encounter in practice.

Evidence about uncertainty in the decision-making process emerges from the experience of practising decision-makers and from the way in which actual projects are implemented on the coast. The discourse of those decision-makers and the evidence from those projects constitutes raw data on which descriptive analysis of uncertainty in decision-making can be based. Yet, to obtain that data requires active involvement and enquiry by the investigator.

In the study described, a hermeneutic perspective which recognises the role of the interpreter in the analysis of any social phenomenon has been adopted. Hermeneutics originated in the study of sacred text, from the debate between those who believe that the text has meaning that exists independent of the process by which it is interpreted and those who believe that meaning emerges as a result of the relationship between text and interpreter in the process of interpretation. Heidegger adopted the view of hermeneutics in which the meaning of the text is contextual. It depends on the specific environment in space and time, in which the interpretation takes place as well as the concerns the interpreter brings to the situation. The investigator and the subject interact and, furthermore, the investigator carries a heavy responsibility for generating meaning in the evidence which is unearthed. That is not to suggest that the investigator has complete freedom to generate theory. On the contrary, it will become clear from the description of this study that there is much emphasis on being faithful to the raw data and constructing theory in a transparent and auditable way.

The study proceeded as follows:

1. Contacting a range of experts and practitioners.
2. Conducting of tape recorded interviews with each of the experts/practitioners.
3. Analysing of the tape recordings using Grounded Theory.
4. Ordering the findings of the Grounded Theory analysis and constructing conceptual models of the system under consideration.
5. Circulating the findings to the interviewees for review and comment.
6. Revising the interpretations in the light of comments from the interviewees.

The work did not proceed in a strictly ordered fashion, rather it involved successive phases of iteration, particularly between the second, third and fourth stages listed above: models of the decision-making process began to emerge after the first interview and were subsequently reinforced, modified or refuted as the interview programme and analysis proceeded. The interview data was used both as a source of basic information and perspectives and as the data set for testing and corroborating hypotheses and models.

Subsequent to the interview programme and analysis, during the early summer of 1998 the House of Commons Agriculture Committee addressed the issue of flood and coastal defence. The report, minutes and evidence were published in July 1998 (House of Commons, 1998). Although not specifically focused on uncertainty and decision-making this probing study, which involved representations from all of the key organisations with an interest in flood and coast defence, provided revealing evidence to compare with the evidence obtained in this research. The independent report on the Easter 1998 floods (Bye and Horner, 1998) which relates to fluvial rather than coastal flood defence also contains evidence about institutional arrangements in the Environment Agency which is germane to this research.

2.2.1 A note on the terminology

The approach adopted in this study has its roots in the social sciences whilst the subject was coastal engineering. It is not surprising therefore that there is some confusion over terminology at the interface of these two disciplines.

The term 'environment' will be used in the social science sense. The environment is the set of surrounding influences within which a system operates. It may for example include cultural, institutional, political and legal aspects. The term 'physical environment' will be used to refer to the natural and built environment to which coastal engineers are customarily referring when they talk about environmental impacts or influences.

Similarly, the term 'process' will be used to refer to a set of activities, for example a design process or a consultation process. What coastal engineers refer to as 'coastal processes' or simply 'processes' will be labelled 'physical processes'.

2.3 The Grounded Theory research

2.3.1 Background to Grounded Theory

Grounded Theory is a methodology developed in the social sciences for analysis and interpretation of qualitative data (Glaser and Strauss, 1967). Data may be in the form of interview or focus group transcripts or participant observation. The word 'grounded' derives from the concept that the interpretation generated is 'grounded' in data. The interpretation, or theory, is in the form of conceptual models of the phenomenon under consideration, which emerge from the data. The need to demonstrate that these models are faithful to the empirical data is emphasised.

The present study was based on semi-structured interviews with eight experts and practitioners with key decision-making roles in UK coastal management. Interviewees were encouraged to expand on issues that they considered to be of importance. The resulting data was therefore rather wide in its scope and diverse in perspective. The transcripts of the interviews were long and complex and required detailed examination in order to identify common themes.

Grounded Theory analysis contrasts with mechanical content or protocol analysis (Ericsson and Simon, 1984) in which the frequency of occurrence of particular themes or words is analysed statistically. Content analysis was neither possible nor appropriate because of the small number of interviews and because of the unstructured nature of the interview process. Grounded Theory, by contrast, is an interpretative process in which the analyst takes responsibility for perceiving and creating order in the interview data. This requires identifying both what is relevant as well as what is irrelevant in the data, whilst at the same time faithfully reflecting the significant complexities inherent in the material.

2.3.2 Application of Grounded Theory in the current study

The core steps in conducting a Grounded Theory analysis with qualitative data are illustrated in Figure 2.1. The aim is to move from a set of unstructured data to a collection of theoretical concepts with relationships between them.

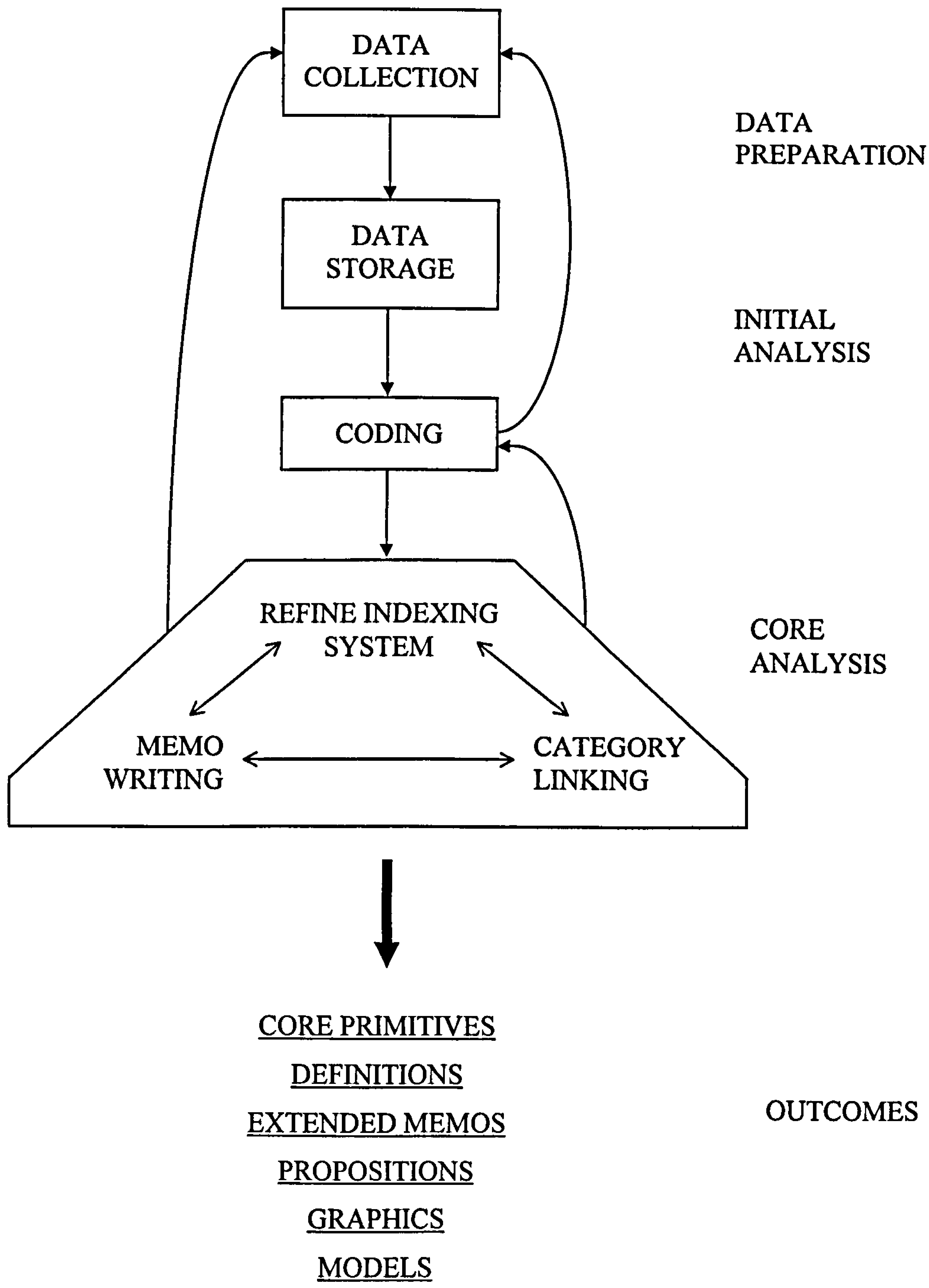

Figure 2.1 Steps in Grounded Theory analysis (after Pidgeon et al., 1991)

Data collection and storage

A total of eight tape recorded interviews were conducted between October 1996 and January 1997 with representatives from the following organisations:

1. MAFF (South West Region)

2. Environment Agency (Anglian Region)
3. North Norfolk District Council
4. HR Wallingford (two representatives)
5. ABP Research and Consultancy
6. Sir William Halcrow and Partners
7. Posford Duvivier

The interviewees were selected in order to represent perspectives from clients, designers and government. All of the interviewees had considerable experience and expertise in their fields. In the interests of confidentiality the names of the interviewees will not be referred to. The interviewees will be referred to as A to H in an order that does not correspond to the order of the above list.

The conduct of unstructured and semi-structured interviews is an uneasy balance between allowing the interviewee to freely explore aspects of interest and guiding them towards the substantive areas that are the subject of the interviewer's interest. The importance of the relaxed nature of semi-structured interviews cannot be over-emphasised. Rapport must be established between the participants, and confidentiality assured so that the interviewee feels free to say what really happens rather than what the organisation corporately says happens. The objectives of the interviews were to elicit information on:

- decision-making processes during planning, designing, implementing and managing coastal defences;
- the sources of uncertainty in the above processes;
- current methods of taking account of and managing uncertainty in coastal defence.

Interviews lasted between one and two hours each and were conducted in the offices of the interviewees. The general approach was to embark on each interview with a prepared list of topics to be discussed, but without specific questions. The list of topics evolved in the light of the first two or three interviews and then converged on a fairly stable set of issues, which are summarised in Table 2.1. In the early interviews the interviewees were specifically asked to describe in general terms the activities involved in the coastal defence process. In practice the interviewees were not particularly clear about the sense in which the word 'process' was being used. This approach therefore required more input on the part of the interviewer. In subsequent interviews, interviewees were encouraged to describe examples of projects, in which case the processes involved and the sources of uncertainty emerged from discussions more naturally.

Table 2.1 Summary of issues addressed during interviews

Technical uncertainty
Talk through the technical decision process. Describe a typical project, to identify where uncertainty may be significant and how it may be coped with.
Collecting data (hydraulic, morphological, coastal defences)
Is lack of data (historic data on beach erosion for example) frequently a problem?
Are authorities doing enough in terms of data collection, analysis and storage?
How do you decide what to collect and what to do with it?
Do you do any quantitative analysis of the value of information?
Analysing (physical and numerical modelling)
Comparing results from different models
How is systems uncertainty taken into account?
How could it be done better?
Probabilistic design.

Uncertainty in economic appraisal
1. costs - is uncertainty quantified?
2. benefits (flooding, coastal erosion) is uncertainty quantified? Do flood risk costs take account of flood warning and evacuation? Any back verification from actual floods?
3. unquantifiables - are there many attempts to quantify amenity/environment? Is uncertainty taken into account? To what extent is the Ministry prepared to recognise these benefits?

How is the capacity of the management system taken into account?

Political uncertainty (i.e. local interests) - is this a big issue?
How do you cope with it?
Can it be taken into account in quantitative terms?

Weighing up values
What are the criteria/objectives?
Are they explicit?
Is it possible to compare them?
How are disparate criteria (economic, public interest (visual, amenity, privacy, business), environmental etc.) balanced?
Is it possible to apply them transparently/consistently?
How are conflicting interests balanced?
How are temporal conflicts taken into account?

Uncertainty in the implementation process
Obtaining finance
Construction risk. Time/cost over-runs

Uncertainty in flood warning and emergency works. Uncertainty in the planning process. Any other areas of uncertainty

Shoreline management planning
Are Shoreline Management Plans (SMPs) being translated into action? i.e. is decision-making becoming more strategic?
Is there any quantification of uncertainty in SMPs?
How are conflicting time scales (public perception, economics, biodiversity, morphology) accounted for?
Do administrative/funding arrangements encourage a strategic approach?

Observational approaches for coping with uncertainty
Are dynamic/reactive solutions
1. a good way forward?
2. being implemented in practice? Is there inertia on the part of operating authorities?

How do you see coastal defences developing over the next ten years?
Policy changes
Changes in practice
Changes in the way decisions are made (changes in appraisal)

Basic errors in conducting interviews include failure to establish a rapport with the interviewee, inadvertently dominating the interview session or asking leading questions (Burges, 1983). The interviewer has to make sufficient contribution at the start of the interview to establish a rapport whilst not predetermining the path of subsequent discussions. In expert interviews the interviewer is much less prone to dominate the discussion than in situations where the interviewee is less self-confident. In the current interviews all of the interviewees were older and more experienced in their particular field than the interviewer. The interviewees were wary of leading questions and were confident enough to contradict suggestions and propositions from the interviewer. Consider, for example, the following extract from the early part of one interview, when the interviewee is already taking control of the situation and resisting influence on the part of the interviewer:

Interviewer: "Probably the best thing is if you talk through a project and what you think was done about the technical uncertainty and how it was taken into account in the decision-making."

Interviewee: "I think I would come at it in a different way and say what are the technical uncertainties which we experience..."

The taped records of the interviews were transcribed to form a permanent easily accessible record. The process of transcribing what amounted to a very rich body of data was highly labour-intensive. In total some 65,000 words of interview data were transcribed. The interviews were labelled according to date and source and the paragraphs in the text were numbered for reference in the subsequent analysis.

Initial analysis: coding

Having collected, transcribed and labelled the interview data the next task was to begin to identify important themes and build an indexing system for the data. Initial indexing, referred to as *coding*, proceeded by means of tentative labelling of phenomena considered to be of potential relevance. Each paragraph of the transcripts was scrutinised and significant concepts were noted in the margin of the text. This initial note was the first indicator of a concept emerging from the text.

As this coding process continued the list of concepts rapidly expanded and recurrent concepts began to emerge. Particularly dominant or relevant concepts were noted as entries in a computer database together with a reference to the paragraph(s) in which the concept was articulated. In previous Grounded Theory studies it has been usual to enter concepts in a card index. In keeping with this tradition the database entries will be referred to as 'cards'. Typical cards are reproduced in this chapter. There is clearly an element of judgement associated with this labelling process. For the purpose of subsequent analysis it is important to recognise that the aim was not to record *all* the instances where a particular concept occurs, rather it was to collect a range of indicators that pointed to the multiple qualitative facets of a potentially significant concept (Pidgeon *et al.*, 1991).

The success of the initial coding depended in part on choosing an appropriate level of abstraction for the concepts in question. The interview data contained an eclectic mix of information, which ranged from technical descriptions of specific analytical methods to reflections on political and cultural issues. The use of particular terms that are derived directly from the interviewee's discourse tended to tie the analysis to the specific context of the particular interview. The aim was, however, to identify more general concepts for subsequent theorising.

The different perspective of the different interviewees meant that it was at times difficult to find common concepts that were grounded in the interview data even though the interviewees were talking about a common subject. Occasionally there was direct conflict between the testimony of different interviewees. This is understandable for interviewees with different perspectives but also occurred for interviewees from the same perspective (*e.g.* consulting engineers). Where this occurred the conflict itself was considered to be a relevant phenomenon and worthy of note.

Core analysis

As the amount of data expanded and the emergent themes became reinforced the index system was refined and links were generated between concepts. The indexing system was refined by improving the fit of the categories to the indicator incidents and by providing careful definitions of important categories.

As the indexing system evolved certain categories became *saturated*, to use Glaser and Strauss' (1967) term. That is, a point was reached where the collection and coding of additional data no longer contributed further significantly variant incidents to the card. At this point a definition of the concept was written which stated explicitly the qualities which had already been recognised in some implicit manner when a new case had been classified into the category concerned (Turner, 1981).

The process of producing definitions ran parallel to, and stimulated the *writing of theoretical memos*. The objective of writing memos was to capture and externalise the thoughts of the analyst generated by close contact with the data whilst coding and producing definitions. Memos provided an opportunity to generate and develop explanations of the emerging concepts and to discern some of the interrelationships that existed between them. This ultimately led to *integration of emerging concepts* by creating links between them. Emerging concepts were tested by revisiting the interview material to ensure that the concepts did 'fit' the interview data. The remainder of this chapter details the concepts and linkages that were discovered in the interview data.

2.4 An overview of coastal defence processes

The first level of analysis involved least interpretation on the part of the analyst and was more akin to content analysis than Grounded Theory. The aim was to extract from the interview data inventories of processes and roles in the field of enquiry: coast defence. From this it was possible to establish some basic factual models of the coastal defence system.

2.4.1 Roles

Extracting the set of roles listed in Table 2.1 from the data set was a straightforward activity but an important first step in understanding the coastal defence system. Although individuals in the organisations listed in Table 2.1 may be engaged in the coastal defence system on a full time basis, coastal defence work is not necessarily the only or indeed the primary activity of the organisations listed. It is perhaps obvious but also important to recognise that each of the actors in the coastal defence system have non-coast defence objectives which may or may not be in conflict with their coast defence activities.

Table 2.2 Actors in the coastal system

Actor	Role in provision of flood and coast defence
Maritime District Councils (MDCs)	Client body for coast protection: commissions studies and works.
	Manages formation of local coastal policy via SMP procedure.
	Monitoring.
Environment Agency (EA)	Client body for flood defence: commissions studies and works.
	Manages formation of local coastal policy via SMP procedure.
	Monitoring
	Emergency works and some maintenance activities.
Ministry of Agriculture, Fisheries and Food (MAFF)	Approving projects for Grant Aid
	Administering Grant Aid
	Development and dissemination of policy
	Administering R&D funding
Consultants	Analysis
	Design
	Construction supervision
Contractors	Construction and maintenance
Statutory consultees	Stakeholders in decision-making
Influential pressure groups	Stakeholders in decision-making
Local populations	Stakeholders in decision-making

2.4.2 Processes

The interview data was rich in references to activities that are carried out in order to deliver coastal defence infrastructure. In the first pass through the data, references to process were noted and grouped into clusters of related processes. At this stage no formal links or precedence relationships were established between the processes. Indeed the evidence suggested that any given process interacted in some sense with most other processes, so to draw out these links would be uninformative.

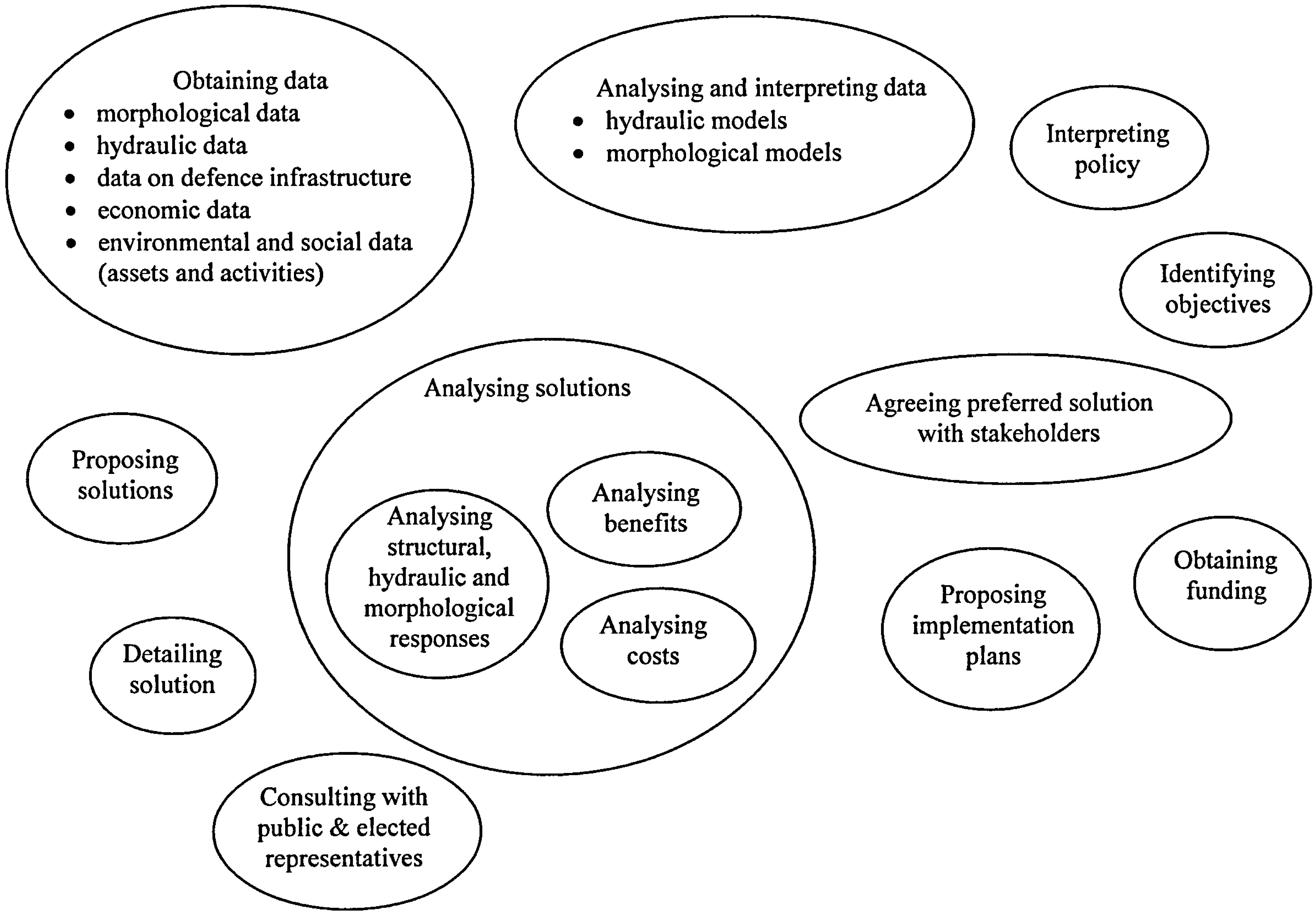

Figure 2.2 Planning, design and appraisal process

Figures 2.2 to 2.4 summarise the information on processes obtained from the first pass through the interview data. The Figures show essentially the same system from rather different perspectives. Figure 2.2 is drawn from a technocratic perspective. It shows the activities carried out in the delivery of coastal defence plans and designs. The interviewees mentioned many specific types of data and analysis, which could be included to add detail to the overall model of the design process. However, the interview data demonstrated that in practice the activities involved in a given design or study will be specifically selected depending on the objectives of that study.

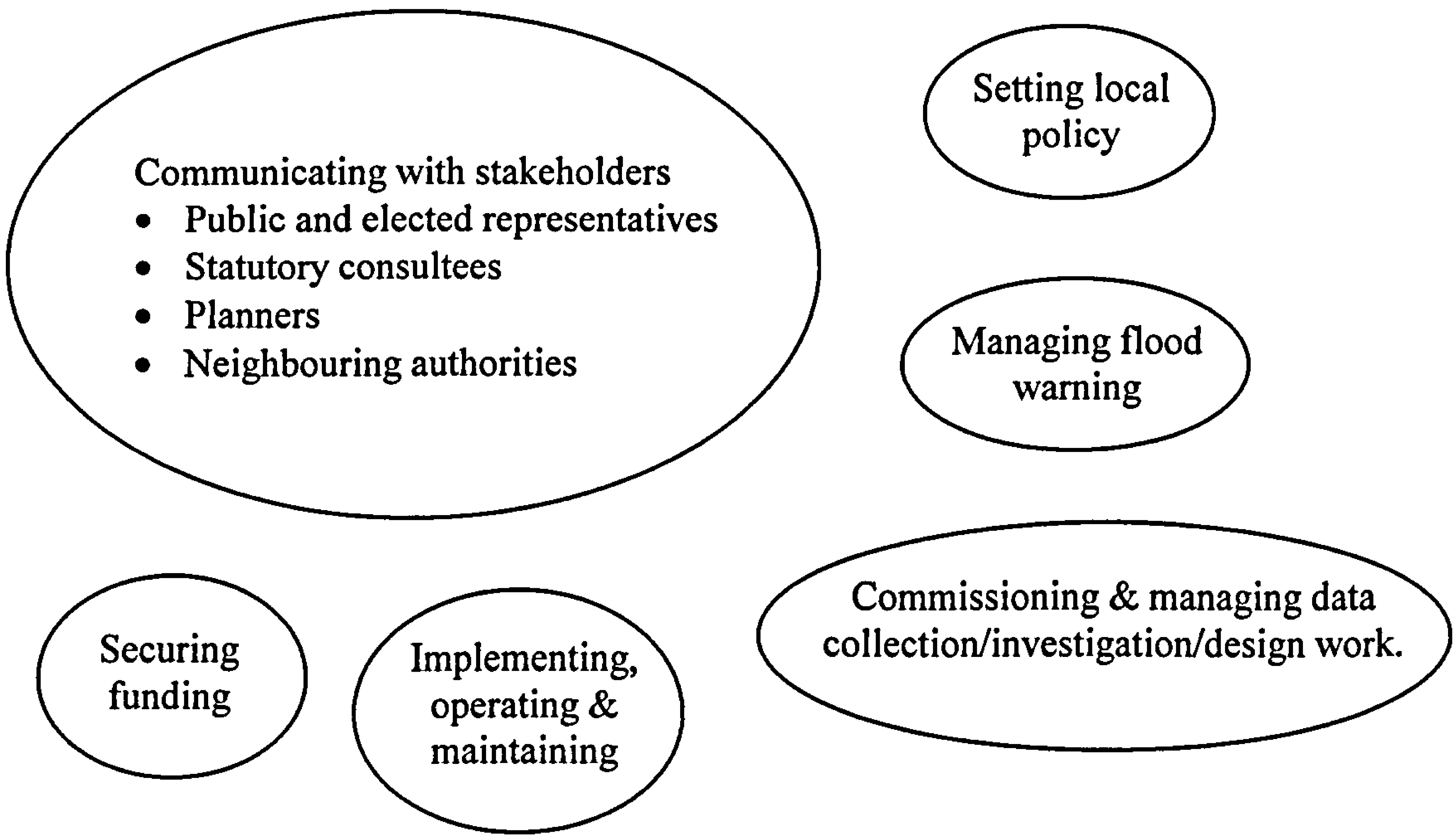

Figure 2.3 Maritime District Council and Environment Agency processes

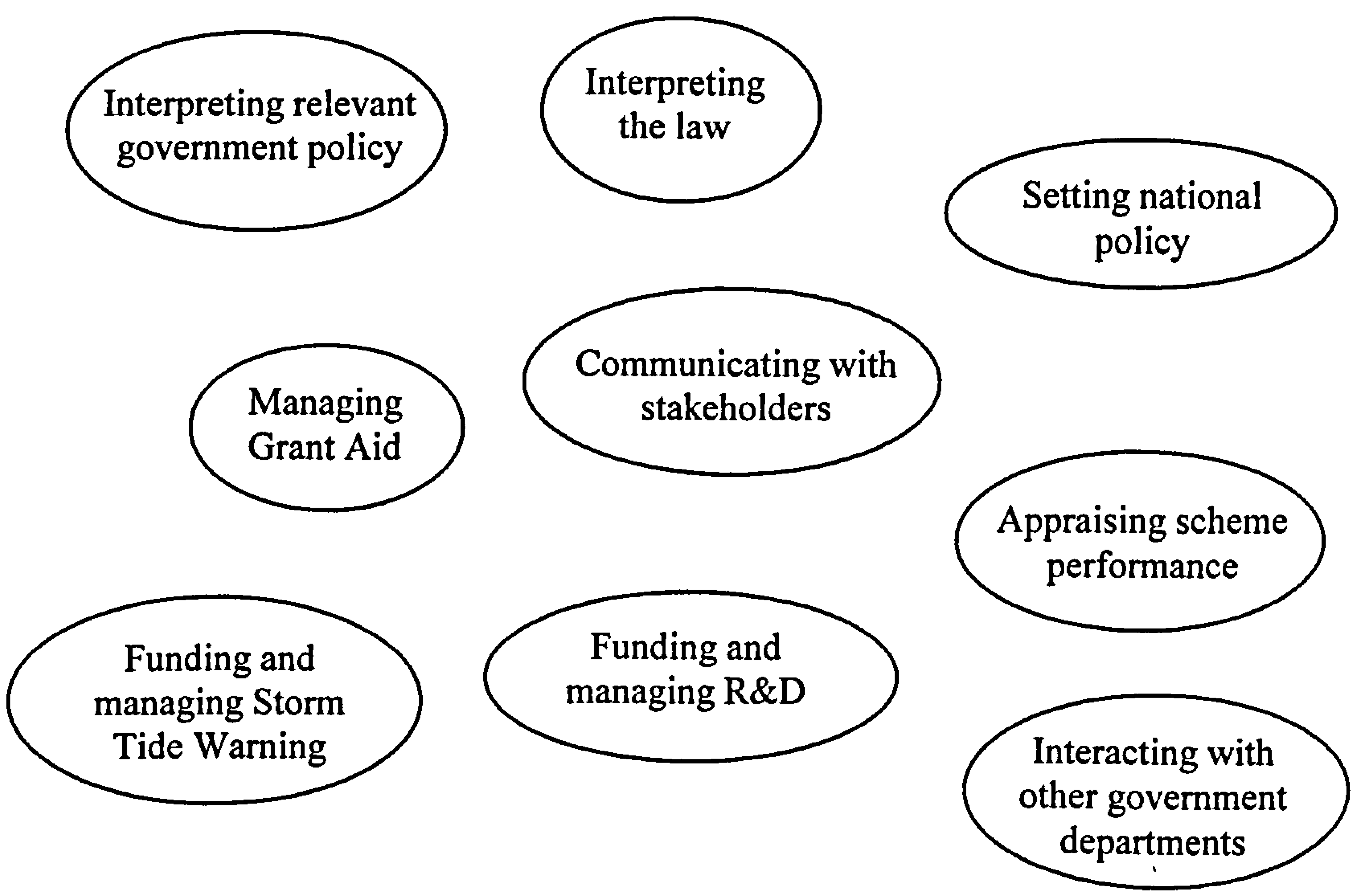

Figure 2.4 MAFF processes

Figure 2.3 is drawn from the perspective of the institutions responsible for local implementation of coast defence. It represents the current situation where the institutions responsible for coastal defence at a local level are on the whole commissioning bodies. They have in-house expertise but design and implementation are commissioned from consultants and contractors. Figure 2.4 is drawn from the point of view of MAFF.

2.5 Decision-making processes

In this section key characteristics of the decision-making process which were evident in the interview data are summarised.

2.5.1 Decision-making processes are instance-specific

From analysis of the interview data it was clear that decision-making processes were specific to the projects under discussion. The interviewees described rather project-specific modes of working.

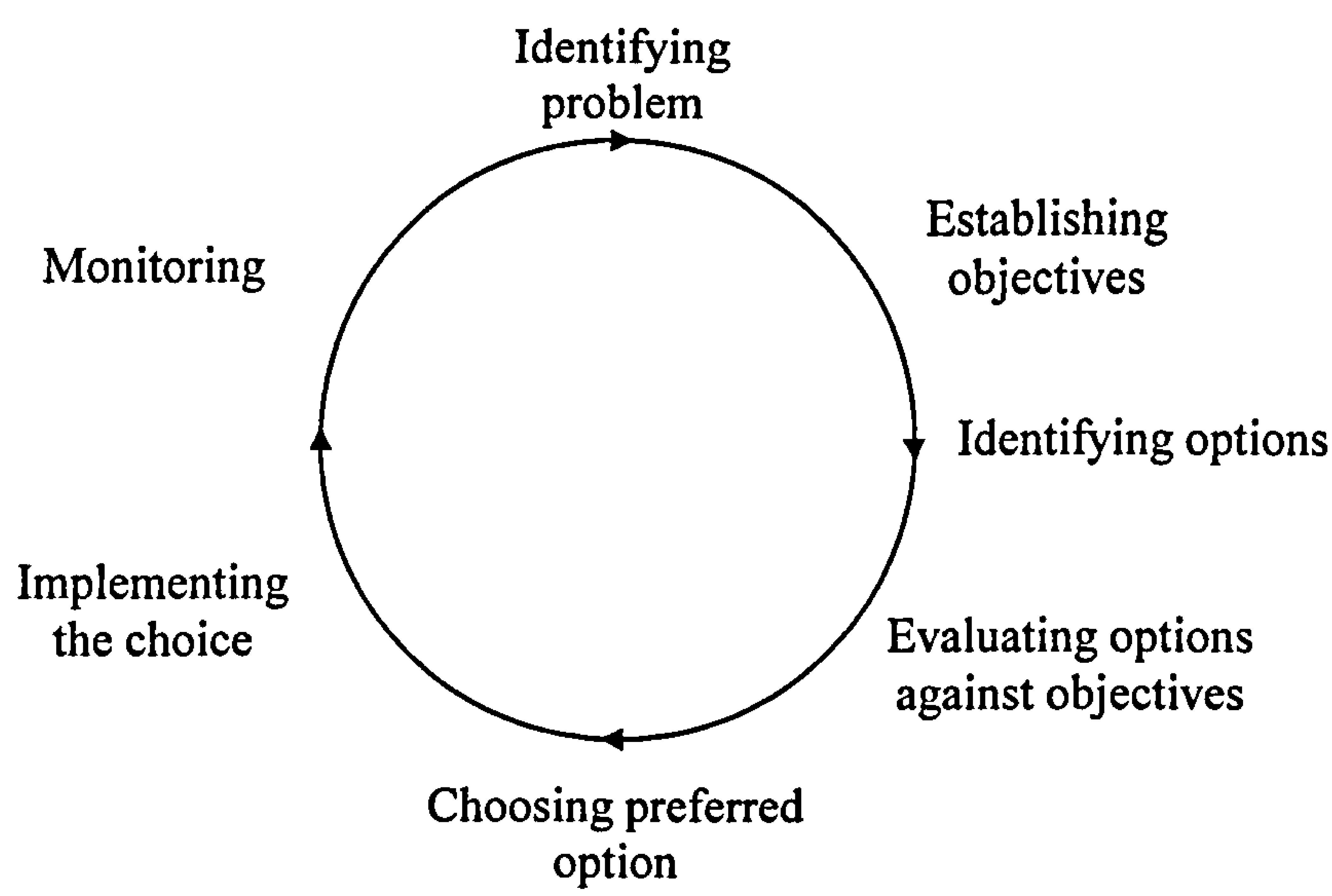

Figure 2.5 The decision-making process

At the most general level the interviewees corroborated the decision-making cycle (Figure 2.5) proposed by many decision theorists, which is discussed in more detail in Chapter 5. However, in practice there were often intermediate loops, sometimes with long delays between stages, with previous stages being revisited before the choice of preferred option.

Decision-making processes can become particularly complex, and specific to a project, when the public are involved. There is growing, but not universal consensus, that it is important to engage stakeholders in decision-making:

> F, Paragraph 43 "You've really got to talk to them. It's this getting the consensus, getting everybody behind the scheme, to get ownership of the scheme that you're promoting so that they're... routing for it just as you are so that it becomes their scheme."

> G, Paragraph 32 "Two schools of thought [on involving the public]. One, you shouldn't let them near any decision whatsoever because you will be there for ever, and the other is that you ignore local information and knowledge at your peril."

Involving the public in the decision-making process gives it legitimacy that purely technocratic decisions do not necessarily possess. However, by moving out of the technocratic domain the decision-making process becomes more complex and possibly less structured. Thus the move towards engaging stakeholders in decision-making is tending to make the decision-making process more instance-specific.

2.5.2 Decision-making is based on a multi-disciplinary set of issues.

The interviewees referred to a complex set of socio-technical issues that were taken into account during decision-making.

> G, Paragraph 4 "...the job they have to deal with is so diverse these days, they've got to be an economist, they've got to be an environmentalist, they've got to be an expert in geotechnics, in materials..."

It was clear that whilst some of these issues were explicitly stated as decision objectives or constraints, others were not. The political nature of some of these issues is reflected in Card 36 which is reproduced below.

Concept No	Concept Title
36	**Political issues are perceived to have a major influence on some schemes. Political issues are considered to be largely outside the control of technical experts.** E, Paragraph 14 "The decision, particularly on the cases that are the tricky ones, is quite often political." E, Paragraph 40 "Yes the most difficult component is the political one, because that can often be completely irrational." A, Paragraph 29 "The decision-making as to which direction to go in, whether there is a need for a scheme, the direction the scheme should go in, is very heavily orientated to public acceptability, politics, will probably be increasingly more so in the future." H, Paragraph 34 "... I do know through the SMPs there's a number of cases where in particular areas people have said 'I don't care what the process is doing we're going to defend it, it doesn't matter what the coast is doing we're going to protect it because my boss upstairs he's being leaned on by the Councillors and says we're going to protect it, end of story'". B, Paragraph 33 "I suspect that at the end of the day the other factors, what you might describe as the inter-personal factors and the political factors, I'm not saying political in the narrow sense of what the governing party wants to do but in the much broader sense of the wheeling and dealing of life - whoever shouts the loudest or puts the most pressure on the situation can lead to something happening or not happening."
MEMO	*Consultants seem to be more anxious than EA/MDCs/MAFF about political influence deflecting projects from the path of technical optimisation. Consultants are perhaps more technocratic in their approach, whilst governmental or semi-governmental organisations are more conscious of issues of legitimacy.*
LINKS	
Card 19	Formation of public opinion

The need for professional experience in order to weigh up the issues involved in decision-making was stressed (see also Section 2.7.1). This implies that decision-making is not entirely based on explicit criteria which could be set out for an inexperienced colleague. Clearly the tacit knowledge (Polanyi, 1967) that is gained from experience is believed to be an important aspect of decision-making.

2.5.3 Decision-makers are not unitary

In Sections 1.2.3 and 2.4.1 the various roles in coastal defence were introduced. MAFF, the EA, MDCs, consultants, contractors and consultees all have a stake in the decision-making process. Which of these parties plays the lead role depends on the decision at hand, for example

- MAFF sanctions Grant Aid;
- the EA and MDCs decide which consultant to appoint;
- consultants are responsible for detailed design decisions;
- contractors are responsible for decisions relating to their method of work.

However, this is clearly a simplified view and a number of bodies may have a stake in each of the above decisions. For example statutory consultees can influence decisions on design and clients instructions can influence the contractor's method of work. Thus the decision-making process is characterised by a number of stakeholders who are involved to a greater or lesser extent. This is a significant departure from normative decision theory (see Chapter 6), which considers the decision-maker to be unitary with a unique set of objectives.

2.5.4 Decision-making is part of an ongoing management process

The interviewees referred to decision-making in the context of ongoing processes and as part of a hierarchy of decision-making cascading from national policy to Shoreline Management Plans, to strategy plans, to individual schemes. The interaction between these hierarchical levels was not always clear. In particular there appeared to be some inconsistencies between decision-making in SMPs and other decisions. This is an issue which was also raised by the Agriculture Committee (House of Commons, 1998).

The need for continuing attention to the coast and for responsive modes of decision-making was widely recognised. Decision-making forms part of an ongoing process of observation, interpretation and action.

2.6 Sources of uncertainty in decision-making

The issues which interviewees took into account when they were making decisions and in particular their perception of uncertainty during decision-making were explored in some detail. The product of this discussion was a disordered set of issues at very different levels of resolution, varying from specific technical issues to vague cultural aspects. The objective of this section is to develop, on the basis of the interview data, some generic understanding about the sources of uncertainty in decision-making and how they interact.

It had been intended to also establish an inventory of sources of uncertainty. However, it became clear that the list would be both uninformative and very difficult to compile. Every piece of information and every model used by a decision-maker has uncertainty associated with it to a greater or lesser extent. Thus a list of uncertainties would amount to a very general but equally uninformative catalogue. What is important is uncertainty in the context of specific decisions.

Two stages always featured in descriptions of decision-making processes

- analysis and interpretation of alternatives;
- selection of the preferred alternative.

The former stage may be considered to be a modelling activity, the latter a choice according to some preference, value or taste. As will be evidenced below, the interview data repeatedly demonstrated that both analysis of alternatives and the criteria for selection of the preferred alternative were fundamental sources of uncertainty. These two fundamental sources of uncertainty will be referred to as

(1) *modelling* issues, and

(2) *values* issues.

The analysis of alternatives and the weighing up of values are carried out by a range of different people. Transmission and interpretation of information across discipline boundaries and between different people is the cause of a different type of uncertainty which will be referred to as

(3) *communications* uncertainty.

Finally, there are a number of issues referred to by decision-makers as being sources of uncertainty which were beyond their control. These were issues outside the local system in which they operated and in some circumstances were outside the coastal defence system in general. An example which illustrates this issue well is the influence of institutional changes within both MDCs and the EA. Decision-makers felt that institutional changes which were beyond their control had influence the quality of decision-making in their organisations. These are

(4) *environmental and cultural* issues.

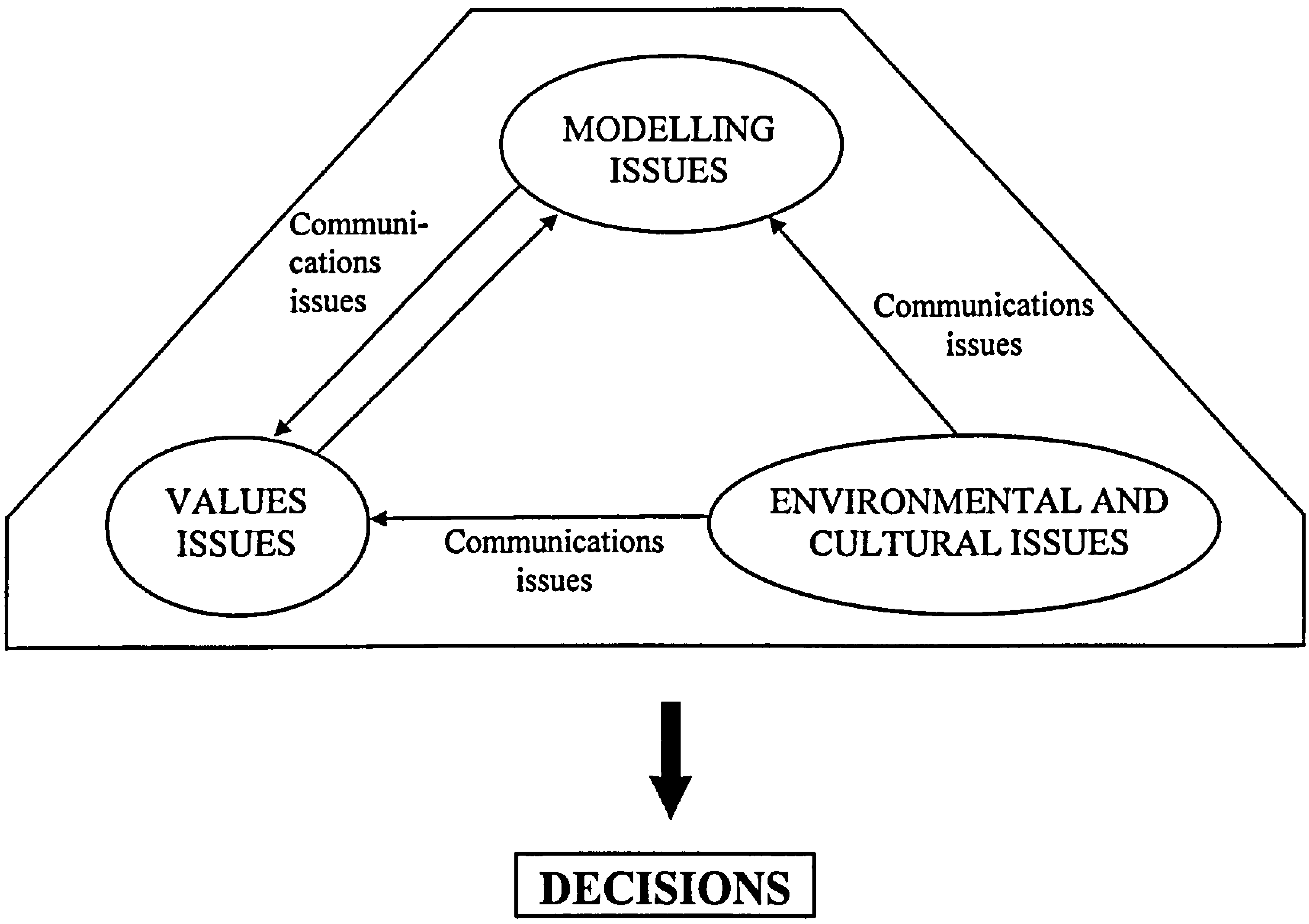

Figure 2.6 Fundamental sources of uncertainty

One of the participants in the research, when presented with the model in Figure 2.6 for comment, proposed their own perspective (Figure 2.7), together with the following commentary:

> *Category 1 is the uncertainty in our understanding of the nature of the problem, which given that the problem is generally brought about by the forces of nature is likely to be high. Category 2 I would characterise as political uncertainties and in this I would include the vagaries of the system used to consult, justify, finance and implement coastal engineering works. Finally, Category 3 is the uncertainty in the design and construction process. Given experience and further research the uncertainties of 1 and 3 can be reduced but I would expect 1 to remain at its present level of uncertainty. Better systems could mitigate this category of uncertainty, but I believe that the judgement of how much is spent, and on what, will always be subject to varying political pressures.*

In the vocabulary of Figure 2.6, *Category 1* uncertainties are modelling issues and *Category 2* uncertainties are a mix of values and environmental issues. *Category 3* relates to the process of designing and implementing coastal projects. It is an uncertainty inasmuch as some prediction of the effectiveness of that process (for example in terms of time and cost) needs to be made in advance of it taking place. In the vocabulary adopted here, uncertainties in prediction are modelling uncertainties.

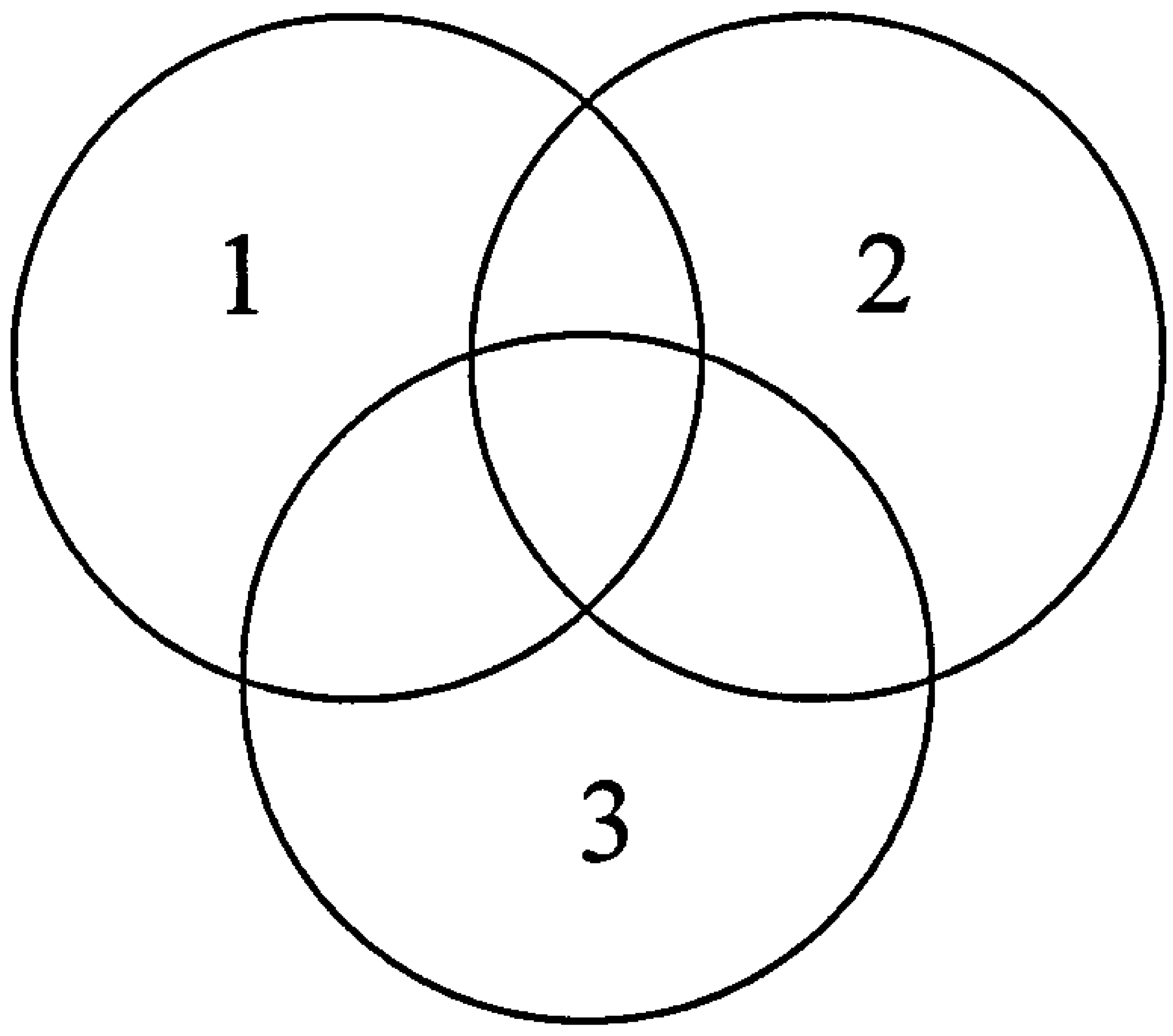

Figure 2.7 Model of the uncertainties inherent in coastal engineering, as proposed by one of the research subjects

Each of the uncertainties identified in Figure 2.6 will now be examined in more detail.

2.6.1 Modelling issues

The term modelling is used in the most general sense of the word. Modelling relates to any abstraction of reality that a decision-maker generates in order to assess the performance of a decision alternative. So for example a decision-maker may, on the basis of a consultation exercise, construct a mental model of the political reaction which could be expected upon construction of a proposed coast defence scheme. There is significant uncertainty associated with this as with many other models.

The titles of Grounded Theory cards relating to modelling issues are listed in Table 2.3. The titles illustrate the very general nature of modelling issues. They demonstrate general doubts, concerns and uncertainties about the appropriateness and incompleteness of the models employed in decision-making. It may not be immediately obvious that some of the cards are directly related to modelling problems. Take for example Card 10, which relates to the diversity of different types of models which a decision-maker is expected to use. So, for example, the decision-maker has to build up understanding (mental models) of consultation processes, political implications and impacts on the natural environment as well as models of physical processes.

In contrast to the findings of this analysis, the Agriculture Committee (House of Commons, 1998) paid only limited attention to issues of modelling uncertainty (other than in the context of climate change). For example the Committee dwelt on the issue of controlling development in areas at risk from flooding, apparently assuming that the risk could be precisely defined. However, the independent enquiry into the fluvial flooding at Easter 1998 (Bye and Horner, 1998) specifically highlighted the problems of modelling flood risk. Mr Bye's comments are consistent with the evidence obtained in this research. The Agriculture Committee gave much consideration to the relative merits of hard and soft coastal engineering without reaching a clear view on what precisely these entailed or addressing the modelling and uncertainty implications of these contrasting approaches.

Table 2.3 Grounded theory cards relating to modelling issues

Card	Title
1	The diversity and complexity of the physical environment on the coast is perceived to be a significant source of uncertainty. Models of physical and natural processes are recognised as being questionable due to lack of site specific data and lack of physical understanding.
2	The design/implementation process is specific to the site and the objectives of the scheme in question. Although some general processes can be identified, at a resolution specific enough to be of practical use, processes are too complex and site specific to be modelled in a generic way.
3	Construction of mental models of the coastal defence system is a key stage in problem solving. This interpretative process is recognised as being subjective.
4	The robustness of models of the natural coastal environment is difficult to test and consequently they are regarded with scepticism by experts.
10	Decision-making is based on an expanding multi-disciplinary set of issues. Amongst these are technical, economic, safety, environmental and political considerations.
11	Coastal engineers are concerned about the quantity and quality of field data.
13	Coastal engineers stress that their models are only partial representations of complex physical processes and that the coast has the capacity to behave in ways which may not have been foreseen.
15	Conservatism is used to manage uncertainty. Conservatism is manifest in factors of safety and other margins applied to design and other decisions.
17	Linkages and remote impacts may be perceived but not evaluated. The importance of remote impacts is recognised but they are considered to be difficult to evaluate in quantitative terms.
20	The need for flexible response is recognised. This involves having active monitoring programmes and the capacity to implement or adjust works in response to monitoring.
25	Models of the human system are recognised to give variable results. Institutional uncertainty and uncertainty over human development of the coast are very difficult to model.
26	Probabilistic methods of analysis and design are not widely applied.
28	The use of environmental economics is limited by Ministry reluctance (which perhaps originates in the Treasury) to accept economic valuations of environmental benefits, and by the cost of studies.
35	There is an inconsistency in relevant time scales. Morphological, environmental, economic and political time scales amongst others are inconsistent, introducing uncertainty and scope for conflict.
42	Opinions differ on the reliability of costings. Costing is a model which is essential to assess scheme viability.
51	Design is driven by failure mechanisms rather than by probabilities of failure. Response functions with which to compute probabilities of failure are often unavailable.
54	Uncertainty in the impacts of SMP policies has received little attention. This has been the case even where significant changes to the coastal defence strategy have been proposed.

2.6.2 Values issues

Values issues relate to the problem of weighing up values and resolving value conflicts. These are ethical problems. The most striking example are conflicts over the primacy of economic, natural environment and safety priorities. The problem originates because each individual in a population can have different preferences and, moreover, may find it difficult to express their preferences or may express inconsistent or unstable preferences. Thus, as well as being based on ethical dilemmas, values issues involve questions of perception and communication. Communication is dealt with specifically in Section 2.6.2 and is closely linked with issues of values.

Government provides a framework for resolving value conflicts in the form of policy. MAFF policy sets priorities for the types of schemes it will fund, which reflect safety, economic and natural environment considerations. This policy is interpreted on a site-specific basis. It is in the interpretation policy in the light of site-specific concerns that value issues trouble decision-makers.

Table 2.4 Grounded theory cards relating to values issues

Card	Title
10	Decision-making is based on an expanding multi-disciplinary set of issues. Amongst these are technical, economic, safety, environmental and political considerations.
18	Public consultation is part of the procedure for scheme planning and consultation. Attitudes differ on involving the public in scheme development. Public consultation can influence and inform the decision-making process.
23	Guidance on values issues is vague. Interpretation of the guidance causes difficulties.
28	The use of environmental economics is limited by Ministry reluctance (which perhaps originates in the Treasury) to accept economic valuations of environmental benefits, and by the cost of studies.
29	Conflict over values and objectives is a root cause of project difficulties. Stakeholders can have widely different objectives.
31	SMPs have become a vehicle for value disputes. Conflict over values is a consequence of addressing policy on a wide scale and examining a sometimes controversial range of options.
33	There is confusion over quantification of objectives. This relates to agreeing objectives before a decision and measuring performance during and after implementation.
35	There is an inconsistency in relevant time scales. Morphological, environmental, economic and political time scales amongst others are inconsistent, introducing uncertainty and scope for conflict.
48	The value of information is not computed and may not be appreciated. There can be disagreement over the appropriate level of investment in investigation.
52	The primary decision objective at project appraisal is economic. Other objectives can be taken into account provided economic criteria have been met.
53	Compensation arrangements are a value issue which inhibit good coastal management.

The primarily economic perspective of PAGN repeatedly featured in the deliberations of the Agriculture Committee (House of Commons, 1998). The Committee recommended that social and environmental values be more effectively included in the decision-making framework but gave little guidance on how this could be achieved, save for vague reference to multi-attribute decision-making – an approach which is discussed in detail in Chapter 6 of this thesis. In the proceedings of the Committee there were some stark contrasts in the values of the individuals who gave evidence, most strikingly between representatives of the National Farmer's Union and representatives of the WWF-UK, the Wildlife Trusts and the Royal Society for the Protection of Birds.

2.6.3 Environmental and cultural issues

Environmental issues are an influence on decision-making, yet because they are outside the decision-maker's immediate control it was felt that they were a major source of uncertainty. The extent to which environmental factors are outside the control of decision-makers clearly depends on the power wielded by the decision-maker. One decision-maker's environmental constraint is the product of another's decision. This is the case for example with government policy, which amongst the interviewees was on the whole considered to be an environmental constraint even though they may at times have attempted to influence formation of policy. Whilst decision-makers within government clearly have power to set specific policy they too are influenced by the political environment (Card 22, Table 2.5) which is largely outside their control.

The proceedings of the Agriculture Committee (House of Commons, 1998) dwell upon institutional and political issues. In principle the Committee favoured removal of environmental constraints by

- introducing more local flexibility in funding;
- bringing maintenance costs within the Grant Aid mechanism;
- reviewing PAGN and MAFF's priority scoring system;
- improving arrangements for compensation or stewardship payments at managed retreat sites;
- ending the administrative distinction between flood and coast defence and removing the inconsistency between natural processes and administrative boundaries.

Many of the environmental issues listed in Table 2.5 can be thought of as being cultural issues, which are difficult to influence even by individuals with power. For example Card 15 states that "conservatism is used to manage uncertainty". The tendency towards conservatism is part of the culture of professional civil engineering designers. The culture is one of finding practical, safe and as far as possible simple solutions. Conservatism is an ingrained approach to achieving safety and simplicity (though not necessarily efficiency). This is not necessarily a criticism. It is a cultural observation that is borne out by the interview data. Similarly the emphasis on experience (Card 14) is an understandable response to the complexity of many coastal engineering problems and to the novelty of coastal engineering science.

Table 2.5 Grounded theory cards relating to environmental and cultural issues

Card	Title
6	Managing authorities suffer from a lack of suitably qualified/experienced in-house staff. The number of experienced in-house staff has been reduced during various reorganisations and the remaining staff have broadening and arguably over stretched responsibilities.
7	The client's perception of the consultant's culture is to preserve commercial interests. Profit and limitation of liability are perceived to be motivating factors which influence consultant's decision-making and the way they interact with clients.
9	Consultants treatment of uncertainty is shaped by their perception of the client's needs. Consultants perceive a reluctance on the part of clients to recognise the magnitude of technical uncertainty.
12	Decision-makers have to strike a compromise between the need to reduce uncertainty and the need for action. The pressure to act is combined with fear of failure.
14	Experience is highly valued amongst practitioners. The need for experience is a response to the complexity and the multi-disciplinary nature of the system.
15	Conservatism is used to manage uncertainty. Conservatism is manifest in factors of safety and other margins applied to design and other decisions.
16	Cost-benefit assessment is a key environmental constraint on decision-makers. The conduct of cost-benefit assessment is driven by the need to obtain MAFF approval for Grant Aid.
18	Public consultation is part of the procedure for scheme planning and consultation. Attitudes differ on involving the public in scheme development. Public consultation can influence and inform the decision-making process.
21	Budgetary pressures are an environmental influence on uncertainty management. This is particularly true in the case of criteria for Grant Aid, and funding of studies, monitoring and maintenance.
22	Current flood and coast defence policy is a consequence of public opinion. Occasional disasters shape public perceptions.
30	The relationship between SMPs and projects is not clearly defined. Opinion is divided as to whether project decision-making is consistent with SMPs.
32	Institutional arrangements constrain decision-making. Funding arrangements are a key determinant of the decision-making process.
36	Political issues are perceived to have a major influence on some schemes. These issues are considered to be largely outside the control of technical experts.
37	The fragmented nature of the flood and coast defence industry is a barrier to organisational learning. Institutional changes have resulted in a loss of organisational knowledge.
38	Coastal management continues to be largely reactive in some areas. Reactive approaches are a consequence of a combination of institutional weakness and lack of understanding of coastal processes and best practice for managing them.
40	There is a need to educate stakeholders in relevant issues. Education is essential to improve communication and build common understanding.
44	SMPs are resulting in rather fragmented strategies in some areas. This often reflects fragmented settlement patterns but does not necessarily reflect physical coastal processes.
50	Designers strive for confidence that their design is safe. The emphasis on safety is part of the culture of design engineers.

Recent and current changes in the organisational cultures of MDCs and the EA were felt to be a major influence on decision-making, introducing considerable uncertainty. Interviewees regretted the lack of suitably qualified and experienced in-house staff (Card 6), a situation which is a consequence of market testing and compulsory competitive tendering policies which were well beyond their control, other than via the ballot box (the interviews were held before the change of government). The scarcity of appropriate in-house expertise in the Environment Agency was highlighted in the Independent Review of the Easter 1998 floods (Bye and Horner, 1998). Corporate memory and the capacity to share best practice has apparently been diminished by the loss of experienced staff.

Cards 7 and 9 relate to how communication of uncertainty is influenced by the culture of the client/consultant relationship. Stated in very crude terms, the culture is for the consultant to endeavour to satisfy the client's perceived need for confidence whilst at the same time limiting liability for the consequences of providing information/advice which results in incorrect decisions. The two influences ideally result in a balanced communication of uncertainty which will assist decision-makers. If one or other of the influences dominates the result is over or under-estimation of the scope and implications of uncertainty.

2.6.4 Communications issues

Communications issues relate to how information concerning models and values is communicated amongst stakeholder. It also relates to how environmental constraints and influences affect decision-makers.

Communication relating to risk and uncertainty is particularly difficult. As Card 8 (Table 2.6) testifies, managers try to avoid uncertainty in communication with the public in order to promote public confidence. They have difficulty communicating uncertainty even when it is a major aspect of scheme design, such as in soft engineering projects. Two of the passages recorded on the card reinforce the communication difficulty:

> G, Paragraph 4 "If you go to the public for example and say there are... wildly different ideas and facts and figures that could be applied to the same problem, all we've done is had our best shot and come up with something, the public doesn't have a lot of confidence."

> D, Paragraph 27 "We go and put that beach in and it's there in 1998 or so and along comes winter 1998/99 and we have a big storm and there it goes. 'Told you it wouldn't stay' say the fishermen. How do you convince them?... How [do] you get that message across so that they understand the way in which you've designed for it?"

The problem is of course not restricted to communicating with the public. Table 2.6 evidences the current difficulties in communicating uncertainty in technical information amongst professionals.

Arguably communication issues are a branch of modelling issues. Communication involves the transfer of information. Each individual recipient of that information constructs a mental model on the basis of the information. The mental models of the individuals transmitting and receiving information will differ. Communications issues originate from this discrepancy in mental models.

Table 2.6 Grounded theory cards relating to communications issues

Card	Title
5	The uncertainty in technical information is not well communicated. Communication problems originate both in the presentation of uncertainty by technical experts and in the interpretation of technical information.
8	Managers try to avoid uncertainty in communication with the public in order to promote public confidence. They have difficulty communicating uncertainty even when it is a major aspect of scheme design, such as in soft engineering projects.
19	Risk communication and the formation of public opinion are not well understood. Public opinion is seen as the wild card in the decision-making process.
24	Development of government policy lacks transparency.

2.7 Accounting for uncertainty in decision-making

When asked about ways in which uncertainty was taken into account in decision-making interviewees mentioned:

- sensitivity testing,
- experience,
- using established analytical methods,
- factors of safety,
- conservatism,
- risk registers and qualitative risk assessment,
- scenario modelling,
- probabilistic design and probabilistic risk assessment.

2.7.1 Experience

Intuitive and implicit methods of taking uncertainty into account in decision-making are currently much more prevalent than explicit methods. To apply these approaches requires experience, which is highly valued. The importance of experience is closely linked with the apparent culture of conservatism which is discussed next.

Concept No	Concept Title
14	**Experience is highly valued amongst practitioners. The need for experience is a response to the complexity and the multi-disciplinary nature of the system.** G, Paragraph 20 "I think the vast majority of [projects] do go smoothly and I think a lot of that is really just using tried and tested approaches." G, Paragraph 22 on freeboard "It's arbitrary but quite often it's based on local practice." C, Paragraph 56 "If it's based on past experience it seems to work." D stresses the need for experience amongst designers and contractors who work in coastal engineering. H, Paragraph 10 "It's only really through experience that you can make the choice as to what are appropriate models to use in any particular situation." F, Paragraph 22 "There is no substitute for experience, there really isn't. Better and better techniques are becoming available, more and more models to model all sorts of things..."
MEMO	*Experience is a dominant issue amongst more traditional consultants and clients. The valuing of experience is a sign of the culture in these organisations.*
LINKS	
Card 1	Complexity of physical processes
Card 4	Scepticism over models
Card 15	Conservatism to manage uncertainty
Card 10	Multi-disciplinary problems
Card 15	Concern over loss of experienced staff

2.7.2 Conservatism

Conservatism is currently one of the key mechanisms for managing uncertainty. Conservatism embodies essentially heuristic approaches to decision-making and consequently suffers from a lack of transparency.

Concept No	Concept Title
15	**Conservatism is used to manage uncertainty. Conservatism is manifest in factors of safety and other margins applied to design and other decisions.** G, Paragraph 20 "You come up with some answer to the nth decimal place and then you say 'I'm going to add half a metre on top of it' to cover your risk, and it does cover your risk." G, Paragraph 50 "... there's so many uncertainties there, you're not going to air on the side of skimming things down..." E, Paragraph 57 "As to unknowns, I think they're just that and I think the only way one takes account of them, any of the steps in modelling, design of a structure, assessment of impacts on the rest of the coast or the rest of the estuary or something, one takes account of it by the factor of safety or the way one errs in a particular direction. I don't think there's any structured approach to that at the moment." D, Paragraph 15 "... the coastal side was eliminated more by 'if we think of a number and double it then we're bound to be big enough.'" D, Paragraph 19 "There was no ability to build in large factors of safety, so in that sense you've got to do a little bit more analysis." D, Paragraph 77 "... no matter how much data you get, so you are still going to have you large factors of safety which we will do our best to build in subject to MAFF project appraisal...." D, Paragraph 88 "... you could always use your contingency sum and you could put in some semi-fictitious items in your bill of quantities to allow for uncertainties when you get on site." F, Paragraph 10 "Well you've got to proceed with your design in the knowledge that is may well be exceeded,.. you will leave a tolerance, you will leave an allowance for freeboard,... It doesn't actually appear really formally anywhere how you put these

	things into practice,..."
MEMO	*D, Paragraph 19 implies that when the room for conservatism is limited then uncertainty must be reduced via more analysis.* *D, Paragraph 77 links the use of conservatism with the constraints provided by PAGN. The feeling is that the model provided by PAGN does not account for uncertainty in a way which a practising engineer would like to do.* *Means to reduce uncertainty may not be formally accounted for.*
LINKS	
Card 14	Experience
Card 12	Fear of failure
Card 42	Dependability of costings

2.7.3 Probabilistic methods

Most of the consultants interviewed recognised the benefits of probabilistic methods but said that they rarely employed them in their own practice. This was because of:

- lack of time,
- shortage of sufficiently trained staff,
- miss-trust of results,
- probabilistic methods being inappropriate for situations where responses are poorly understood or response functions are unavailable.

It was explained that lack of time was a consequence of competitive tendering for consultancy work. It was felt that clients were not sufficiently familiar with the benefits of probabilistic methods to pay for the additional staff time that they entail.

2.7.4 Action versus uncertainty

Decision-makers have to strike a compromise between the need to reduce uncertainty and the need for action. The compromise is struck largely on the basis of intuition. Therefore, in specific cases decision-makers differ on the relative need to act and the need to reduce uncertainty. Card 12, which relates to this concept, is reproduced below.

2.7.5 Value of information

Explicit evaluation of the value of information to reduce uncertainty in a decision is rarely carried out. The appropriate level of studies is left to the judgement of the client advised by their consultant. Opinions often differ on the appropriate level of studies. The following quote is typical.

> Consultant: "... if people are prepared to spend an extra ten or twenty thousand up front you could be looking to save a hundred or two hundred thousand on the construction by continually optimising it. That's just something you've got to get across to the clients

> though and we're continually banging the table and saying that but things aren't going to change in the short term."

Yet the clients interviewed also stressed the importance of an appropriate investment in design and investigation and expressed regret that monitoring did not receive the attention it deserved:

> "And one of the things which is always the first thing to go is monitoring. It's the first thing to get cut. You set it all up, you get it going, the information comes in, your designs improve, the way you approach it improves and you're going along and you say 'well we've got all the information we don't need to do it any more, cut the monitoring.' And you cut that monitoring and another ten or twenty years down the line you say 'why did they ever stop doing that?'"

Concept No	Concept Title
12	**Decision-makers have to strike a compromise between the need to reduce uncertainty and the need for action. The pressure to act is combined with fear of failure.** G, Paragraph 16 "You cannot afford to sit back and do nothing and wait around for twenty years or fifty years or a hundred years waiting for the best thing since sliced bread, you've got to make a decision." G, Paragraph 46 ".I think one of the differences now is the fear of failure,... political risk of something not working out." C, Paragraph 44 " went to one conference, I think it was Prof. Brunsden on the Dorset coastline, was suggesting that we shouldn't be making decisions unless we've got the information on which to base these decisions... we need more information. As I see it that's an easy way of putting off any decisions, you just say 'I haven't got enough information'. You've got to make these decisions now at some time for the future and you've got to make them on the basis of the information to hand." B, Paragraph 33 "So I think in terms of local comfort I think people would prefer a local authority which was actively involved and had a degree of duty and responsibility to actually maintain the defences..." A, Paragraph 11 "there isn't the option for them to say 'well hang about a bit... we're not really sure what is the best policy for this bit of coast', and therefore we're going to have to select one of these four: retreat, sustain, advance whatever, and therefore we're going to have to go for this one."
MEMO	*The need for action and the fear of failure are features of the culture. It is related to public perception.* *Fear of failure results in conservatism.*
LINKS	
Card 8	Decision-makers try to avoid uncertainty in communication with the public
Card 18	Involving the public in scheme development
Card 15	Conservatism to manage uncertainty
Card 19	Formation of public opinion
Card 48	Value of information

2.8 Implications for uncertainty management

The interview study helped to establish the following guidelines for the research:

- It is important to take a comprehensive view of uncertainty and address all of the four sources of uncertainty identified in the Grounded Theory analysis. The interview data demonstrated that the tendency to focus exclusively on technical and economic issues has resulted in unproductive conflicts and unsuccessful projects. There is a need for a vocabulary that can communicate the values and organisational aspects as well as the technical aspects of the problems which coastal engineers confront.

- It is important to recognise the context of decision-making. Individual coastal engineering decisions take place in a context of ongoing coastal management.

- Techniques aimed at improving the management of uncertainty in coastal engineering need to be able to handle a wide range of types of uncertain information. This includes precise modelling data but also much more vague information about values and organisational constraints. It will be a challenge to reflect the uncertainty in such a wide range of types of information, not least because the uncertainty is not always expressed. Indeed it took laborious analysis to identify some of the uncertainties implicit in the discourse of the interviewees.

- Processes are project-specific. Generic process models are useful at a high level. They may also be of use as checklists with large areas of the model being omitted for specific instances. For most practical purposes project-specific process models will need to be constructed.

- The process of assembling evidence in the lead-up to a decision is a complex one, often combining a multi-disciplinary range of activities. It desirable that all of these activities can be represented in one model to provide a clear picture of the interaction of processes in the lead-up to a decision. Some rather detailed modelling and analysis processes are undertaken as part of coastal engineering projects. It is important to be able to represent the uncertainty in these activities that sometimes contribute crucial evidence to decisions. Thus it will be necessary to represent processes at very different levels of definition.

- Improvements to the communication of uncertainty will be welcome. In particular it will be helpful if the implicit use of expert judgements in engineering decision-making can be made more explicit. Given the complexity of the problems which coastal engineers face, the need for expert judgement cannot be avoided, but tools to help externalise and communicate expert judgement will be of value.

- Effective communication between stakeholders requires a vocabulary of uncertainty that is reasonably comprehensible. Decision-makers have difficulty in communicating uncertainty-related issues between each other and with stakeholders.

- There is an increasing need for transparency in decision-making. To achieve this requires tools for externalising expert judgements and improving communication, mentioned above. However, it also requires that methods are rigorous and repeatable as on occasions they will be subjected to detailed scrutiny.

2.9 Conclusions

1. A descriptive study has identified key sources of uncertainty in decision-making for coastal managers in the UK and has investigated the common responses to uncertainty. A complex set of socio-technical issues is taken into account during decision-making, only some of which are explicitly stated.

2. The study adopted Grounded Theory as an investigative technique, progressing in an interactive and iterative manner from unstructured interview data to conceptual constructs that are well grounded in that interview data. The interview subjects have confirmed that the theory is a reasonable reflection of their beliefs and behaviour.

3. Sources of uncertainty in decision-making are diverse but can be categorised in essence as being modelling issues, values issues, communication, and environmental constraints. Modelling issues relate to the analysis and interpretation of decision alternatives on the coast. Values issues relate to the, sometimes conflicting, objectives that drive decision-making on the coast. Communications issues relate to the transmission and interpretation of information within and across discipline boundaries and between stakeholders. The term 'environmental' is used here to refer to surrounding institutional, political and cultural issues outside the immediate control of the decision-maker, rather than to the natural environment.

4. Intuitive and implicit methods of taking uncertainty into account in decision-making are currently much more prevalent than explicit methods. There is no evidence to suggest that these intuitive approaches, which are dominated by heuristics and a culture of conservatism, will continue to be effective if decision-makers become increasingly overloaded with information or are subject to heightening expectations.

5. Descriptive analysis of the type described in this chapter is an essential precursor to implementing feasible and desirable improvements to the coastal management decision-making process. Principles have been established that will guide the development of uncertainty management techniques.

2.10 References

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CHAPTER 3

Systems and processes

3.1 Objectives of Chapter 3

- to explain the limitations of reductionist analysis of systems;
- to introduce the key concepts of systems thinking;
- to propose process modelling as a way of describing hard and soft systems, and to identify its potential benefits.

3.2 Introduction

The background to the UK coastal defence system was introduced in Chapter 1 and the human and organisational aspects were explored in more detail in the empirical study described in Chapter 2. This chapter addresses systems in more theoretical terms. It aims to develop generic methods of describing systems, both hard and soft.

Hard and soft coastal engineering were introduced in Chapter 1. Dynamic engineering solutions, which endeavour to work with nature, are referred to as soft coastal engineering, whereas hard, often concrete, engineering works which prevent natural processes are referred to as hard coastal engineering. In the systems terms the labels 'hard' and 'soft' are used in a rather different sense to distinguish between physical aspects of the natural and built environment, and human/organisational aspects, respectively. An aim of systems approaches is to treat these hard and soft systems in generic terms. It will be argued that the distinction between the physical and human domains is in many respects unproductive. This is consistent with the view of the coastal defence system developed in Chapter 1, in which the need for integrated approaches was stressed. Soft coastal engineering requires that coastal management systems become increasingly integrated with built coastal defence works. The human (and computer-aided) activities of monitoring and decision-making are as fundamental to soft coastal engineering as the physical changes made to the coast by, for example, mechanically placing sediments on a beach. The other integrative aspect introduced in Chapter 1 is the joint determinism between the developed and natural environment, which is so evident on the coast, characterised as it is by remote interactions on spatial and temporal scales.

The highly interactive complex system on the coast is typical of the 'messy' or 'wicked' problems that have proved to be a particular challenge to the established methods of problem-solving based on reductionist analysis. The limitations of reductionist approaches are described below before introducing the principles of a systems approach, which endeavours to address some of these limitations. Process modelling is then introduced as a means of describing systems and forming a basis for desirable and achievable change to the way in which human processes are enacted. This brief introduction to systems thinking and approaches to process modelling prepares the ground for the new developments in uncertainty representation in process models, which are introduced in subsequent chapters.

3.3 The limits of reductionism

Reductionist analysis involves breaking a problem down into its component parts, analysing the behaviour of each of the parts separately, and then combining the behaviour of each of the parts to make inferences about the problem as a whole. The idea has its origins in the work of René Descartes and is now deeply embodied in Western thinking and in particular in the sciences. Scientific endeavour has been tremendously successful at making sense of complex and formerly incomprehensible phenomena. To do so scientists have had to select some items out of all those that could be examined and address them in relative isolation, perhaps in a laboratory. There is also a tradition in the sciences, which has its origins in the work of William of Ockham, of requiring logically coherent and economical explanation. Parsimony is a highly regarded characteristic of scientific theories, so scientists often go to lengths to minimise the amount of elaboration required for facts to be explained.

The reductionist ideal suggests that the behaviour of any system can be explained in terms of the behaviour of its component parts. So, for example, biological phenomena can be explained in terms of chemistry and physics.

> *The reductionist ideal would be an explanation of the social sciences in terms of psychology, of psychology in terms of biology, of biology in terms of chemistry, and of chemistry in terms of physics the most basic of sciences. (This is the ideal that underlies Lord Rutherford's famous remark: 'There is physics and there is stamp collecting'.)*
>
> (Checkland, 1981)

The assumption is not only that the behaviour of the parts is the key to the behaviour of the whole, but also that the principles governing the assembly the components into the whole are themselves intelligible and reasonably straightforward.

The success of the reductionist approach should not be underestimated but on the other hand it is no panacea. Its limitations become clear when the sorts of wicked problems found on the socio-technical interface are addressed. These limitations have their origins both in the fundamental characteristics of the reductionist approach and in the way in which it is applied in practice.

It seems that there are aspects of systems behaviour that are not reflected in reductionist models, that the behaviour of the whole system is not necessarily equal to the sum of the parts – the fundamental assumption of reductionist approaches. In other words there are aspects of systems behaviour that *emerge* when the system is considered as a whole, which are not readily interpreted in terms of interaction of the parts. Individual elements of a system do not necessarily behave the same when examined singly as when they are playing a part in the whole.

Consider for example behaviour of the human body as a system. The human body can be reduced to an interacting population of cells, which in turn can be analysed in terms of the biochemical reactions between molecules in those cells. Yet there are aspects of human behaviour, for example speech or walking, which emerge when the human body system is addressed at a high level. These behaviours are not an obvious behaviour to expect when populations of cells or molecules are being analysed. They are 'emergent properties' of the system. Not only do these emergent properties not exist at the lower level, they are meaningless in the language appropriate to the lower level.

On the coast there are macro-scale behaviours which are difficult to interpret in terms of small scale physical processes. For example the nearshore zone of the muddy Northeast coast of South America has a series of 'macro-ripples' with a wavelength of 20-30km and amplitude of a few metres which slowly migrate along the coast (Abernethy, 1980). Analysis of the physical processes of cohesive sediment transport has failed to predict this emergent property of the coastal system.

It is in the social sciences that reductionist methods are most problematic. The problems of the social sciences are particularly difficult because the subject of investigation (human beings and societies) have a level of intentionality which does not appear in the inanimate world. Moreover the social scientist himself or herself forms part of the phenomenon of interest. Only a fairly narrow range of organisational problems have succumbed to reductionist approaches like Operations Research.

Human organisations exhibit characteristics that are more than merely the sum of the behaviour of the individuals engaged in those organisations. Organisational culture emerges from the collective behaviour of an organisation (Handy, 1993) and it is clear from the analysis described in Chapter 2 that organisational culture is a major influence on the effectiveness of the coastal defence system. Emergent properties are clearly of great interest if sense is to be made of the coastal defence system.

Another reason why the weakness of reductionist approaches often becomes clear when they are used to tackle messy problems is, paradoxically, a consequence of their very success. Reductionist methods have become so established in science and engineering that expectation of their predictive capacity can be over-optimistic. Reductionist methods are based on models of systems that are inevitably incomplete representations of the system behaviour. Through the reduction of complex phenomena to reasonably intelligible simple models, systematic bias is introduced. Under some circumstances, for example in well-behaved and relatively self-contained physical systems, the departure of the model from reality will be insignificant. However, real coastal engineering problems seldom conform to this ideal. The evidence from the interviews described in Chapter 2 suggests that experienced coastal engineers will willingly acknowledge the limitations of the models of the coastal system which they employ. However, all too often it does seem that what will be referred to here as 'naïve reductionism' persists, whereby reductionist models are used as a basis for decision-making without full recognition of their limitations.

Blockley (1980) argues that naïve reductionism is encouraged by the education of engineers, which tends to dwell upon reductionist theories of highly idealised systems and avoid the complex open problems which engineers encounter in practice. The 'technical rationality' of reductionism (Blockley, 1992, Dias and Blockley, 1995) is a necessary but not sufficient requirement for good engineering practice. 'Reflective practice' was identified by Schön (1983) as characteristic of the way effective practitioners behave, and has been explored in the context of civil engineering by Dias and Blockley (Blockley, 1992, Dias and Blockley, 1995). Reflective practice embraces a wider range of skills than technical rationality, including responsible reflection on the limitations of models used as a basis for decision-making.

Herbert Simon used the term 'bounded rationality' to refer to the tendency of decision-makers to address only those aspects of a decision that can be tackled with the available models, whilst neglecting those aspects that are not amenable to analysis (Simon, 1965). In the interview data analysed in Chapter 2 and in the case studies described in detail in Chapter 7 it became clear that those aspects of a coastal engineering decision that are amenable to the available analytical tools are subject to much more detailed analysis than those complex, often socio-technical problems, that are not. There are examples in Chapter 7 of absurdly detailed analysis of phenomena that are of extremely marginal relevance to the decision in hand. This absurd reductionism is the converse to a systems approach that begin by addressing the problem situation as a whole, working back from the objectives of the decision-maker.

3.4 The systems movement

The systems movement arose in the 1950s as a response to some of the problems of reductionism. A systems approach can in general terms be thought of as one that aims to give as complete as possible a description of the relevant aspects of the system. It stresses the connectiveness of different parts of the system and aims to address the behaviours that emerge from interaction of different parts of the system. It also recognises the *meta-system* within which the system under consideration functions.

According to Checkland (1981)

> *The central concept 'system' embodies the idea of a set of elements connected together which form a whole, this showing properties which are properties of the whole, rather than properties of its component parts.*

A system can be addressed at a number of levels. At a given level there are properties that emerge at that level and seem to be irreducible. Key systems concepts can be summarised as follows (Checkland, 1981):

1. Emergence. Systems thinking recognises that emergent properties are important characteristics of systems.

2. Hierarchy. It is both meaningful and useful to describe systems in hierarchical terms, *i.e.* at a number of different levels of definition. This is a departure from the reductionist approach, which tends to address a system at one basic level of resolution of constituent elements. Any system must be thought of in the context of its meta-system *i.e.* the higher level system within which it is contained.

3. Communication and Control. To preserve a hierarchically ordered system in a dynamic environment requires information (in some sense) to be passed around the system for the purpose of regulation or control.

Emergence and hierarchy together lead to the concept of a *holon* (Koestler, 1967), which is both a part and a whole. Systems are made of interacting holons. Looking up the hierarchy a holon is part of a larger system, so in that sense it is a part. Looking down the hierarchy the holon is itself made up of sub-systems and in that sense is a whole.

One problem faced by the systems movement is that 'systems' is a term used in many senses by a wide range of disciplines, not to mention the various factions within the systems movement itself. Writers on systems range from the theoretical work on general systems theory (von Bertalanffy, 1968) to the descriptive 'active research' conducted by soft systems methodologists (Checkland, 1981, Checkland and Scholes, 1990, Fortune and Peters, 1995). As systems ideas become more widely accepted it is inevitable that this diversity of interpretations will proliferate, which is a welcome indication of the richness of a complex topic, yet also calls for careful use of vocabulary.

The systems movement has diversified and advanced both in theoretical terms and in terms of applications of systems thinking to real world problems. The emphasis in the applied stream, particularly as it has been applied to human and organisational systems, has been on understanding the systems behaviour rather than on prediction. Whilst soft systems may be too complex for reductionist predictive tools, systems thinking can provide perspectives on these complex phenomena which lead towards feasible and desirable improvements to the system. Systems thinking does not, however, eliminate the need for predictive models based in reductionist principles. Predictive models provide very important evidence for decision-makers. However, in open systems that evidence will inevitably be incomplete, so the decision-maker needs an appreciation of the system he or she is engaged in. What is called for is a merger of views, with systems thinking informing the use of predictive models.

3.5 A process paradigm

Process modelling is aimed at providing a generic language to represent systems. It aims to be generic in the sense that it can be used to describe systems that involve physical artefacts, human beings and organisations. So, for example, it aims to address the messy problems that occur at the interface between human and technological systems (Blockley, 1999).

The term 'process', like 'system', is given rather different meaning in different disciplines and seems to be increasingly prevalent. Business process re-engineering (Hammer and Champey, 1993, Blockley, 1996) is an idea that emerged in the early 1990s aimed at improving the efficiency of business organisations by rational analysis of the way they work and re-organisation to become more focussed on organisational objectives. Meanwhile the terminology of process is central to information technology, particularly in the Object Oriented paradigm. To add further confusion, coastal engineers primarily think of processes as the physical processes at work on the coast. The hypothesis proposed by Blockley (Blockley, 1999, Blockley and Godfrey, 1999) is that it is useful to think of everything as a holon and as a process. The various domain-specific conceptions of a process, be it a business process, a software process or a coastal process, are instances of a more general view of process as any purposeful activity. Indeed Blockley argues that it is meaningful to think of an artefact (a product) as a process because it changes through time and interacts with its environment.

3.6 Approaches to process modelling

A process model represents the relationship between a set of holons in a system. It describes the transformations that take place as a result of interactions between the holons in the system. It provides a means of representing and structuring information that is relevant to a situation of interest. That information is encapsulated in the relationships between processes, which are indicated by links (sometimes directional) in the process diagram, and embedded in the process attributes.

Process modelling is a rather broad concept. Many existing techniques for modelling projects, the flow of information, design activities and decision-making could be classified as process models, and all of these approaches have some common characteristics. One such characteristic is the emphasis on transformations between inputs and outputs. Another characteristic, which is central to some approaches to process modelling (though neglected in others), is hierarchy. Hierarchy enables processes to be analysed at different levels of definition.

Constructing a process model requires logical thinking about the flow of information and the relationship between the different holons in the system of interest. There is a strong integration between process model structure and the flow of information, particularly when modelling the process of assembling evidence in the lead up to a decision – one of the main objectives of this thesis. An element of rational analysis will help to identify lower level processes that are necessary for the success of a higher level process. However, there is also a need for synthetic thinking to ensure that the system of interest is described in as complete a way as possible. Tension will inevitably exist between an analytical stream, which is needed to establish a logical model structure based on the flow of information, and a synthetic stream which is needed to identify softer relationships and emergent properties. If the rational stream dominates, as it does in work flow analysis, PERT charts and Petri nets (see below), then important properties of the system may be neglected.

Process models may be constructed from descriptive or normative perspectives. Process modelling from the descriptive perspective is concerned with modelling how processes in practice materialise. It is of use because

- it helps individuals involved in the process to understand their role and the contribution that their work makes to the overall process;
- it aids communication and co-ordination between different players, a benefit that is particularly valuable in complex multi-disciplinary situations;
- it provides understanding of the current situation, which is a prerequisite for installation of process support technology such as management information systems and for process re-engineering.

Normative models describe the way individuals or organisations should behave rather than how they do behave. By comparing a descriptive model with a normative model, managers may be able to design and implement process re-engineering.

The main objective of the process modelling in this thesis is to facilitate uncertainty modelling, and is most closely associated with the descriptive perspective. The process is modelled in descriptive terms. The model is then linked with an appropriate way of representing uncertainty in order to identify key sources of uncertainty and sensitivities in decision-making. The aim is to support decision-makers by demonstrating the implications of uncertainty in a decision and the origins of uncertainty in the processes leading up to a decision.

It will not be possible to identify an appropriate structure for uncertainty modelling until the nature of uncertainty has been explored in more detail. A variety of model structures are introduced at this stage to demonstrate the range of interpretations of process modelling and to support with examples some of the abstract concepts that have been introduced.

PERT charts

PERT charts have for many years been associated with project management. A PERT chart is a graphical representation of the various activities that make up a project. A project consists of a number of activities, some of which need to be completed before other activities start. By associating time with each activity PERT can be used for project programming and in particular the critical path method, where it can be used interchangeably with a Gantt chart. The activities are represented graphically by a node. Arcs are used to connect activity nodes to show precedence requirements.

PERT is used in both the normative and descriptive modes. Indeed, as is typical for most of the methods discussed, the normative and descriptive modes can become confused.

Petri nets

Petri nets are most commonly used for modelling of digital systems, especially concurrent systems. Petri nets are represented as *places* and *transitions.* Directed *arcs* (arrows) connect the places and transitions. The net structure represents the logical requirements for a transition to take place. Thus in Figure 3.1, the three places to the left of the transition bar are input places to the transition – resources that must be available for the transition to take place – and the two places to the right are output places – resources that will be made available by the operation of the transition.

A Petri net can be used in a similar way as a PERT chart, though without providing the facilities for modelling project timing and progress (Peterson, 1981). However, McMahon *et al.* (1995) argue that the advantage of the Petri net approach over PERT is the clear separation of process (transition) and information model (place).

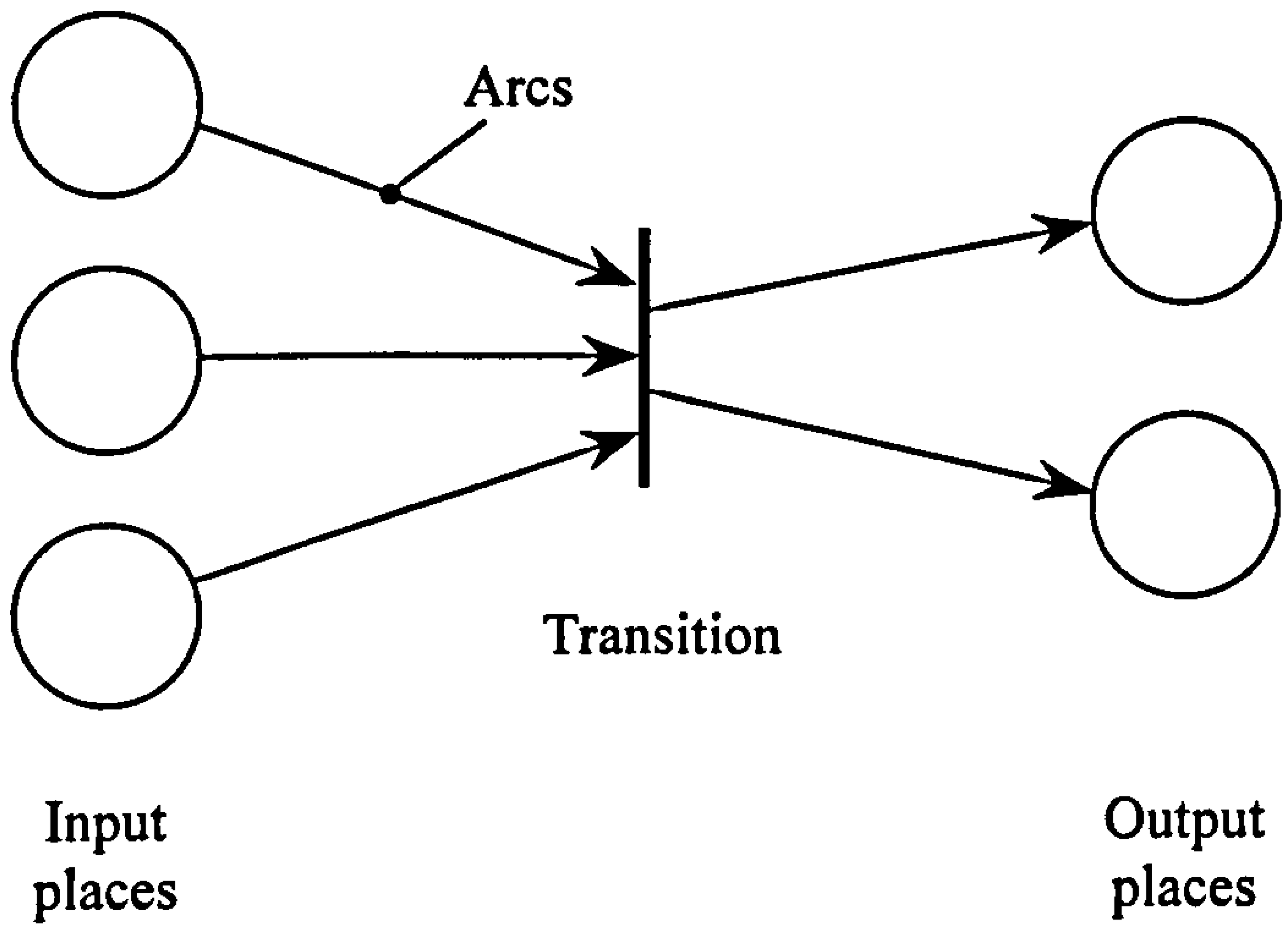

Figure 3.1 Example of a Petri net

Directed Acyclic Graphs (DAGs)

Directed Acyclic Graphs are diagrams composed of *nodes* and *arcs*, which have been used to represent and, through the application of graph theory, analyse a range of complex systems. Arrows on the arcs indicate the direction of progression through the graph. They are acyclic in the sense that any arrangement of arrows is permitted provided it does not form a loop that it is possible to cycle round.

DAGs have been used to structure arguments of causal reasoning so recur in the discussion of Bayesian probabilities in the Chapter 5. In that case the direction on the arcs is used to portray the direction of inference from one node to another, the inferential influence of one node on another, or the probabilistic dependence among nodes (Schum, 1994).

Process Modelling Language (PML)

Process modelling language was developed as part of the Integrated Process Support Environments IPSE 2.5 project (Snowdon, 1989). The process model consists of a time-ordered network of activities and entities. It is intended to form a framework for support of management activities through IPSE and its successor Processwise.

Role Activity Diagrams (RADs)

Role Activity Diagrams (Platt, 1994) stress the roles and responsibilities involved in the enactment of a process. They are structured around the various actors in the process and demonstrate the communication of information and the negotiation of objectives and resources. Nested within the role streams of control are sequential models of processes that resemble PERT approaches. RADs can therefore support concurrency and assist in co-ordinating collaborative work (Platt, 1994).

Free-format process mapping tools

A number of computer-based tools are emerging to support a flexible range of approaches to process modelling. These range from what are essentially drawing packages for computer-based mind mapping (for example VisiMap) to tools which can be integrated with other computer based knowledge bases and applications. One such tool of the latter variety that has been used in the course of this research is Flowmap (Howard, 1992) which provides a flexible workbench for process mapping but can also enable links with text-based information, internet sites and applications. It proved to be less convenient as a platform for uncertainty modelling, which was eventually implemented through a custom-built package. ARIS (Scheer, 1994) is another such tool, which integrates a number of process modelling approaches.

Hierarchical process models (HPMs)

A hierarchical process model consists of processes ordered according to their precision of definition. At the top of the hierarchy are the processes which are the most vague and imprecise, at the bottom are those which are as precisely defined as is required for the problem (Blockley and Henderson, 1988). Each level of the hierarchy is thought of as representing the whole system at that level of definition (Figure 3.2). The hierarchy is extended downwards and upwards as far as is appropriate in the given context. Note how in Figure 3.3 each process is named using the participle of a verb ("establishing", "assessing", "calculating") to signify the dynamics of the process.

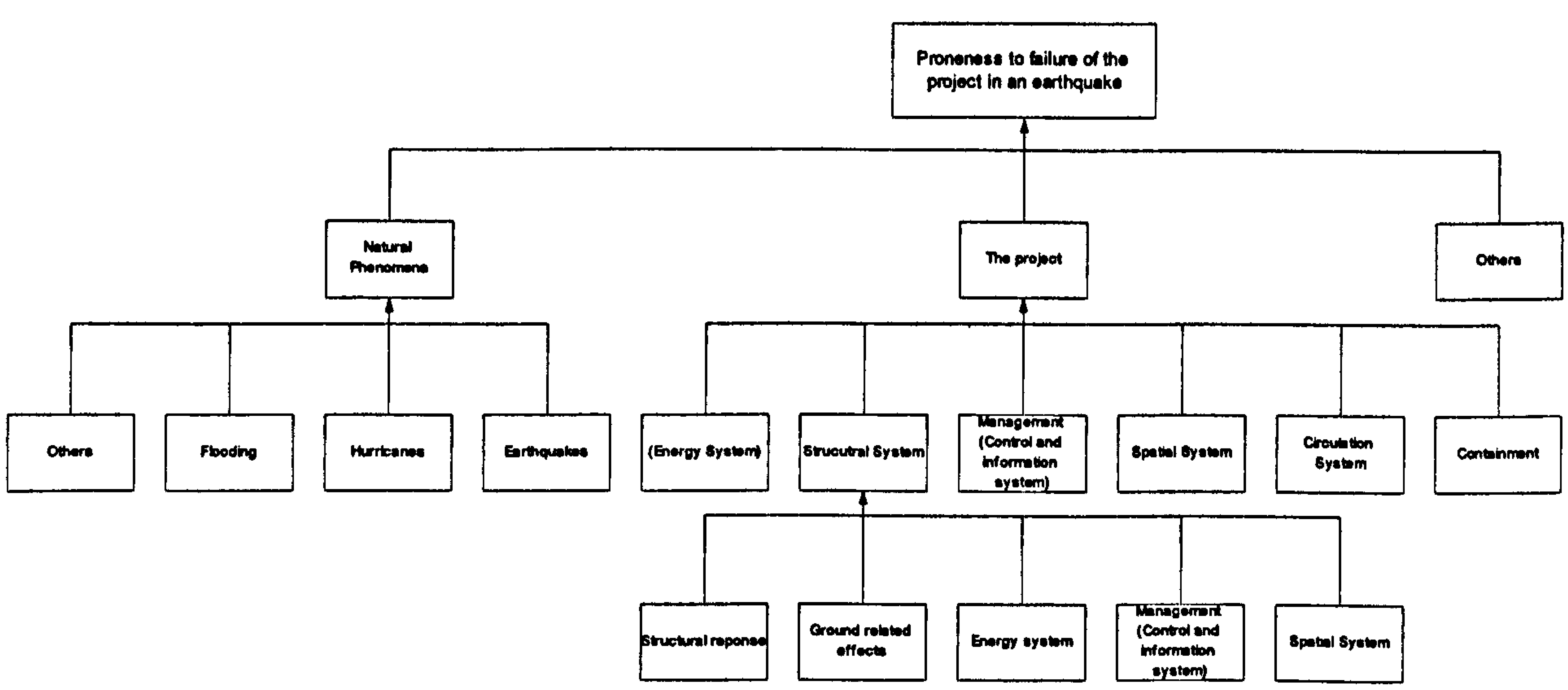

Figure 3.2 Example of a hierarchical model (after Sánchez-Silva at al., 1996)

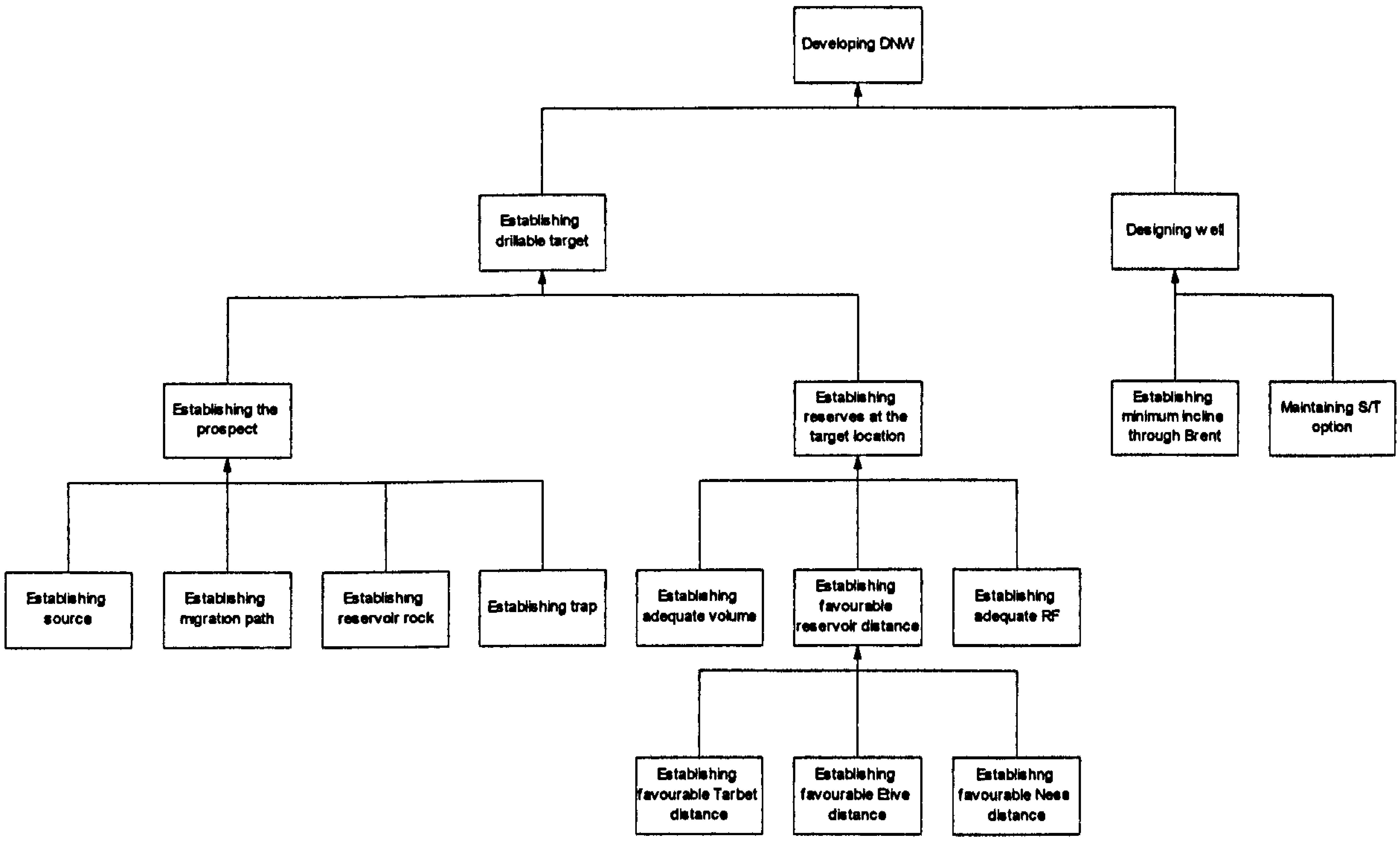

Figure 3.3 Example of a process model from the petroleum industry (after Ball et al., 1996)

Table 3.1 Some attributes of process (after Blockley, 1999)

T	TRANSFORMATION	Describes how the process transforms the state variables
H	HAZARD	Threats to the control of the process
E	ENVIRONMENT,	Context within which the process is enacted
	ETHIC	Values used to evaluate options and scenarios
P	PLAYER,	The individuals who inhabit roles
	POINT OF VIEW	Perspective of the process owner
R	ROLE,	The set of responsibilities required to accomplish the process
	RISK	Scenarios which can lead to failure
O	OWNER,	Person responsible for the process
	OBJECTIVES	Desired future states
C	CLIENT,	To whom the process is delivered
	COSTS	Typical state variable for a commercial process
E	END/BEGINNING,	Finish/start states
	EVIDENCE	Information used to evaluate process and hazard
S	STAKEHOLDERS,	Those with an interest in the process but no direct part to play
	SCENARIOS	Potential futures against which alternative solutions are tested
S	STATE VARIABLES,	Parameterises current state of the process
	SUCCESS	Criteria against which the process will be judged

3.7 Process attributes

The attributes of a process are those items of information that support the understanding and enactment of the process. Attributes will include the state variables of the system. So for example the attributes of an economic appraisal process will include the option costs, benefits and discount rate used for the analysis. The attributes will also describe the context in which the process is to be enacted (*e.g.* the process owner and stakeholders). If process modelling is used to support uncertainty management a key attribute will be some indicator of the uncertainty in the process. Some attributes of process are illustrated in Table 3.1. However, the relevant attributes will be specific to the context of the process and the objectives of the process modeller.

An example of an attributes window from the process modelling tool adopted and developed during the course of this research is shown in Figure 3.4. Note how the attributes window can be used to link the process model to other knowledge bases, for example through the internet, and can be used to launch other applications.

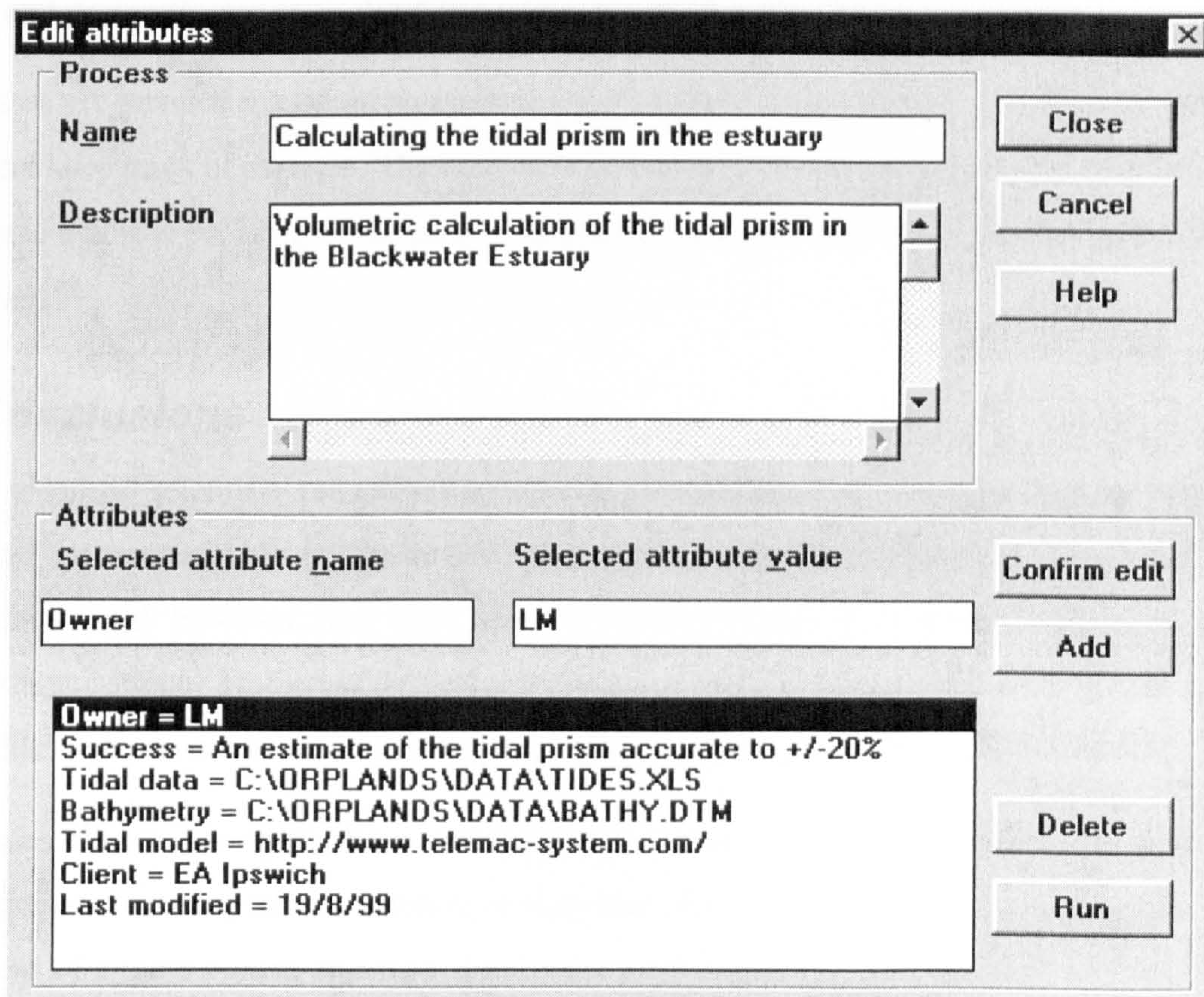

Figure 3.4 An attributes window

3.8 Towards process integration and process enactment

One of the aims of process modelling is to capture the dynamics of the system in which the engineers (or decision-makers in general) find themselves. Yet most process models, be they paper or computer-based, rely on manual updating of the model structure and process attributes. The information contained in them is essentially passive, so does not change without conscious human intervention. As models become more complex, updating becomes more and more burdensome, particularly in highly dynamic situations. Information technology now provides the potential for much more active knowledge bases integrated in process models and for process enactment through the process model.

Process enactment means that the transformations that the process model represents are actually implemented through via the process model. A simple example would be a process with involves communicating some digital information to another player. That information could be embedded as a process attribute through an internet address or directory path, and the communication process be enacted via the process model by launching an email application from the process model. Many of the analysis and design processes associated with coastal management are now enacted electronically. Geographical Information Systems are holding an increasing proportion of coastal monitoring data (Deakin, 1994) and are becoming integrated with the numerical models used for analysis and design (Lowe *et al.*, 1994, Hoogweg *et al.*, 1996, van Stijn *et al.*, 1996).

If processes are enacted via the process model there is then scope for the model to automatically update and keep track of changes. The information embedded in the model thereby becomes active in the sense that it is evolving as the process is enacted and does not require human intervention to be updated.

3.9 Conclusions

1. The ideas of scientific reductionism are deeply embedded in Western thinking and have proved to tremendously powerful tools for understanding and predicting phenomena in the physical world. However, they have proved to be less successful at dealing with messy human and organisational problems. Reductionist methods tend to address problems at one level of definition and do not acknowledge the importance of emergent phenomena.

2. Systems thinking stresses the connectivities between the different parts of a system and recognises that the system as a whole is more than the sum of its parts. Systems operate in the context of a meta-system and their description will reflect the point of view of the observer. Key systems concepts are hierarchy, emergence, communication and control.

3. Process modelling aims to represent systems and to provide a generic vocabulary that can be used to describe hard and soft systems. Process modelling can be conducted from a descriptive point of view, in which case it aims to support players in the process by demonstrating how their activities contribute to the overall process objectives and by aiding communication between players. It can be used as a tool for normative process re-engineering, which aims to change the way processes are enacted in order to make them more efficient. Process models

can also be used as a platform for uncertainty modelling with the aim of demonstrating the sources, sensitivities and implications of uncertainty in the process. It is this latter mode which is the focus of this thesis.

4. There are many ways of representing process, with different representations emphasising different aspects of the process. Transformation processes between inputs and outputs are a primitive construct of process models. Indeed Petri nets and DAGs are devoted exclusively to modelling transformation process, and their structure is governed by the sequence of transformation processes. Hierarchical process models enable systems to be described at a range of different levels of definition.

5. Information is conveyed in process models through the structure of the model and the process attributes that are embedded in the model. Attributes will be the state variables of the process and other relevant items of information about the process. If process modelling is used to support uncertainty management a key attribute will be some indicator of the uncertainty in the process. The nature of uncertainty and its expression as a process attribute in mathematical terms are the subject of Chapters 4 and 5 respectively.

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CHAPTER 4

Uncertainty, information and decision-making

4.1 Objectives of Chapter 4

- to develop a philosophical grounding for engineering decision-making, based on the concept of dependability;
- to analyse and classify the characteristics of uncertainty;
- to introduce generic concepts of models and relations;
- to describe the decision-making process;
- to introduce models of the process of assembling evidence in the lead-up to a decision, which will form the basis for uncertainty management.

4.2 Introduction

This chapter continues the conceptual developments of Chapter 3 by addressing issues of uncertainty, information and decision-making. Uncertainty management involves identifying the sources of uncertainty in a decision and sensitivities of the decision to uncertainty, and taking account of uncertainty in the decision-making process. To achieve the aims of uncertainty management requires an understanding of

- the nature of uncertainty;
- the decision-making process and in particular the place that evidence about decision options plays in that process;
- the origins of the evidence used in a decision and the processes by which it is manipulated.

Most of this chapter is concerned with developing conceptual, and often philosophical, understanding of these issues. Towards the end of the chapter the concepts that have been introduced will be used to develop a structure for representing uncertainty in the process of assembling evidence in the lead-up to a decision. The issue of how the uncertainty can be measured and how measures of uncertainty can be manipulated is the subject of Chapter 5.

4.3 Towards a philosophical basis for handling uncertainty

Notions of information, uncertainty and decision-making in engineering can be interpreted from the point of view of several philosophical traditions. The rationalist tradition is of great value in its use of reason, logic and mathematics. Yet there is also a strong flavour of empiricism in the reliance on observations and evidence as a basis for decision-making. The subjective nature of observations has already been acknowledged in the Grounded Theory work in Chapter 2. Above all, there is a creative tension between scepticism, which if taken to extremes can demolish all our assumptions of truth, and a pragmatism that endeavours to build a practical basis for engineering behaviour from the debris left by scepticism. This section questions the basis for belief in any item of knowledge and specifically in notions of causality. It aims to establish a philosophical grounding for the concepts of uncertainty, information and decision-making, which are developed in subsequent sections.

Plato used the allegory of the philosopher's cave (in *Republic* Book VII), in which the images on the wall were but poor representations of a more solid metaphysical existence, to suggest that the material world of senses was illusory. According to Plato reality and knowledge is in the mind, where there exists a realm of perfect 'forms' of which the world of material objects detected by the senses is an imitation. Knowledge, as distinct from belief or opinion, is possible only of the real world of unchanging and eternal Forms. Later philosophers (for example Hume and Kant) disposed of the idea that the metaphysical world is knowable, but Plato's notion that the messages received from the senses may be illusory has proved to be more enduring.

For Plato, mathematical knowledge was the ideal. A great deal of knowledge could be established by reasoning alone. Although it now seems that the extent of the knowledge that can be deduced by rational reasoning is more circumscribed than Plato might have hoped, powers of reasoning are invaluable in making the most of the little information that may be available to a decision-maker. Thus mathematics and, somewhat less formally, the reasoned analysis of process models for coherency and consistency, are tools of great value in uncertainty management.

Although influenced by Plato's rationalism, Aristotle displayed more empirical leanings. He founded a system of logic, central to which was the syllogism, which was the basis of logical studies until the nineteenth century. Yet, on the other hand he conducted very wide-ranging investigations of nature. This combination of rational thought with empirical study was to prove to be immensely productive. Galileo, for example, was more of a rationalist thinker than an experimentalist, yet saw observation as a starting point in scientific enquiry.

The immensely influential scientific investigations of the seventeenth century were conducted in search of an underlying and incontrovertible set of (deterministic) mathematical laws that governed nature. Newton's theories appeared to have gone a long way towards achieving this ideal. They have provided engineers with enduring theoretical tools. Yet the very generality and simplicity of mechanics may have stifled the appreciation of uncertainty in engineering by engendering a culture of determinism, to be reinforced by Laplace who suggested that it should be possible to predict the state of the universe at any point in time from the position and momentum of particles. Despite the advent of quantum mechanics, this culture of determinism has proved to be difficult to dispel.

René Descartes (1591-1650) has already been mentioned in Chapter 3 as the founder of the reductionist scientific method. Descartes' insistence that analysis should be complete with nothing omitted is important and problematic from the point of view of uncertainty analysis. Descartes held great store by mathematical reasoning with clear distinct ideas, free from contradiction. It requires complete analysis to conduct such reasoning, but beyond the domain of mathematics, complete analysis is not achievable.

Descartes is also central to a discussion of uncertainty in his assertion that any idea that does not pass the test of doubt should be eliminated. This sceptical view led him to conclude that the proposition *Cogito, ergo sum* is the only idea unshakeable by doubt. With the *cogito* Descartes initiated a theme that has been very important in modern philosophy, that it is human perception of existence which is the starting point for enquiry. The exploration of how human perceptions are shaped by evolution, gender, culture, and indeed the unequal ownership of the means of production, has occupied generations of philosophers and a proliferation of other disciplines since. Post-modernism indicates that there are no easy answers.

Meanwhile the British empiricists stressed the importance of observation and experience above the rationalism that prevailed on the Continent at the time. John Locke (1632-1704) asserted that human knowledge is ultimately derived from sense experience. Yet he was also enamoured of the Newtonian concept of underlying structure and order in nature. His philosophy therefore on the one hand employs notions of external physical processes affecting our senses and, on the other, asserts that we can have no direct knowledge of such processes. George Berkeley (1685-1753) overcame this inconsistency by rejecting the notion of an underlying order and asserting that "to be is to be perceived". According the Berkeley all the objects we perceive and ordinarily take to exist in the world outside ourselves are simply collections of ideas existing only in minds. His rejection of the factual truth of the observations of science was unacceptable to Berkeley's contemporaries but has found favour with twentieth century philosophers of science. Berkeley was not requiring that we attempt to make the bizarre transformation of human existence in which we treat the world and our own bodies as if they were in some sense not really there. What he did wish to transform is the conceptual apparatus that, in the attempt to explain the nature of things, had produced logical incongruities. It is in the same spirit that this thesis is written. It would indeed be bizarre to write an engineering thesis whilst studiously avoiding reference to the physical world. But the Newtonian notion of underlying order has led to grave misapprehensions about the nature of uncertainty in engineering. By reflecting on philosophical objections to the persistent engineering culture of determinism it is possible to develop a richer view of uncertainty and of engineering activity.

David Hume (1711-1777) anticipated the conclusion of the modern logical positivist by arguing against the relation between cause and effect. Hume argued that however many times event *A* is observed to be followed by event *B* it does not logically follow that *B* will always follow *A*. That the sun has been observed to rise on every day in the past provides no proof that it will rise tomorrow morning. We may expect the sun to rise tomorrow morning, but this, according to Hume, is a matter of psychology, not logic. In other words causality is merely a habit of mind developed through our experience of things. For a philosopher writing at the time when the Newtonian model of nature held sway, these were revolutionary ideas. As far as Immanuel Kant was concerned, David Hume had 'proved irrefutably' that notions of causality had no rational grounding (Russell, 1946). Russell himself argued that causation plays no role in physics proper and offered to purge the word from the language of science (Pearl, 1997).

As well as being sceptical, Hume was in the empirical tradition, because he gave credence only to claims that could be analysed to show that they refer in the first instance to sense impressions. Hume distinguished between 'impressions' and 'ideas' in a way that is analogous to the treatment of information and models later in this chapter. For Hume impressions are the messages received by the senses whilst ideas are the 'faint images of these [impressions] in our thinking and reasoning' (Collinson, 1987). Hume also referred to 'relations of ideas' and 'matters of fact'. Any meaningful proposition must either express some kind of relationship between ideas and be necessarily true or false (for example an algebraic statement), or state a putative fact which is only contingently and not necessarily true or false. These notions of Hume's have been a major influence on the treatment of models and uncertainty in this and subsequent chapters.

It was left to Kant (1724-1808), whom Hume had wakened from his 'dogmatic slumbers', to bring together empiricism and rationalism. He recognised the strength of the empiricist claim that sense experience is the source of all our beliefs but could not accept its sceptical conclusion that those beliefs cannot be justified. At the same time he rejected the rationalist claim that factual truths about what does and does not exist can be conclusively established by reason alone. Kant asserted that knowledge is founded on subjective experiences that are produced by external entities that affect the senses. However, what those external entities are in themselves remains completely unknowable. All of our knowledge is merely based on the appearance of those external objects.

Our knowledge is therefore as much a function of the mode of experiencing as it is of the nature of the objects in themselves.

Just as Hume distinguished between 'relations of ideas' and 'matters of fact', Kant distinguished between 'synthetic' and 'analytic' propositions. Analytic propositions are true without reference to any other knowledge because their subject is contained in their predicate. For example the statement 'a rainy day is a wet day' is analytic because the concept of wetness is contained in the concept of a rainy day. Synthetic propositions, meanwhile, are based on empirical observations. Kant argued that humans have a set of pure concepts of understanding, such as causality, which are used to make sense of the world. He called these 'synthetic *a priori* propositions'. They are synthetic because they say something about the world but are *a priori* in the sense that they are not derived from sense perception. They are part of the human mind, and indeed essential in order to make human experience possible.

Kant's ideas about human reasoning set in place productive movements both in psychology and philosophy. His models of human reasoning have now been superseded by modern and post-modern philosophers and psychologists, but by combining empiricism with the subjectivism of Descartes' *cogito* he set up a philosophical framework which is stable enough to support much of the treatment of information and modelling in this thesis.

A treatment of uncertainty and information which starts with Kant's epistemology of an external world that is unknowable is fundamentally different to one which starts with the assumption that there is an external truth which would be knowable if only there were enough information available. The notion of a knowable external world, one that may even be reasonably well ordered, drives an ambitious quest for models that are an increasingly close approximation to some stable external reality, without recognising that models are human constructs to make sense of a selective set of perceptions. It is more sound to start from the unarguable empirical point of view that we all as individuals seem to be receiving signals from our senses. As conscious beings, when we receive information from our senses we try to make it intelligible by constructing models. These may be mental models or external shared models. Either way, following Hume, models can only be seen as being pragmatic habits of mind, which help us to make decisions. It may be possible to make logical deductions from the propositions that models represent, and those deductions can be tested for consistency with other available information. However, such tests can provide no verification that our models represent reality, they can only demonstrate that the models are consistent, or inconsistent, with other observations. This is one of the most important themes in Popper's philosophy.

4.4 Karl Popper

Twentieth century philosophers have only been referred to in passing. Sir Karl Popper (1902-1994), who for Blockley (1995) is "the most important philosopher of this century for engineers", will be the exception. Popper is important because he provides a workable escape from the philosophical problems of causality and induction raised by Hume. Popper represents the middle ground that has emerged between the remnants of dogmatists (in the tradition of Plato and Locke) and the sceptics (and their modern counterparts, the radical relativists). Popper recognised that a theory can never be proved to be true, but he argued that it is at least possible to be sure that a theory is false, thus allowing more by way of absolute knowledge than a radical sceptic will normally admit (Smithson, 1988). Popper thereby opens up a complicated middle ground of partial knowledge, which is the terrain that most engineering activity inhabits.

In Hume's time, the scientific method based on inductive reasoning was well established. Francis Bacon (1561-1626) had argued that general laws and principles could be derived from a number of particular instances by using inductive reasoning. Bacon supported the organised collection, through experiment, of sets of data to support or refute a hypothesis. This led to the development of a scientific method, which was refined and applied by Newton, and can be simply stated as

1. observation and experiment,
2. inductive generalisation,
3. hypothesis,
4. attempted verification of hypothesis,
5. proof or disproof, and finally
6. knowledge.

Not only does this model rely on inductive reasoning, which is problematic in logical terms, it is also inconsistent with Thomas Kuhn's (1962) studies of the way scientists work in practice. Kuhn saw that observation and experiment is not done speculatively but is driven by recognised problems and paradigms.

Popper recognised that a scientific hypothesis cannot be logically proven, regardless of the number of supporting instances are identified. However, it only takes one counter instance to falsify the hypothesis. Thus all scientific hypotheses are transient. Science proceeds in an evolutionary mode, with falsified hypotheses being periodically superseded by more general and more powerful hypotheses (Magee, 1973). This results in a view of science that is rather different to Bacon's, which can be summarised as follows:

1. identified problem with existing theories;

2. proposed solution, in the form of a new theory;
3. deduction of testable propositions from the theory;
4. tests, *i.e.* attempts to refute the theory by, among other things, observation and experiment;
5. preference established amongst competing theories.

The final stage leads to the theory being adopted by the scientific community as a paradigm in which to conduct their normal science (Kuhn, 1962), until new problems begin to emerge. This problem solving approach to scientific evolution is in many ways analogous to engineering problem solving. However, Blockley (1992*a*) argues that science and engineering are driven by different values, because of the differing consequences of error. Scientific values are truth and precision, whilst engineers are interested in dependability and safety.

Popper proposed that the information content of a theory or hypothesis determines its testability. A theory that makes very precise predictions contains a great deal of information. It is also highly testable. On the other hand a vague proposition contains less information and is more difficult to refute. A statement that a person's height is 'about six feet' contains less information than the statement that they are 5'11". Because the latter statement is more precise and contains more information it is easier to refute.

Popper was very impressed by Einstein's theory of relativity because of its precise predictions, which were highly testable. An ingenious test (by Eddington) did not manage to falsify Einstein's theory. For Popper the difference between science and non-science is that scientific knowledge is testable (Popper, 1963). Popper's emphasis on the ingenuity of the test is closer to Francis Bacon's view of corroboration than to Rudolf Carnap's. Bacon believed that the variety of instances was the best indication of corroboration of a hypothesis, whilst Carnap argued in favour of the multiplicity of instances (Cohen, 1989). In his analysis of the modelling of uncertainty in expert systems Buxton (1989) argues that the support provided by a set of evidence depends on three main factors:

1. degree of compatibility between the evidence and hypothesis;
2. amount of evidence; and
3. variety of evidence.

The relationship between the content of a theory and its testability (or, according to Popper 'corroborability' (Magee, 1973)) is of great relevance to engineers. Theories with high information content are very useful to engineers. Engineers are also interested in the extent to which theories have been tested in circumstances similar to the ones they are working on. Newton's theory of gravitation, for example, has a very high information content. A large number of propositions can be deduced from this theory, which correspond to observations. The theory has been falsified, but, as will become clear in the following section, the truth of a theory is an issue that is not necessarily of concern to engineers.

4.5 Engineering from dependability

4.5.1 Dependability

Engineers therefore seem to be in a puzzling situation. The scientific theories that they use to construct models and support decision-making cannot be proved to be true. Furthermore, some, if not many, of the theories that are useful to engineers (notably Newton's theory of gravitation), have been proved to be false. Confronted with this paradox, engineers seem to be prepared to follow Kant and behave *as if* their theories were true. This, on the face of it, may appear to be reckless. Not so, according to Blockley (1980) who argues that engineers are not necessarily interested in the truth of theory. What they are interested in is its dependability, in other words the extent to which it provides a sound basis for decision-making. Truth, according to Blockley, is a sufficient but not necessary condition for dependability. Abandoning the scientific value of truth is no excuse for a lack of rigour. Engineers clearly need a rigorous means by which to judge the dependability of the information that is used to make a decision. The problem of establishing the dependability of information initiates new and perhaps more complex problems.

A responsible engineer will want to know how near to the truth a theory is, and under what conditions, if at all, it has been falsified. Thus an engineer will be very interested in how well tested a theory is and the results of those tests, in other words the *evidence* for or against the dependability of the theory. Blockley (1980) argued that in a good test of dependability

1. the experiment must be repeatable;
2. the output states must be repeatable;
3. the states of the system must be clear and distinct;
4. the magnitude of perception of the states must be repeatable.

It is no coincidence that some of these conditions, illustrated in Figure 4.1, coincide with what is usually referred to as being *objective* information. An observation in a test or experiment is usually considered to be objective if it is independent of the observer. The tenant that observations should be independent of the observer is central to the pursuit of traditional science. A good scientist is expected to perform and report experiments in such a way that another scientist who sets up the experiment according to the instructions will obtain exactly the same results. If that is the case, then all four of Blockley's conditions will have been satisfied. Objectivity is not connected to truth, rather it is to do with the way observations are made and reported. The conventions of measurement are a common way of securing objective observations. But measurement is no more than a convention, albeit a very useful one, that enables different observers to obtain the same results in an experiment. Whilst some measures, like weight, length and time are standardised, there are many others (for example the number of unemployed) that can be more controversial.

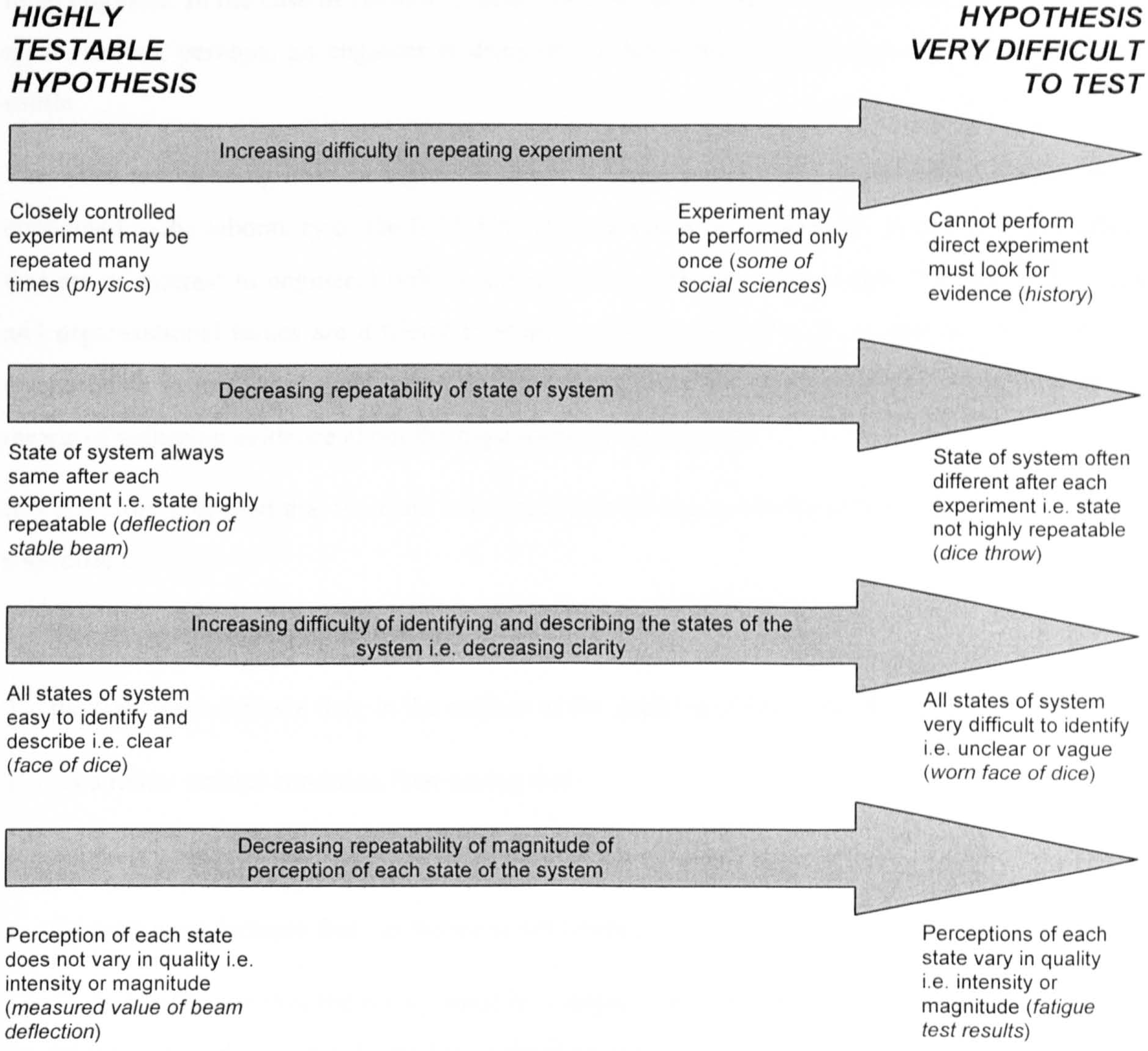

Figure 4.1 Conditions for dependable information (after Blockley, 1980)

In the social sciences it is much more difficult than in the physical sciences to ensure that the observation is independent of the observer. The semi-structured interviews reported in Chapter 2 were highly dependent on the role played by the interviewer, even though an effort was made to achieve rigour and consistency in the analysis of the interview data. Indeed only in special cases can the role of the observer be effectively overlooked when handling information. Contemporary philosophy and science is stressing this. For example, in Derrida's post-modern view, text only has sense when interpreted by the reader, and in the Copenhagen interpretation of quantum theory the presence or absence of an observer can influence what is being observed.

Naturally, to ensure dependability it is not merely the case that the hypothesis should have been through exacting tests – the theory should not be refuted by tests which are relevant to the conditions in which the theory is to be used. There is an important element of judgement here. The discussion of Newton's theories indicates that some refutations of a theory may not be of concern to an engineer. In the case of Newton's theory of gravitation this is a straightforward judgement to make, unless, perhaps, an engineer is designing a spaceship. Most judgements are rather more subtle.

The word test is being used in a general sense. It includes scientific experiments, which may be conducted in the laboratory or the field. However, as Figure 4.1 illustrates, not all of the situations that are of interest to engineers will be amenable to scientific experiments. For example, human and organisational issues are difficult to study and the results of such studies are almost always questionable in methodological terms. In the more general sense a test can be thought of as any means of gathering evidence about the dependability of a theory.

It is therefore proposed that there are two conditions for dependability of a theory in the context of a specific decision.

1. The theory must be well tested, and

2. the tests must indicate that, in the context of the decision in hand, the theory is not refuted.

This is a rather stricter condition than saying that

1. the theory must be well tested in the context of the decision, and

2. the tests must indicate that the theory is not refuted.

The former definition that the theory must be comprehensively tested, even beyond the context of the decision, provides more information to the decision-maker.

The conditions for dependability may seem to be complex, yet dependability and evidence are things engineers deal with on a daily basis. Responsible engineers are constantly evaluating the dependability of the models they use in design and other types of decision-making. A significant class of engineering failures are a result of theory that is established for a particular set of conditions being used in significantly different conditions to which it is not applicable. In their quest for longer spans and stronger structures engineers have from time to time exceeded the established limits of the theory they employ (Blockley, 1980). On the other hand, given that every civil engineering structure is in some significant respects a one-off, engineers have on the whole proved their ability to evaluate the dependability of their models in constantly changing conditions.

4.5.2 Incompleteness

Incompleteness has been defined by Blockley (1996) as "the mismatch between models and reality". The sense in which Blockley uses the term incompleteness is to indicate that there may well be phenomena outside our models that nonetheless are of significance to the outcome of our decisions.

Incompleteness is related to the idea of an 'open world'. The term 'open world' is used to refer to the domain of synthetic propositions, which are based on empirical observations. In an open world there is a limitless quantity of information, which derives ultimately from some logically unknowable reality.

Precisely defined and constrained problems can in practice be modelled as being complete. Consider, tossing a coin, which is an activity that is enacted in an open world. A model of tossing a coin, which has two outcome states ('heads' or 'tails'), is incomplete because in an open world there is always some remote possibility of some other outcome (the coin lands on its edge perhaps). However, the outcome is so remote and the consequences of the outcome of so little significance (one has to toss the coin again) that we usually behave as if the two-outcome model were complete. Even though coin tossing takes place in the synthetic world of impressions it is effectively modelled as a closed analytic activity.

In many complex engineering problems the information available is obviously incomplete. A visual inspection of a flood defence embankment yields information that is obviously an incomplete indication of the safety of the embankment. Incompleteness therefore seems to be as much a function of the problem which is being addressed as the available information.

Incompleteness can be due to a number of different causes (Davis and Blockley, 1996):

1. That which cannot be foreseen – this includes the existence of phenomena which were previously unknown
2. That which is foreseeable but is

   a) ignored in error – these effects should be taken into account but are missed by mistake;

   b) ignored by choice – these effects or this data is explicitly considered to be unimportant;

   c) ignored because of lack of resource – these effects are too complex to model at this time or the data required is too costly to collect.

The first type of incompleteness relates to ignorance. Items 2*a* and 2*b* relate to a meta-level judgement of *relevance*. Item 2*c* is a pragmatic consideration.

Perhaps for the same psychological reasons that a deterministic physical world seems to be such an attractive concept, humans seem reluctant to recognise the nature of an open world. Schön (1983) refers to 'selective inattention', the tendency, which is particularly pronounced amongst convergent scientific thinkers, to exclude from their analysis those aspects that are difficult to handle or quantify. The converse of this selective inattention is an open world view, which is characteristic of divergent thinkers, in which the issues of incompleteness are recognised and the possibility of unforeseen phenomena is acknowledged.

Dependability and incompleteness are related concepts that help to deal with the problem of using theories/models in an open world. They are used to indicate how near to the truth a theory/model is, whilst recognising that its absolute truth can never be proven. An estimate of dependability helps a decision-maker to decide whether to hold much store by the results which the theory generates. Dependability is a measure of the incompleteness of a theory in the context of a specific decision.

Dependability and incompleteness are open world judgements. They cannot be deduced from the information generated by a model or from the implicit nature of the model itself. Rather, they are based on a comparison of the model with all of the available evidence about phenomena that are of interest in the context of the decision. A statement of dependability is therefore itself a hypothesis based on a judgement of the quality of the available evidence. It is a hypothesis that will be rather difficult to test, so, to ensure that it is reasonably close to the truth, a statement of dependability often has quite low information content. This is achieved by using linguistic terms or an appropriate mathematization. Chapter 5 is devoted to the development of an appropriate mathematics and structure for statements of dependability.

4.6 The decision-making process

Before looking at the nature of information and uncertainty in more detail it is appropriate to develop the context in which the discussion will be set, which is decision-making. Decision-making is the name by which generic problem-solving activities will be referred. Numerous descriptive investigations of decision-making have identified some broadly similar stages of

- problem definition,
- definition of decision objectives,
- generation and analysis of options,
- choice of the preferred option,
- implementation

(Simon, 1965, Sage, 1986). A number of researchers have recognised that decision-making is not linear, and that implementation of the preferred option is followed by monitoring (be it formal or *ad hoc*), which then leads to identification of new problems (Loasby, 1976, Collingridge, 1980). This leads to the cyclic model shown in Figure 4.2. The cyclic model is attractive because it sees decision-making as an ongoing management process, which is clearly analogous with a generic feedback loop of control. The decision-making loop has strong links with basic systems theory (Checkland, 1988, Senge, 1990). It can be seen as a means of attempting to achieve ones objectives in a dynamic environment. There are also analogies with Popper's view of the scientific method introduced in Section 4.4.

However, the analysis described in Chapter 2 has demonstrated that Figure 4.2 is a rather idealised model. In practice there are often intermediate loops, sometimes with long delays between stages in the process, with previous stages being revisited before the choice of preferred option. In particular there are often loops of analysis followed by option re-identification. Descriptive studies frequently identify searching and screening strategies for decision-making. Where a great deal of ignorance is inherent in the situation a variety of approaches may be used in series, beginning with the simple and highly abstract, to screen out most of the possibilities before undertaking a comprehensive investigation of the few that remain. This helps to explain why later detailed study seems often merely to confirm an apparent predisposition in favour of a particular option (Loasby, 1976).

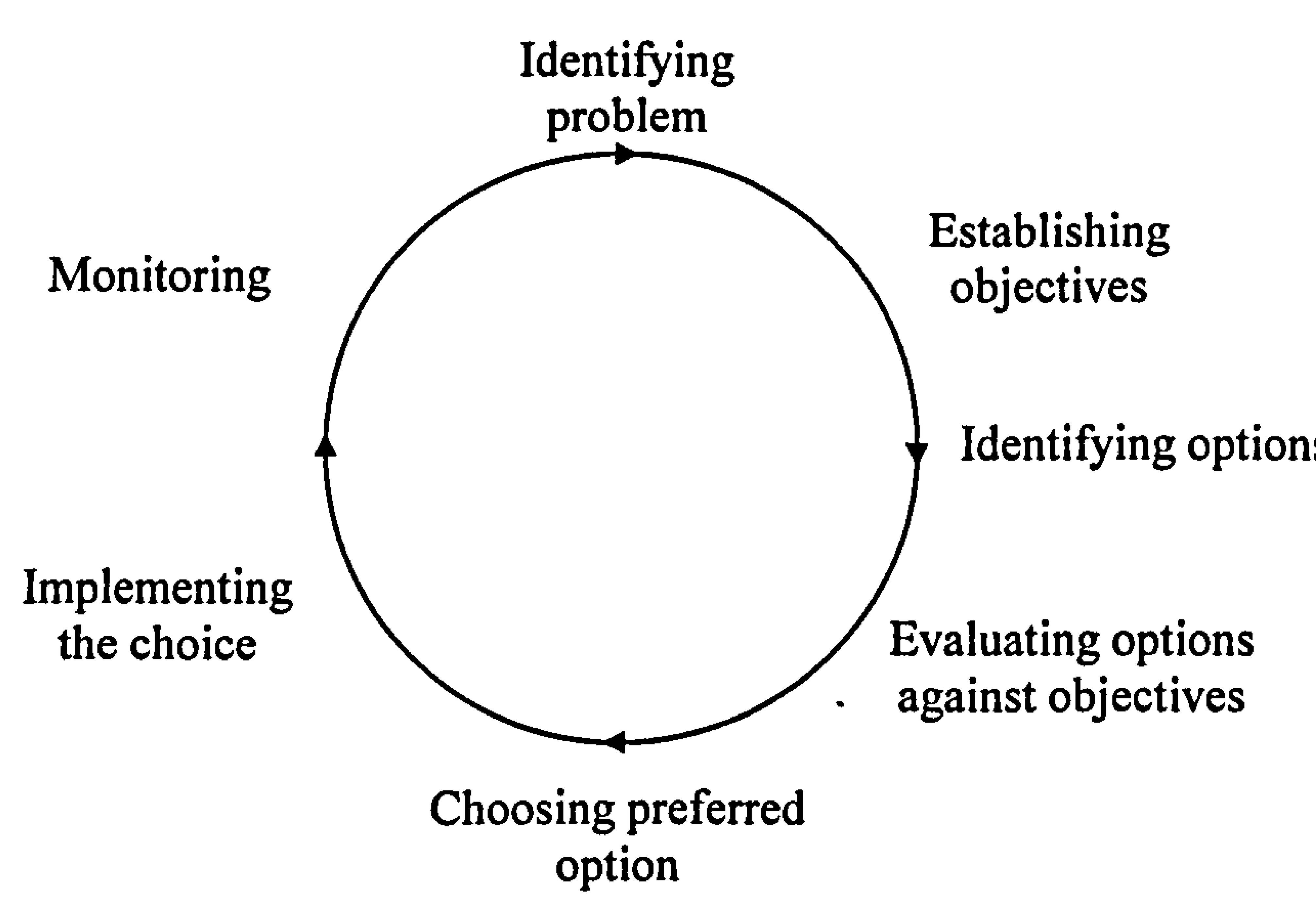

Figure 4.2 The decision-making process

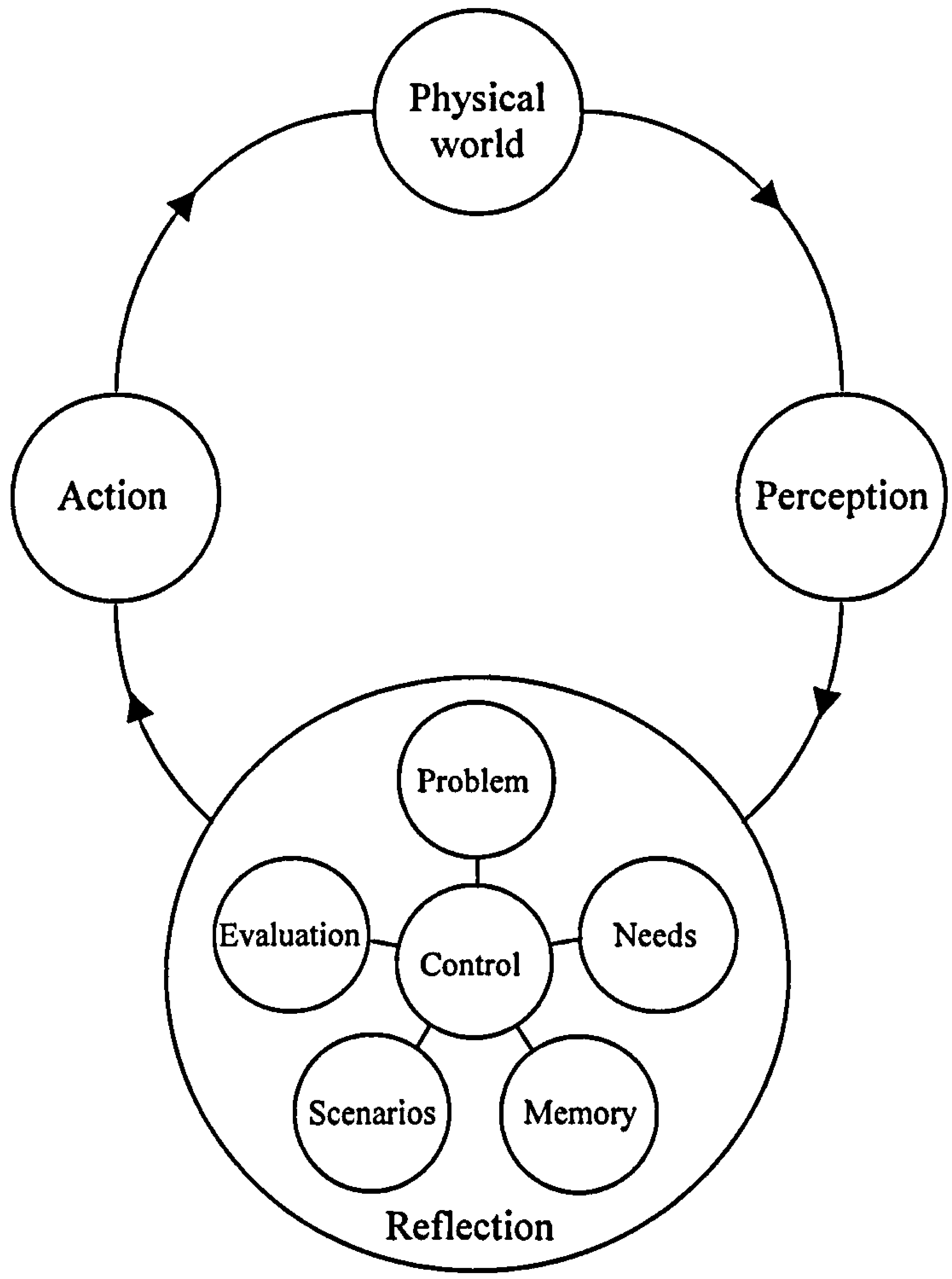

Figure 4.3 Blockley's reflective practice loop

Blockley (Blockley, 1992*b*, Dias and Blockley, 1995) has identified a similar loop as being the basis of professional activity in engineering. Drawing on Schön's (1983) study of how professionals work in practice, Blockley has proposed a reflective practice (RP) loop which links perception, reflection and action (Figure 4.3). The RP loop fits in well with the reasoning adopted in this chapter, in which (inevitably uncertain) signals are perceived and the professional then makes sense of those signals in a period of reflection. They do so by constructing models, which may be mental models or they may be externalised in an informal way, for example as sketches or mind maps (Buzan, 1988), or in a formal way in quantitative models and calculations. This process of reflection enables a preferred course of action to be identified. In this way reflection leads to action and ultimately to more perception.

The stages identified in Figure 4.2 are now examined in more detail. The dependability of a decision-making process will be influenced by each of these sub-processes.

Defining the problem

Problems emerge from interpretation of the signals that the decision-maker is receiving. In order to make sense of signals they need to be interpreted in the context of decision objectives.

Identifying objectives

Objectives are driven by the decision-maker's values, which are interpreted in the context of the decision in hand. If systems are conceptualised in hierarchical terms, then the objectives driving any specific decision are an interpretation of higher level (more vague) objectives, in the context of the decision in hand.

Identifying options

Options are the alternative courses of action that the decision-maker may take. The process of identifying options is a creative activity. Analysis of designers reveals that they go through phases of divergence and convergence during a design process (Ohsuga, 1989, Pugh, 1991) in which options are identified and then rejected.

In a hierarchical system options may be present as some high level prototypes, which can then be refined in the context of a specific decision. However, a hierarchical interpretation does not allow for completely original option generation in the context of a specific low level decision.

Analysing the options

In order to assess the options against the objectives involves some analysis. The options are modelled in some sense in order to analyse to what extent they will meet the objectives. This involves assembling evidence and using it in models to deduce propositions about how the options will perform against objectives. For engineering decisions this process of analysing options can involve vast quantities of evidence with very different levels of dependability and information content. Analysing the options can be thought of as a process of developing, in a concise format, a set of option attributes. Each attribute will relate to the decision objectives, for example maximising cost or achieving target levels of serviceability.

On the one hand the decision-maker needs to have some reasonably condensed information in order to make a choice. Indeed, as will be explained in Chapter 6, the normative choice mechanism requires that the option be expressed in terms of only one measure, a utility, which combines an expression of value with an allowance for the decision-maker's attitude to risk. On the other hand the final measures upon which the decision is based should do justice to the information and analysis that was carried out. In particular it should reflect the extent and type of uncertainty in the types of evidence available to the decision-maker.

A risk assessment is one special way of analysing the options. In a quantitative risk assessment the performance of the options, usually against one or a small number of objectives, is assessed under a range of conditions expressed in probabilistic terms. Appendix 1 is a critique of reliability theory and quantitative risk assessment.

Choosing a preferred option

The moment of choice is when one option is identified as being preferred. A choice represents a commitment to a particular option. Choice is made on the basis of evidence of how the options are expected to perform against the objectives. That evidence may appear in a number of different formats and is bound to be uncertain to a greater or lesser extent. The choice mechanism should reflect the nature and the uncertainty of the available information. This is the principal behind the contingency approach to choice, which is introduced in Chapter 6.

Implementing the solution

Choice is followed by action in order to implement the solution. The implementation phase represents the initiation of another process. That process may require resources, such as constructing built works, carrying out maintenance operations or collecting data. Physical changes to the infrastructure, as well as requiring resources, also constrain the options for future action. Physical changes cannot usually be reversed without commitment of further resources.

A decision need not necessarily represent an immediate commitment to initiating a process. It may equally well merely constrain options for future action. This is the case when a strategy is set for the management of a section of coastline. No physical changes to the infrastructure result immediately from the decision. However, the decision constrains options which are considered during future project-specific decision-making are constrained by the strategic decision. The strategy decision may be reversed but doing so may sacrifice the resources invested in the planning and analysis that was the consequence of the strategic decision.

Monitoring

The materialising solution will itself generate signals that will provide information on its performance and early warning of future problems, initiating another decision-making cycle. The extent to which these signals are detected and acted upon varies, as evidenced in Chapter 2.

The decision-making process discussed above does not proceed in isolation but interacts with processes from other domains. In Chapter 1 coastal management in the UK was presented as a hierarchy of decision-making (Figure 4.4). Although these levels stand out in the hierarchy it is more helpful to think of a continuum of levels of definition from vague high level processes to precise low level processes.

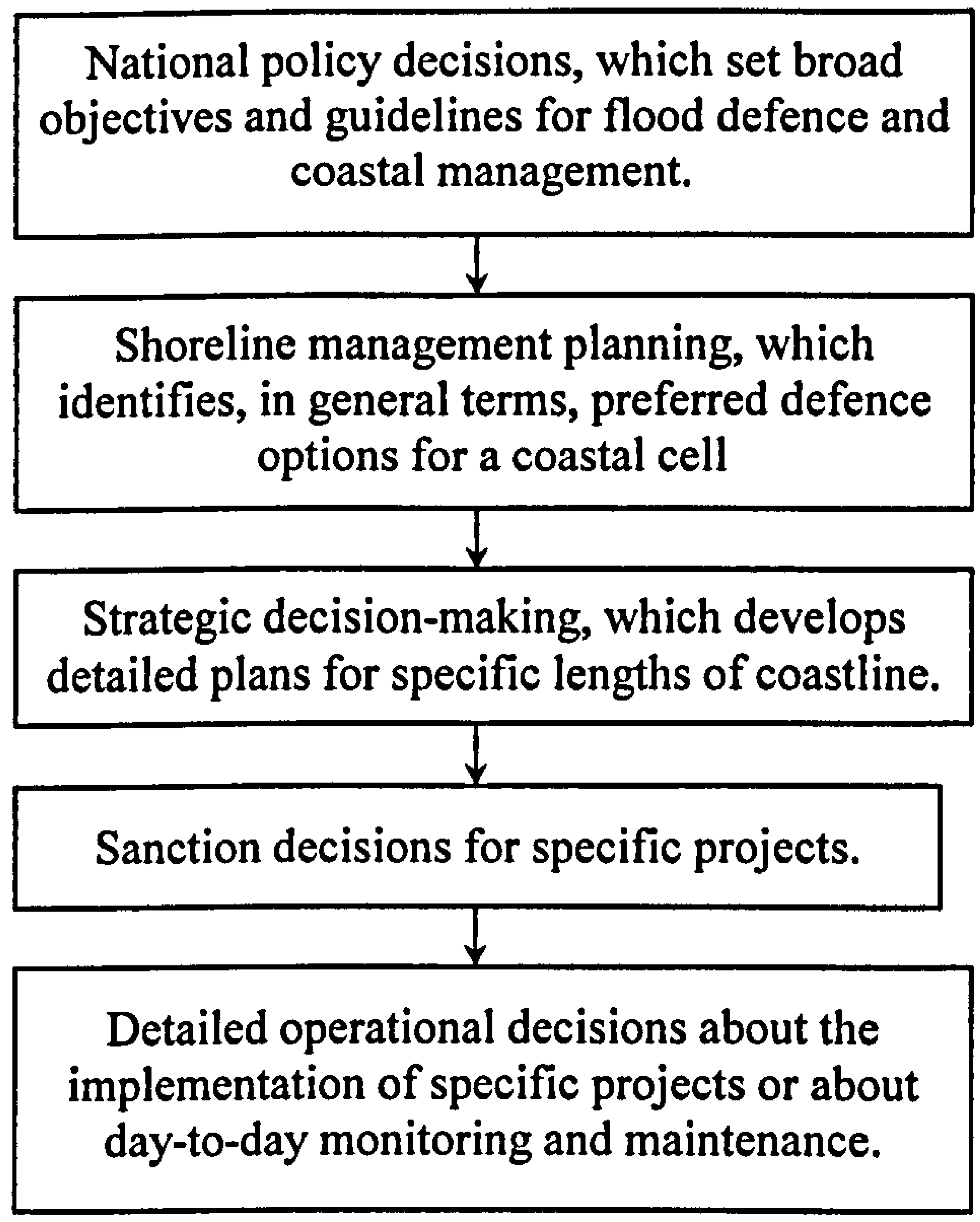

Figure 4.4 Hierarchy of decision-making for coastal management

4.6.1 Information and dependability in the decision-making process

Information plays a critical part in the whole of the decision-making process. This section looks briefly at the way information is used in decision-making and at the flow of information in hierarchical decision-making processes.

The information used in a decision is often uncertain in the following respects.

- Objectives are seldom explicitly articulated and may be conflicting. By expressing values in vague terms it is sometimes possible to avoid conflict in high level policy decisions, but when these vague high level sets of values come to be interpreted as objectives in the context of a specific decision the conflict becomes clear.
- Options are uncertain, often in significant respects. Indeed the exact nature of an option is only known when it materialises after the choice. For example the exact cost of a construction project is only known once the project is completed, not beforehand. Analysis of options is based on uncertain evidence. This evidence can come from a multitude of sources and will be of varying dependability.

The following discussion identifies key streams of information flow in the coastal management process.

Options

Coastal management options and plans become more specific, as one descends the hierarchy from policy to implementation. At policy level coastal management options exist as vague prototypes, which are not site-specific. Prototypes are the set of collectively accepted coastal management options (seawalls, beach nourishment, managed retreat *etc.*) from which specific designs are derived. This set of high level options evolves with time (though there has been little by way of fundamental innovation in the last fifteen years), but in coastal engineering practice designs are developed by refining, selecting and combining these established prototypes. At Shoreline Management Plan level a prototype is selected, but in vague terms on the basis of attributes that are described in linguistic terms or in approximate numerical terms. It is at lower levels in the hierarchy that the proposed management process for a specific stretch of coastline becomes more precisely defined. Thus as the hierarchy is descended the degrees of freedom of potential design options are gradually constrained. The scope of design decisions becomes both more constrained and more precise.

Objectives

Objectives are propagated down the hierarchy from policy towards implementation and operational decisions. The objectives of a particular decision are a consequence of decisions at a higher level in the hierarchy. At policy level objectives are necessarily stated in general linguistic terms. Descending the hierarchy, objectives become more site-specific. Objectives are decomposed into sub-objectives and specific design requirements. The process of selecting and refining objectives takes places in parallel with the process of selecting and refining decision options.

Design objectives and requirements relate to those decisions that have not previously taken place. As the design becomes increasingly defined the scope of objectives relates to the remaining degrees of freedom in the design.

Environmental information

Information relating to the environment in which the options exist (where the term 'environment is used in its most general sense) is accessed at all levels in the hierarchy. It is used to derive option attributes in order to make decisions. At higher levels it is usually manipulated in a more abstracted form. For example at a national level figures for erosion of saltmarsh or flood damage are referred to on a national scale. However, this abstracted information is an aggregation of various specific data collection and manipulation processes.

Data collection and manipulation are transformation processes. The output information may be reused at several points in the hierarchy of decision-making. It may be reused directly or used as input in a higher-level transformation process. Some environmental information processes support multiple decisions. For example the wave height estimation carried out for analysis of strategic options may also be used during detailed design and indeed during the planning of construction work. Thus although a process may appear to be unnecessarily sophisticated or precise for one particular decision it may be appropriate in view of its multiple purposes. Moreover, because processes may be reused in future, a more detailed level of analysis is justified for more promising options than for less promising ones, which is what often happens in practice.

Decision outcomes

Decisions at any level in the hierarchy influence future decisions at both lower and higher levels in the hierarchy. Decisions lower in the hierarchy are constrained by the framework that is set by higher decisions. Decisions lower in the hierarchy also represent a commitment to initiating some process whose existence then needs to be taken into account by higher level decisions. To illustrate this latter point consider the low level decision to repair a short section of seawall. The consequence of a decision is a change in the state of the infrastructure process and is thus a consideration which is taken into account, albeit perhaps in abstracted terms, at higher strategic levels. Even if low-level decisions have not resulted in physical changes on the ground they have still initiated processes which may constrain the scope of higher level decisions.

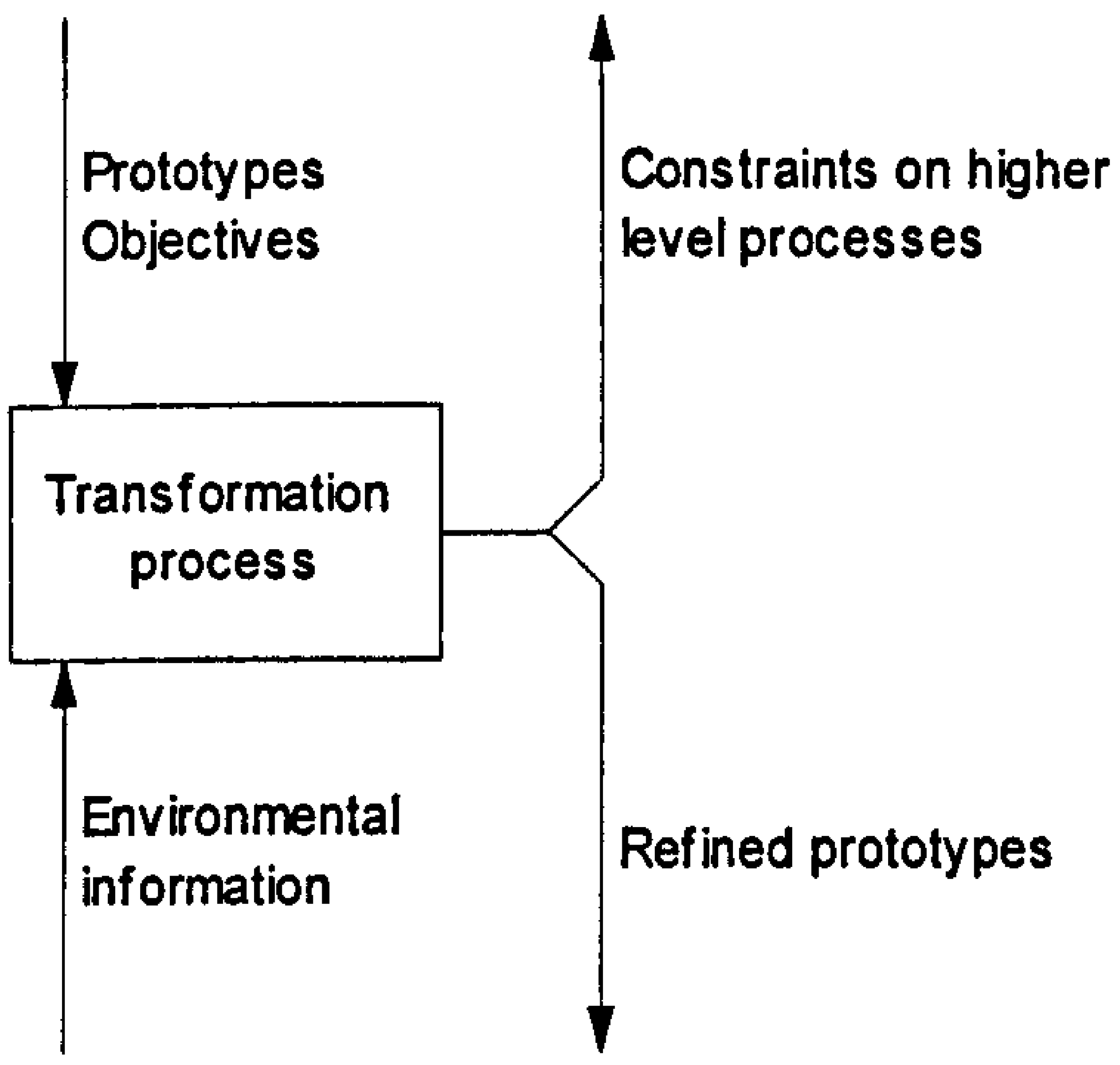

Figure 4.5 Generic hierarchical transformation process

Hierarchical transformation processes with the properties outlined above can be represented by assemblies of the objects shown in Figure 4.5. Figure 4.6 is an example of the hierarchical decision-making process for the Lincshore sea defence scheme, which is discussed in detail in Chapter 7, and is constructed from the objects shown in Figure 4.5. Figure 4.6 illustrates how the

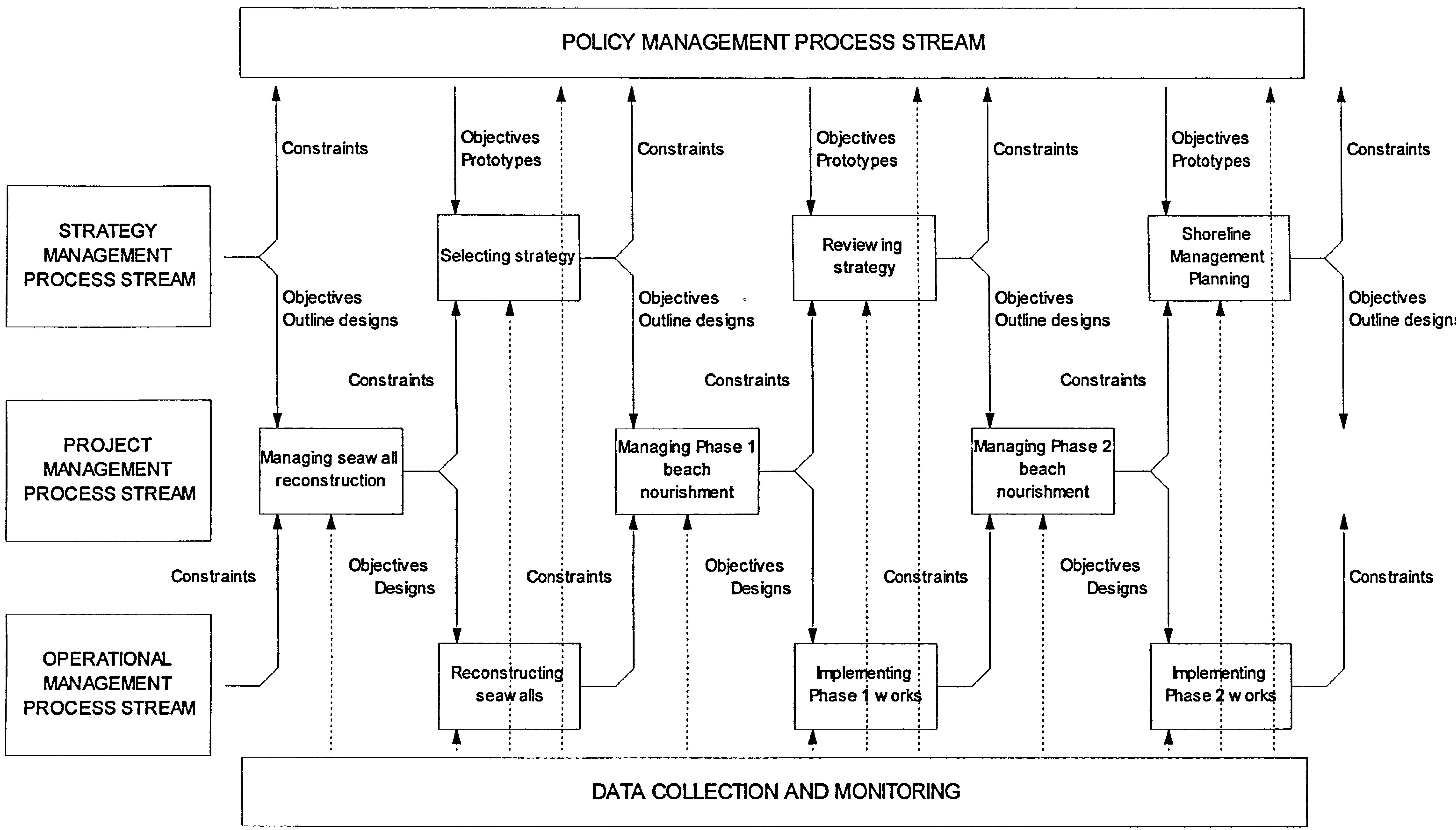

Figure 4.6 Hierarchical decision-making for coastal management

previous local policy of seawall reconstruction was gradually superseded by a policy of beach nourishment. The beach nourishment was implemented in two phases, the first phase being preceded by a major strategic study and the second phase by a review of the strategy.

4.7 Pedigree of evidence

Now that the context of decision-making is established, it is possible to review the nature and sources of uncertainty in a decision. Suppose a decision-maker has assembled some evidence about the extent to which a particular option will meet the decision objectives. That evidence will be input into the choice in order to determine which option is selected. Naturally the decision-maker will be interested in the quality of the evidence. The decision-maker should be concerned with two things

1. the information content of the available evidence, and
2. the degree to which the evidence is dependable in the context of the particular decision.

The first of these two types of evidence can be thought of as the primary evidence, which states how well the options are expected to perform against the objectives. The second is meta-evidence, which expresses the dependability of the primary evidence. Following Popper and Blockley, both of these bodies of evidence are necessary. A decision-maker may have a huge amount of evidence with high information content, but if that evidence is not dependable then it is of little use. On the other hand it is possible to make very vague and non-committal statements which may be highly dependable but are of little practical use because they contain so little information.

The notion of uncertainty adopted in this thesis embraces both the information content of the available evidence and the dependability of that evidence, in the context of decision-making. This combines two common views of uncertainty, which are sometimes seen as being conflicting.

One common view, influenced by communication theory (which is introduced below) is that uncertainty is a characteristic of information. The role of meta-information (evidence about dependability) is not accepted. A contrasting view, which is held by some researchers in the field of quantitative risk assessment, is that uncertainty is confined to issues of dependability (Burmaster and Wilson, 1996). Variations in information content are labelled 'variability'. Yet, because in conventional quantitative risk assessment only probabilistic information is admissible, the concept of variability does not capture the richness of the different types of evidence that may be available to a decision-maker (see Appendix 1).

It should be clear that on its own neither concept of uncertainty is sufficient to capture both of the characteristics of evidence that will be of interest to a decision-maker. Evidence that both has a high information content and is highly dependable is of great value to decision-makers. This evidence will be said to be of high *pedigree.*

Having defined the conditions for dependable evidence in Section 4.5, the information content of evidence will now be examined.

4.8 Types of uncertain information

Communication theory will be used to develop some basic concepts of uncertainty and information. Communication theory is a mathematical treatment of the information content (or conversely the uncertainty) of a signal. It says nothing about the dependability of the information carried by that signal. Thus, although communication theory leads to a classification of uncertainty, that classification is only partial since it leaves out crucial meta-level judgements about uncertainty to do with *incompleteness* and *irrelevance*, which have already been addressed.

Communications theory leads to a classification of types of uncertainty that can be apparent in an item of evidence, or body of evidence, which is relevant to a particular decision. The discussion therefore deals with situations where, for example, a decision-maker has some evidence about the behaviour of a decision option. This evidence may take very different forms, for example numerical model results or the testimony of experts. The discussion will begin however with the simple situation where the information is in the form of a string of digits.

4.8.1 Ambiguous information

Consider the case of a string of decimal digits, so each digit in the string may take a value between 0 and 9 inclusive. The situation is complete, because the possible outcomes are defined. Before a new digit is added to the string there is no information about what number value the digit will assume. After the digit has arrived all but one of the possible values have been eliminated. The uncertainty before the arrival of the digit is proportional to the number of possibilities, in this case 10. The uncertainty is totally resolved when one of the alternatives arrives. The amount of information conveyed by the arrival of the digit may be measured by the difference in uncertainty before and after its arrival (Klir and Folger, 1988).

The situation is uncertain in the sense that the possible outcomes are defined but, before arrival of the next digit, the state it will take is completely unknown. It relates, for example, to the situation where it is known that a revetted embankment has an earth core, but until an investigation has taken place it is unknown whether the core is clay or sand. The situation is therefore one of *ambiguity.* In terms of set theory, there is enough information to define a set of possible values but no information about which sub-set is being referred to. In the following chapter this situation will be addressed in mathematical terms in the discussion of fuzzy measures. Ambiguity is associated with one-to-many mappings.

4.8.2 Probabilistic information

A special case of ambiguity is when there is enough information to provide numerical measures of relative likelihood of the various possible states of information. These measures of relative likelihood are probabilities. The distribution of probability measures across the set of possible outcomes is a probability distribution.

The information to construct a probability distribution may come from statistical evidence or it may be based on subjective belief. Wherever the information to construct a distribution comes from and whatever that distribution happens to be, it represents a situation where there is less uncertainty than the situation of pure ambiguity. This situation will be referred to as being ambiguous in a *probabilistic* sense. Even if the probability distribution across the possible states of nature is uniform (the maximum entropy situation), knowledge of this probabilistic information still represents more information than the situation of pure ambiguity.

Popper defined randomness as 'lack of specific pattern', yet there is a lack of specific pattern in ambiguous information in general, not only when there is enough information to construct a probability distribution. Random models are a type of one-to-many relation. Indeed, it is more usual to think of randomness as class of models that ambiguous data may conform to, to a greater or lesser extent, than as a type of uncertainty.

The best known theory of communication is due to Shannon (1948), which is formulated in terms of probability theory. In Shannon's theory each of the possible states of a signal has a probability attached to it. Shannon entropy is a measure of information content. The Shannon entropy associated with the arrival of an item of information x from a set of possibilities X, is expressed by the function

$$H(p(x)\,|\,x \in X) = -\sum_{x \in X} p(x)\log_2 p(x),$$

so H is on $[0, \infty)$. This is the expected information content of a signal. The first term, $p(x)$, is the weighting which gives the expected value. The second term $\log_2 p(x)$ reflects the information content. Thus if the probability of occurrence of x is very high, say $p(x) = 0.99$, then the actual occurrence of x is almost taken for granted. Its occurrence does not come as a great surprise to the observer, so the observation that x has actually occurred contains very little information. On the other hand when the probability of x is very small then there is a great deal of surprise when it occurs. It carries a large amount of information.

Shannon's theory says nothing about the meaning of information carried on a signal. Because of this it is to Checkland (1988) a theory of data rather than a theory of information. However, it has already been stressed that the information, and conversely the uncertainty, in a signal is only part of the uncertainty in a decision-making problem. A full treatment of uncertainty will address the uncertainty implicit in the evidence, but will also address the meaning and context of the information through consideration of dependability and relevance.

4.8.3 Possibilistic information

Another case of ambiguity is when there is some information about the possibility of the potential outcomes. In the digital example there were ten equally possible outcomes, so it could be inferred that all of the other integers were impossible. A natural extension to this situation is to consider degrees of possibility, when some states are more possible than others.

Possibility is a weaker concept than probability. Zadeh's (1979) now famous example concerns the number of eggs eaten by Hans for breakfast (Table 4.1).

Table 4.1 Possibility and probability distributions for Hans' breakfast menu

No. of eggs	1	2	3	4	5	6	7	8
Possibility	1	1	1	1	0.8	0.6	0.4	0.2
Probability	0.1	0.8	0.1	0	0	0	0	0

Whilst it is possible that Hans could eat five eggs for breakfast it is very improbable. The probability distribution suggests that it is most likely that he will eat two. Whilst something is possible it may be very improbable. On the other hand an impossible event will also be an improbable one. The mathematization of possibility is discussed in Chapter 5.

4.8.4 Vague information

The next type of uncertainty is *vagueness*, which is imprecision of definition. Vagueness is associated with the difficulty of making sharp or precise distinctions in the world. There was no vagueness in the discussion of Shannon's theory. The item of information could take one of ten precisely defined values. However, in many situations, particularly those expressed in natural language, it is more difficult to classify a piece of information because it is imprecisely defined. When introducing fuzzy sets (which are discussed in detail in Chapter 5) Lotfi Zadeh (1965) argued that human beings naturally reason with vague concepts. We talk about a person being "tall" or "fat". These are vague concepts that nonetheless convey meaning. De Bono (1971) referred to these vague words as "porridge" words. The terms 'vagueness' and 'fuzziness' tend to be used interchangeably. Fuzziness and ambiguity are related concepts, but whilst fuzziness is to do with imprecision of definition, ambiguity is to do with absence of information about which of several, possibly precisely defined, states an item of information can take.

4.8.5 Conflicting information

Where more than one item of information relates to a given hypothesis, then it is possible that the information is conflicting. Conflict or dissonance is detected when the information is inconsistent. Conflict will be an indicator that the body of evidence is, to some extent, not dependable.

The information content of a body of evidence that is conflicting may be lower than if it was consistent because of the ambiguous nature of evidence. Therefore the addition of an item of information to a body of evidence need not always increase the total information.

4.9 Classifications of uncertainty

4.9.1 Summary of sources of uncertainty

Having addressed issues of dependability and information content it is now possible to propose a classification of uncertainty in order to establish a vocabulary for subsequent discussion. No claims are made about the definitive nature of the classification of uncertainty summarised in Figure 4.7, and alternatives that appear in the literature are discussed presently. Nonetheless, the classification does capture the aspects and structure of uncertainty that have proved to be important in this research, so it is argued that it is a viable classification in the context of decision-making in coastal engineering.

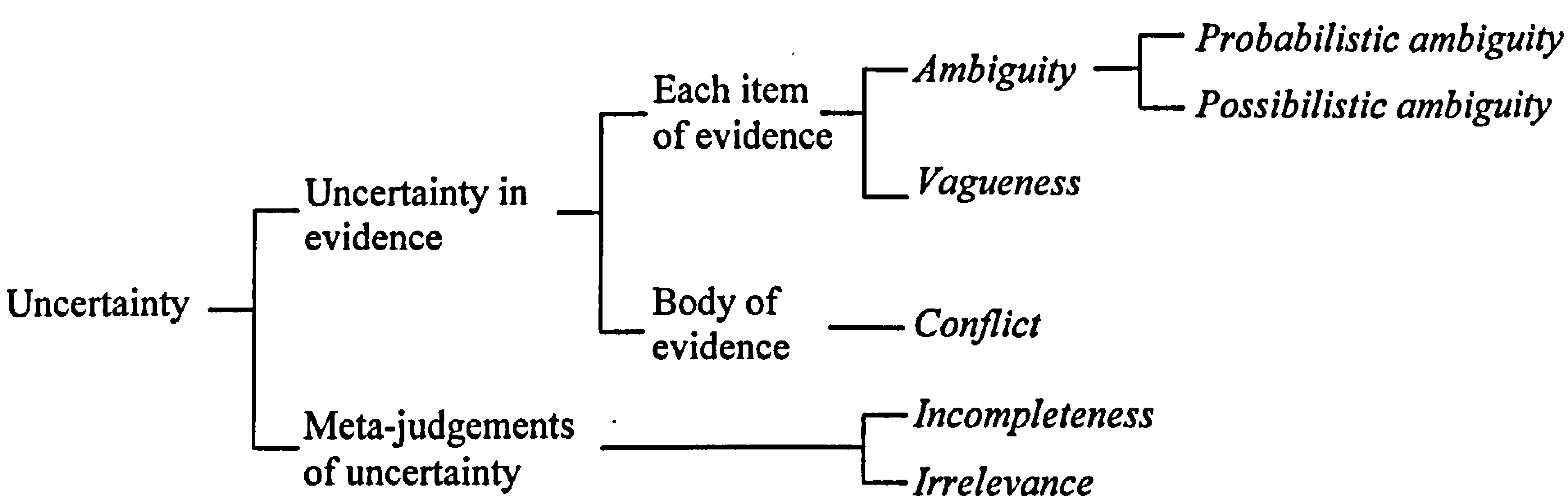

Figure 4.7 Classification of uncertain information

In keeping with the discussion of dependability and information content, uncertainty is divided into two aspects, uncertainty in the evidence and meta-judgements about the evidence. The former are those aspects of uncertainty that are apparent by examination of a single item of evidence or the body of evidence relating to the decision option. The latter relates to judgements about the dependability and relevance of the body of evidence. Because issues of dependability and relevance are reflections, in an open world, on the quality of the body of evidence, they are referred to as meta-judgements of uncertainty.

Uncertainty in the evidence itself can be subdivided into those types of uncertainty that are evident in a single item of evidence, and those that emerge by consideration of the whole body of evidence. Ambiguity and vagueness are characteristics of single items of evidence. In situations of ambiguity there will be some information about possibility and there may also be some information about probability. Issues of conflict emerge by consideration of the body of evidence.

This classification of uncertainty has been influenced by several published treatments of uncertainty (introduced below) and by the testing of these ideas in theory and on the case studies introduced in Chapter 7.

4.9.2 Some classifications of uncertainty from the literature

The systematic theoretical treatment of uncertainty has in historical terms been greatly, some would argue too greatly, influenced by the development of theories of probability and statistics. Shafer (1978) maintains that in the seventeenth century Bernoulli and Lambert distinguished clearly between degree of belief and the, at the time new, mathematization of games of chance. Unfortunately, the distinction has been rather hazy in the minds of some subsequent mathematicians.

Early in the investigation of Artificial Intelligence (AI) it became clear that handling uncertainty was one of the central and most challenging aspects of human reasoning (Shafer, 1987). The classifications of uncertainty which have proved most useful to this research have either emerged from or been influenced by the AI community. Most theorists working in the field of uncertainty have proposed their own definitions of uncertainty. These all testify to the many faces of uncertainty and also to the difficulty of applying semantic classifications to such an abstract concept. The more elaborate classifications endeavour to make quite subtle distinctions between different types of uncertain information. These are distinctions that may well have been apparent in the mind of the authors but are not always well captured by the semantic labels attached to them. On the other hand, the less complex classifications have the attraction of simplicity (which is essential if key concepts are to be communicated to practitioners) but may not do justice to some of the many faces of uncertainty. The definitions of uncertainty that have proved to be of most value in this research are briefly reviewed below.

Blockley (1996) classifies uncertainty as FIR: *fuzziness*, *incompleteness*, and *randomness*, three types of uncertainty that have been introduced above. However, the situation in which information is ambiguous, yet not probabilistic, is not addressed. In the digital example from communication theory, the problem was a completely closed idealisation, so incompleteness was not an issue. The states were precisely defined but there was not necessarily any information about the relative likelihood of the possible states. The situation was clearly uncertain, but not in any sense captured by Blockley's classification.

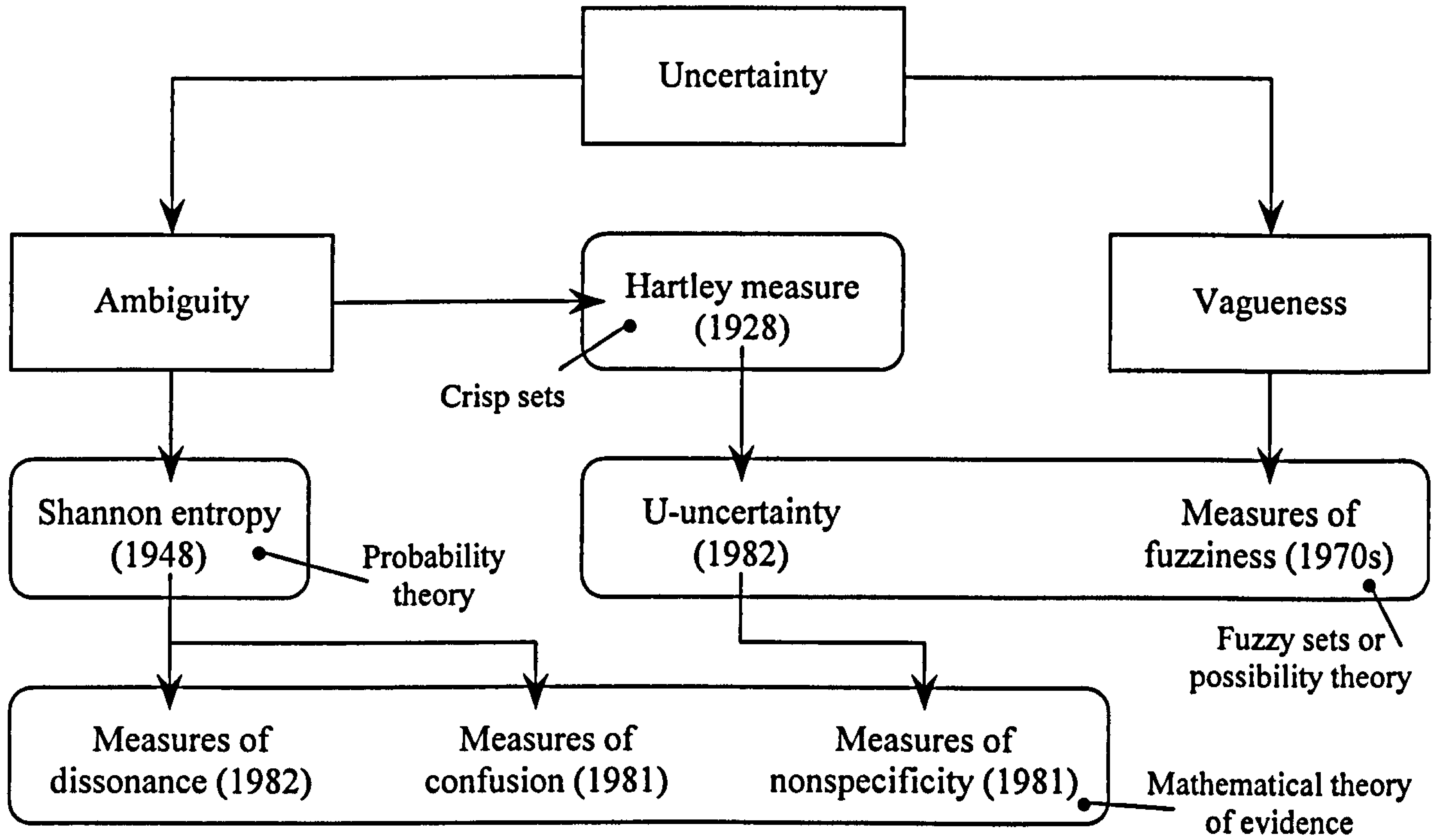

Figure 4.8 An overview of uncertainty measures (after Klir and Folger, 1988)

Klir and Folger's (1988) view of uncertainty is summarised in Figure 4.8 and corresponds closely with a characterisation by Perincherry *et al.* (1993). Their high level classification of uncertainty into the broad categories of ambiguity and vagueness also coincides with Lai and Ayyub (1994). The classification is based on the scope of the mathematical theory of information. As well as probability, it introduces, as types of ambiguity: non-specificity, dissonance and confusion. Their definitions are based on a set theoretic conception of the possible states that an item of information may assume.

- *Non-specificity* is related to the size of the subsets that are designated as a prospective state that the information may assume.
- *Dissonance* is associated with conflicting information.
- *Confusion* is associated with the number of possible states that the information may take.

Based, as it is on the mathematical theory of information, Klir and Folger's view does not include issues of incompleteness and irrelevance, which depend on external judgements of the problem context.

Smithson (1988) regards ignorance as being a more general concept than uncertainty. His "taxonomy of ignorance" is illustrated in Figure 4.9. Ignorance is treated as "the absence or the distortion of 'true' knowledge, and uncertainty as some form of incompleteness in knowledge".

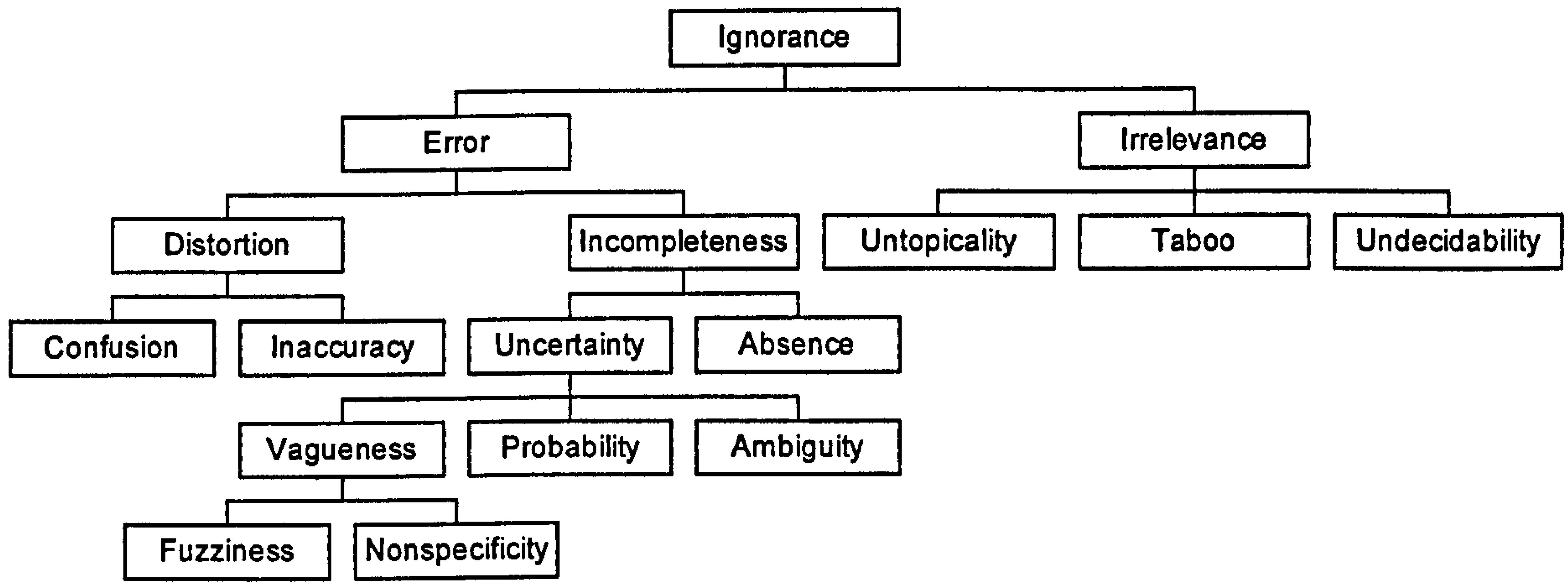

Figure 4.9 Smithson's "taxonomy of ignorance" (after Smithson, 1988)

Uncertainty is thought of as being incompleteness in degree, and absence as being incompleteness in kind. His classification of uncertainty contains categories that have been identified already. The view proposed above is that any body of evidence is inevitably incomplete in an open world, though under some circumstances it may be justifiable to behave as if it were complete. Incompleteness and information uncertainty are seen as joint concepts rather than one being a subset of the other. Smithson's taxonomy does not include the concept of inconsistency, conflict or dissonance, though in his text he addresses these important issues in some detail.

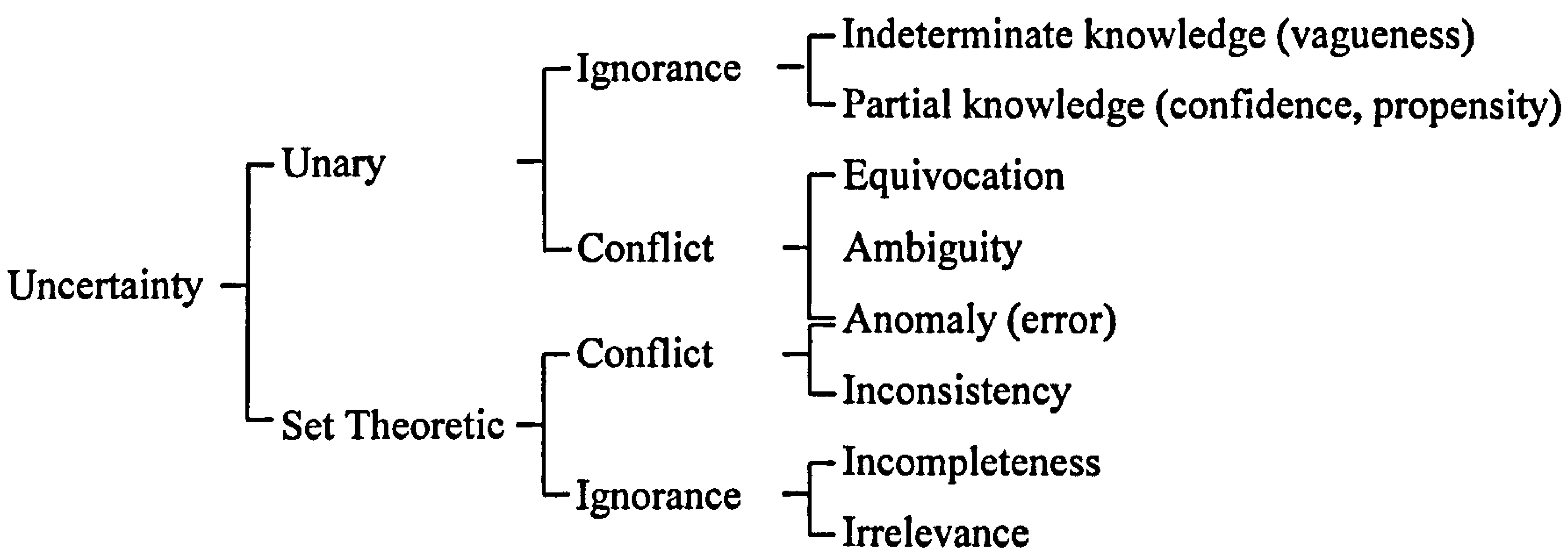

Figure 4.10 Krause and Clark's uncertainty classification for AI systems

Krause and Clark (1993) propose their own classification of uncertainty for AI (Figure 4.10). The high level distinction is between aspects of knowledge that are unary, that is to say apply to individual propositions, and those aspects that are set theoretic, applying to sets of propositions. At the second level the distinction is between ignorance (lack of knowledge) and conflict. Whilst the meaning of conflict is reasonably well established, ignorance is a less meaningful term. Ignorance is to do with lack of knowledge, but in that sense it is not clear how it is distinct from uncertainty in general. Indeed Smithson and Krause and Clark seem to use the terms almost interchangeably.

The main distinctions between the classification proposed in this thesis and Krause and Clark's low-level classification (Table 4.2) is their subdivision of conflict. Their definition of equivocation implies conflict but the term itself is associated with ambiguity.

Table 4.2 Explanation of low level uncertainty concepts in Krause and Clark (1993)

Krause and Clark (1993) term	Meaning	Relation to classification proposed in this thesis
Indeterminate knowledge (vagueness)	Subsumes fuzziness and non-specificity	Vagueness
Partial knowledge	Subjective belief or frequency	Probability
Equivocation	Proposition may simultaneously be both supported and discredited	Possibility, Conflict
Ambiguity	Alternative non-overlapping interpretations of the meaning of the proposition	Ambiguity
Anomaly (error)	Incongruous, but not formally inconsistent, propositions.	Conflict
Inconsistency	Propositions cannot be simultaneously true	Conflict
Incompleteness		Incompleteness
Irrelevance		Irrelevance

4.10 Models and relations

So far in this chapter the uncertain nature of the evidence used in a decision has been discussed. A classification of uncertainty, which combines issues of information content in a body of evidence, together with meta-judgements of dependability and relevance, has been introduced. These are the first steps towards coherent concepts of uncertainty management, which are the end point of the chapter. So far, however, the link between all of the available information that may be of relevance to a decision, and a measure of the extent to which the options satisfy the objectives that can be used as a basis for choice, has yet to be made. Models are used to make this link, in other words, to deduce option attributes that can be used to distinguish between options. Models can be thought of in the most general sense as relations between different items or clusters of information.

4.10.1 Relations

Figure 4.11 shows a relation between two clusters of information, which are relevant to a decision. It indicates that some pattern has been identified which puts set A in some correspondence with set B. The relation is of fairly general type because it contains one-to-many and many-to-one mappings.

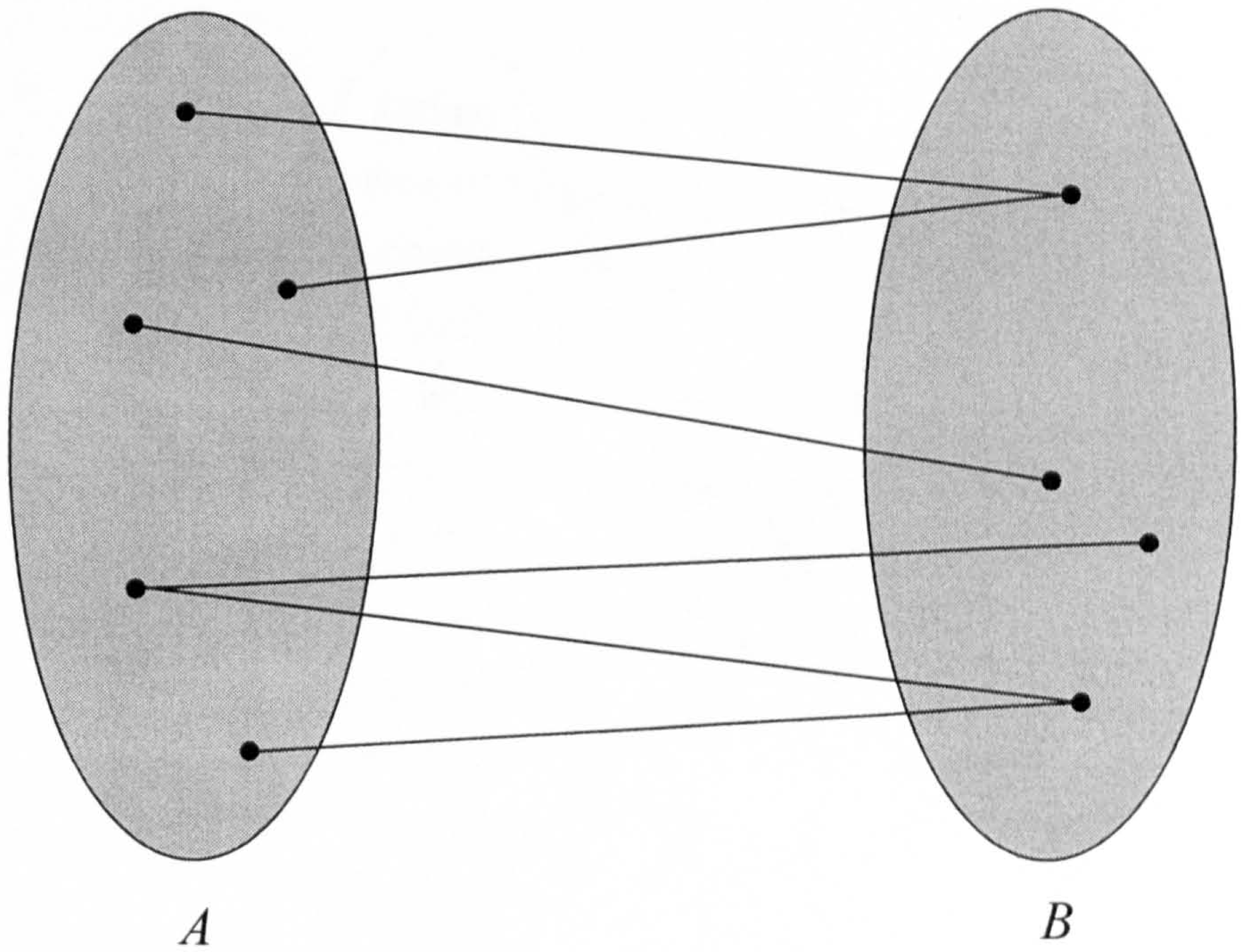

Figure 4.11 A relation

Many familiar models are cases of Figure 4.11. For example the relation between wave length L and period T in small amplitude linear wave theory is the functional relation

$$L = \frac{gT^2}{2\pi}$$

which is a one-to-one mapping and can be drawn as a monotonic function on a Cartesian graph of L against T. The same theory gives a many-to-one mapping of water surface elevation z against time t:

$$z = \frac{H}{2}\sin\frac{2\pi}{T}t$$

where H is the wave height. These are two examples of the many deterministic relations which abound in engineering science. They represent a precise correspondence between two items of information.

One-to-one and many-to-one mappings represent important groups of patterns that are applicable when there is precise and complete information. When the available information does not satisfy these conditions it may be possible to construct a one-to-many relation. So for example, having studied information provided by historic calendars for a number of years, one could construct the relation shown in Figure 4.12. To use more mathematical terms, Figure 4.12 established a relationship between the set of twelve months in the year and the set of integers $\{28,29,30,31\}$.

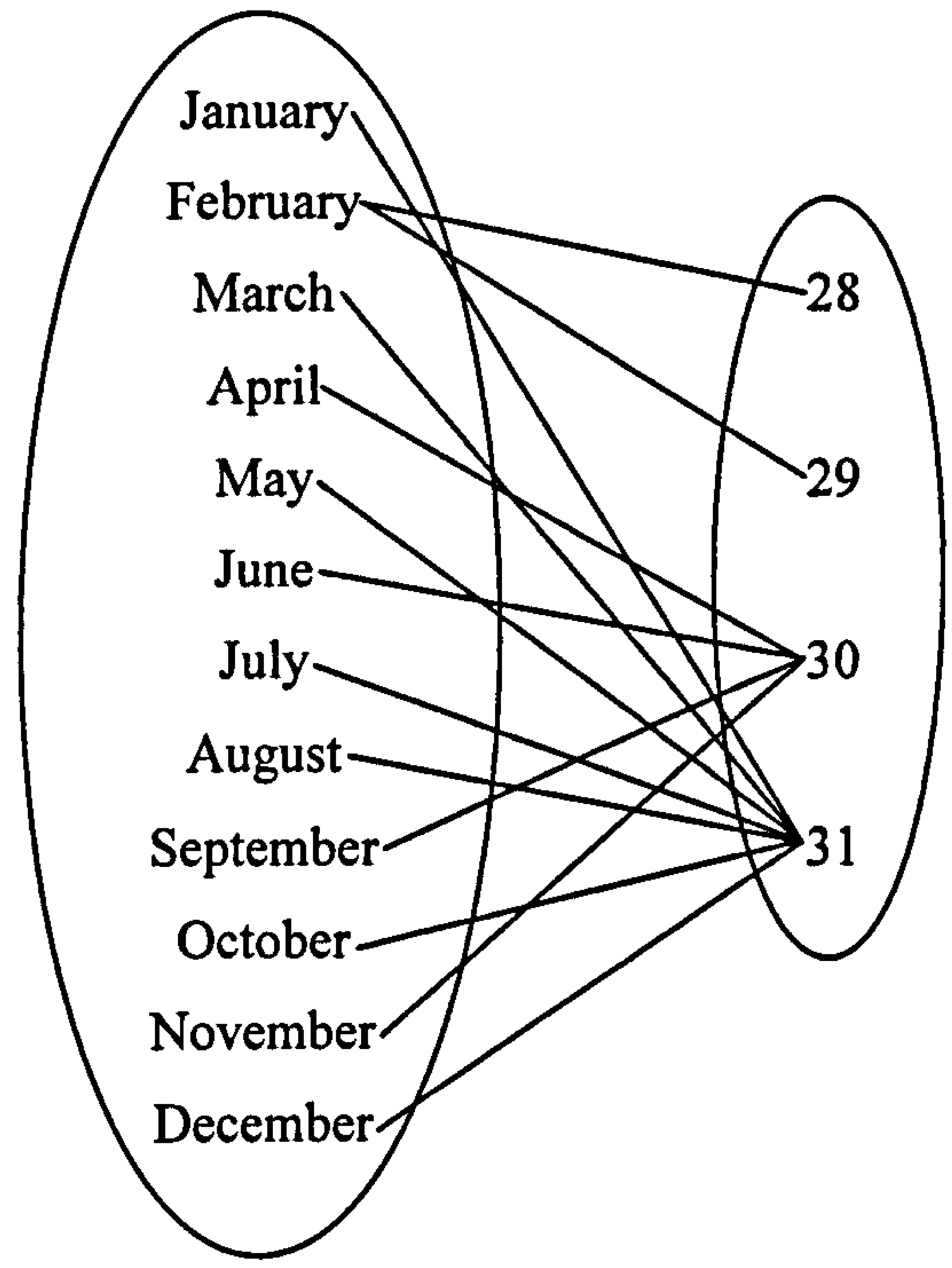

Figure 4.12 Many-to-many mappings

Probability theory addresses one-to-many mappings by providing some additional information about the relative likelihood of the different relations. Thus Figure 4.12 indicates that the month of February may have 28 or 29 days, but says nothing about the relative frequency of the two outcomes. Probability theory goes a stage further by constructing a distribution across the two outcomes. This expresses the relative likelihood of the possible outcomes. However, there may be circumstances when there is enough information to construct a many-to-many mapping without constructing a probability distribution. One may have been told about the existence of a phenomenon called a leap year without any knowledge of the relative frequency with which leap years come round. The ambiguity associated with one-to-many mapping is therefore more general than probabilistic mapping and the associated concept of randomness. Note also that constructing a uniform distribution across the possible outcomes implies significantly more information than merely stating that the mapping is of a one-to-many of many-to-many variety. Probability theorists may use maximum entropy arguments to justify constructing a uniform distribution in the absence of any further information, but such arguments are only necessary from a point of view that probability is the only mathematization of uncertainty. It should be clear that the type of information that lends itself to probability theory is but one of many possible information situations that decision-makers have to make sense of.

If the available information is expressed in fuzzy rather than precise terms then it is possible to construct a fuzzy relation between two sets. A fuzzy relation is an extension of the principles introduced above because it admits the possibility of a partial relationship between information. In

Table 4.3 The number of days in February

	28	29	30	29
Possibility	1	1	0	0
Probability	0.75	0.25	0	0

Table 4.3 although the relation between the month of February and the number of days in the month was ambiguous, it was an exact relation. Month names and integers are precise concepts. The idea of a month being "fairly February" or an integer being "very 28" is nonsense. However, it has already been explained that many of the concepts used in human reasoning and expressed in natural language are vague or fuzzy. Thus observations at a Faculty Board meeting may suggest a relation between grey hair and baldness, in the form, for example, "academics who are *fairly bald* are also *fairly grey*". The general relation could be shown in a Cartesian space of baldness and greyness shown in Table 4.4. The relation shown in Table 4.4 is possibilistic, as in the first row of Table 4.3, but in the case of Table 4.4 there is a partial possibilistic relation. In the following chapter it will be shown how in mathematical terms probability is a more strict theory than possibility, so just as determinism is a special case of probability, probability is a special case of possibility.

Table 4.4 Fuzzy relation between baldness and hair colour

	Not grey	**Slightly grey**	**Fairly grey**	**Very grey**	**Completely grey**
Not bald	1.0	0.8	0.5	0.2	0.1
Slightly bald	1.0	1.0	0.8	0.5	0.3
Fairly bald	0.3	0.5	1.0	1.0	0.5
Completely bald	0.0	0.1	0.5	0.8	1.0

Incompleteness has already been identified as an important characteristic of information. Suppose, that the analysis of months and days conducted in order to formulate Figure 4.12 had mistakenly identified only three possibilities for the number of days in the month {28,30,31}. In that case, regardless of the strength of relation, the information which the model provides about the month of February will be incomplete. If the set of possible outcomes is logically exhaustive then it becomes impossible to miss an outcome in this way. So the relation for the number of days in the month could have been defined on the infinite set of integers greater than zero. Many continuous probability distributions are defined on infinite spaces. It is always possible to construct a logically complete set of propositions by defining a 'catch-all' set for 'any other proposition(s) not otherwise included.'

A more profound issue of incompleteness relates to whether the model or relation is a function of all of the relevant phenomena. The number of days in the month is related to only one other item of information, what month it is. The hair loss and greyness of academics is presumably related to genetic, lifestyle and, perhaps, other phenomena. Indeed it may be interesting to construct a relation between hair loss and some other variable, say number of children. Whilst the first type of incompleteness introduced above was to do with the extent of the axes on relation space, this type of incompleteness is to do with the dimensionality of the relation. At a most extreme level it is at least conceivable to construct a hyperspace of relations between all of the items of information that are received by an individual. As well as being beyond the processing capacity of any individual there is still the distinct possibility that there are phenomena that have not been envisaged.

4.10.2 The influence of relations on uncertainty in decision-making

The results obtained from a model will in general contain uncertainty. Figure 4.13 illustrates how degree and type of uncertainty is influenced by

- the uncertainty in the input information and
- the nature of the relation.

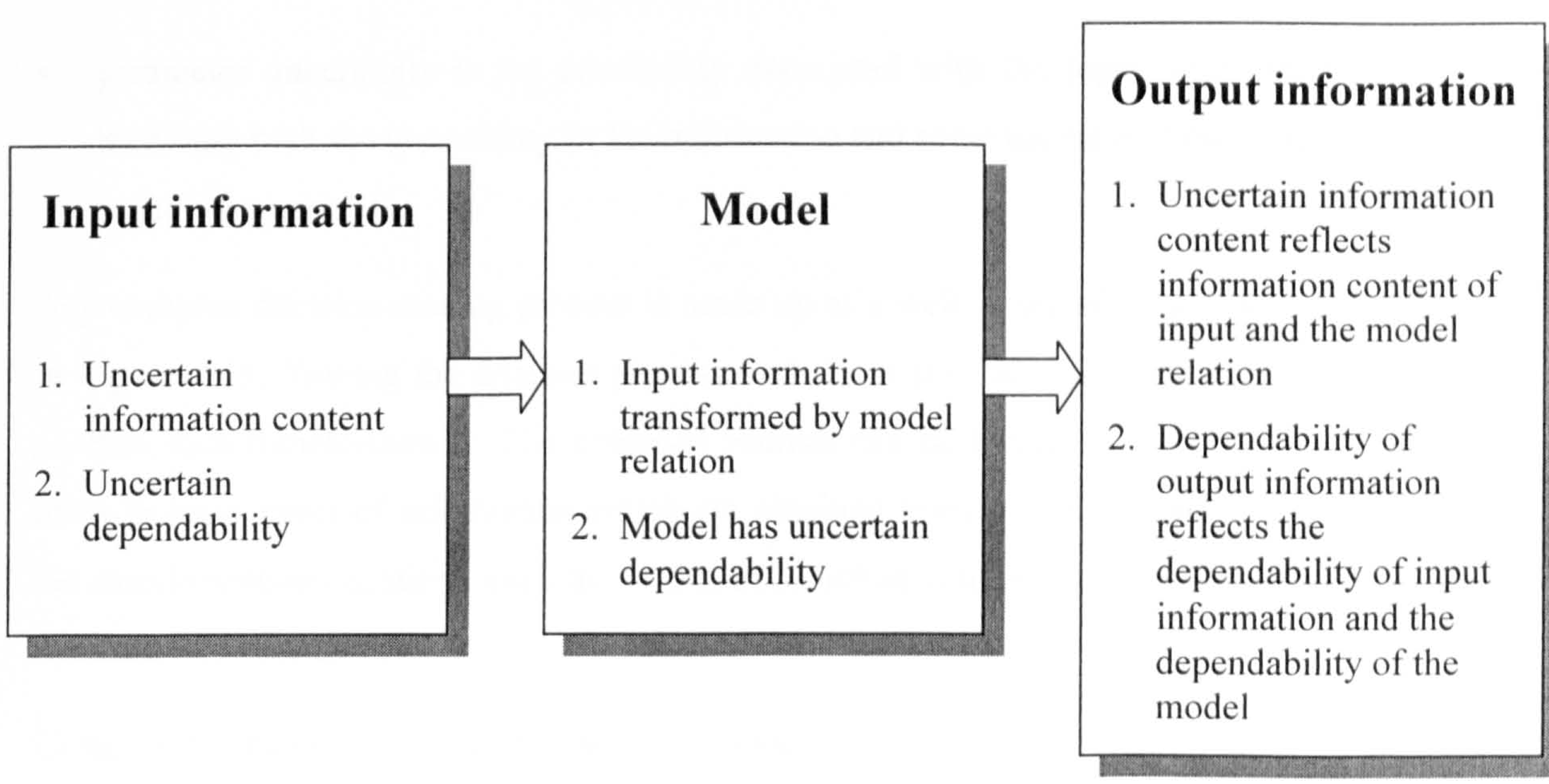

Figure 4.13 Sources of uncertainty in a generic modelling process

A deterministic relation with precise inputs will generate precise outputs. If probabilistic information is input into a deterministic relation then the result, which may be obtained analytically or numerically, will generally be in a probabilistic format. A relation that transforms precise information onto a fuzzy set is known as a fuzzy membership function. It has been shown how one-to-many mappings will generate ambiguous information even if the input information is precise. Joint probability relations will allow a probability distribution to be constructed across that ambiguous information. Thus in many situations the type of uncertainty in the output information will be different to the type of uncertainty in the input.

Equally important is the dependability of the model, which will influence the dependability of the output information. Thus, regardless of the format of the output information, be it precise, probabilistic or fuzzy, the dependability of the output information will be influenced by the dependability of the input information and the dependability of the relation.

Thus, whilst the model output parameters provides evidence about the extent to which a given option satisfies the decision objectives, the dependability stream of evidence provides information about the degree to which the output parameters are providing dependable information for decision-making. Both of these streams of evidence may contain different types of uncertainty, be it fuzzy, probabilistic, possibilistic or conflicting.

Blockley (1980) refers to 'systems' and 'parameter' uncertainty, and some risk assessors refer to 'model' and 'parameter' uncertainty (Finkel, 1990, Carrington, 1996) where:

- systems/model uncertainty is the uncertainty associated with the relation (presumably including both the information uncertainty which may be introduced by the nature of the relation, and the model dependability);
- parameter uncertainty is the uncertainty associated with the input information (presumably including both the uncertainty in the information and some measure of the dependability of the input information).

Any complex decision-making process is made up of a web of transformations of the type shown in Figure 4.13. Tracing the analysis process back from the decision, each input will come from another such transformation. The evidence sources can be traced back as far as is appropriate, often to basic items of information which are obtained from empirical studies. Tracing forwards, the transformations continue until the output information is in an appropriately condensed state to form the basis for choice.

Consider for example a wave overtopping model (Figure 4.14). There are three inputs, each of which is of uncertain dependability. There will also be diverse evidence relating to the dependability of the model. Schum (1994) refers to this evidence relating to the dependability of the model as 'ancillary evidence'. For the overtopping model this may include

- evidence from model verification studies;
- general evidence concerning the dependability of parametric models derived from laboratory model tests;

- general evidence concerning the dependability of overtopping models based on average discharge rates rather than wave-by-wave discharge volumes;
- judgements concerning the similarity between the seawall in question and the idealisations used in the model;
- and so on.

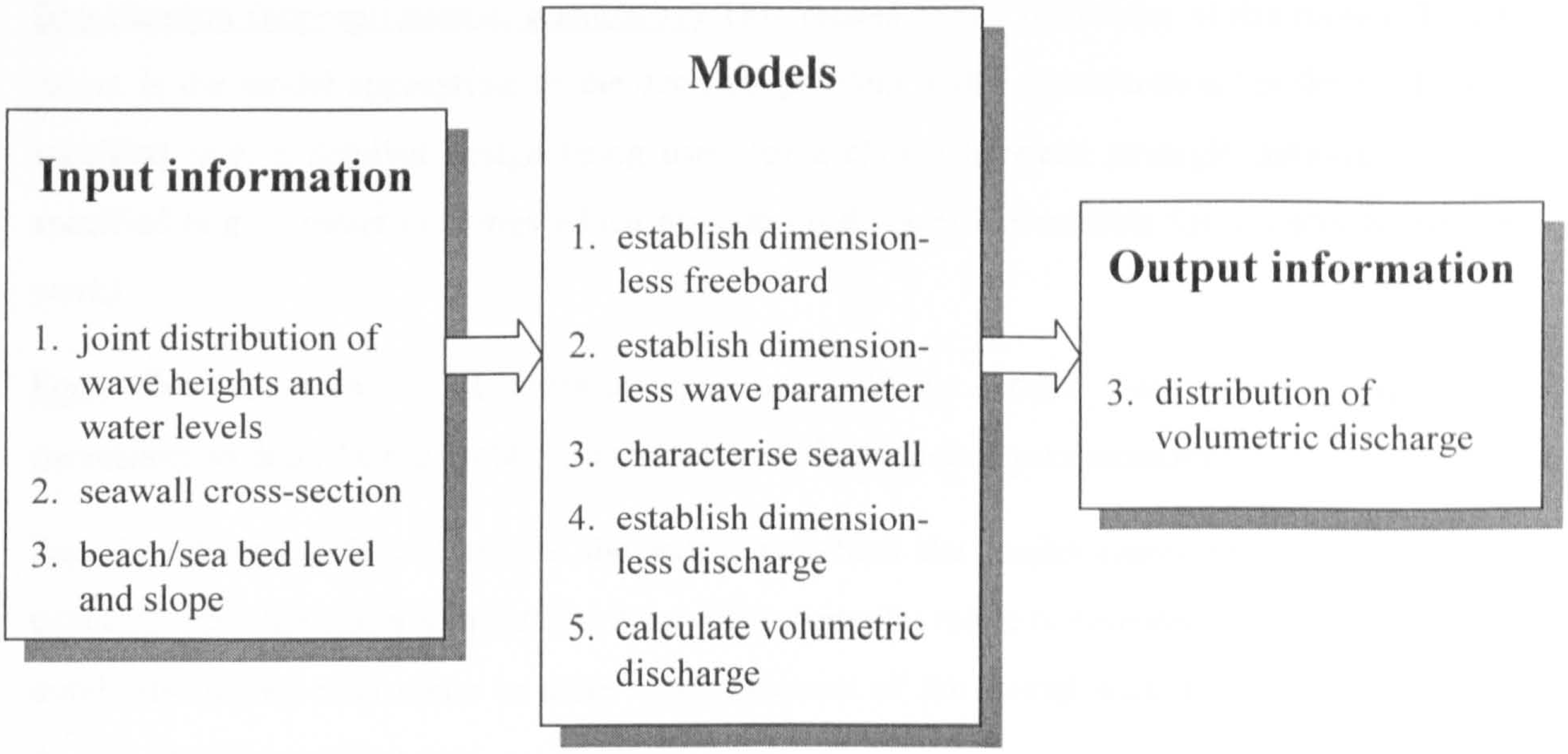

Figure 4.14 Transformation model for calculating wave overtopping

The dependability of the input parameters can be established through the analysis of their origin and subsequent transformations that they have undergone. For example the dependability of the joint distribution of waves and water levels will depend on the wave modelling and statistical analysis of water levels, which will in turn depend on the wind data, from which the wave model was hindcast, and the water level data, which were used for statistical analysis. The ancillary evidence relating to the dependability of the model by contrast relates not to a traceable deductive process but to a process by which models are generated.

Models that are independent of the observer are powerful in the important respect that they are transferable and can form a collective tool. However, in many of the more challenging and important decision-making situations the ideal of a model that is so well accepted that it is independent of the individual who uses it may not be achievable.

Blockley (1992*b*) classifies qualities of scientific and engineering models in terms of

- function,
- form,
- grounding,

- specification,
- applicability,
- outcome.

These can be used as a basis for gathering the ancillary evidence relating to the dependability of a model.

Specification (appropriateness, granularity) This relates to the resolution of the model. To what extent is the model appropriate to the decision problem under consideration? Is the model over-specified (*e.g.* a detailed design being used for a choice between strategic options) or under-specified (*e.g.* contract drawings which provide insufficient information for a tender to price the work)

Form This relates to the attributes and parameters of the model. Does it employ appropriate parameters to describe the problem? Is the choice of parameters parsimonious?

Function/Outcome This relates to the extent to which the model represents behaviour that is expected from the entity in question. It may, for example, relate to response to loads or long term durability. It may also relate to more human aspects of the model such as phasing of works or decision-makers preferences.

Applicability This is the applicability of the model for the purpose for which it is employed. Are input parameters within the range of applicability?

Grounding. This can relate to the robustness of the model. To what extent is the model robust to fluctuations in the input parameters or in the environment?

By systematically addressing these issues it is possible to develop a view on the dependability of the relations used in a modelling process.

4.11 Uncertainty management

Enough concepts are now in place to begin to develop ideas about how engineers can manage uncertainty in decision-making processes. It has been explained that decision-makers will be interested in the information content of the evidence they have about decision options and objectives. The nature of this information, and the implicit uncertainty, is a consequence of the original information sources and the transformations the information has been through in order to condense it into an appropriate format for a choice.

The nature of the uncertainty in this primary evidence is specific to the particular decision. Some information may be expressed in deterministic terms and brought together by a series of precise relations. For example cost estimates are often expressed in precise terms and are brought together by simple arithmetic operations that are one-to-one mappings. It is also customary to express some evidence in probabilistic terms, though it is still manipulated with one-to-one mappings. This is the case for the standard loss-probability analysis of expected flood damage (MAFF, 1993). The primary evidence will often also include expert judgements, which may be expressed in fuzzy terms or mapped onto subjective probabilities or interval numbers. Evaluation against other criteria, for example political criteria, will involve even more vague beliefs. Evidence of this type will be manipulated, at least implicitly, with the more fuzzy relations of mental models. The dependability of those models will often be questionable. This vague evidence is nonetheless relevant to the success of the decision.

It has been argued that equally important is the meta-information about the dependability of this primary evidence. The information about dependability is essential to provide an indication of the closeness to the truth of the primary evidence. Without information on dependability, it is irresponsible to take a decision, behaving as if the primary information were true. In practice engineers intuitively take account of the dependability of the processes for which they have been responsible when they are making decisions. In Chapter 1 it was argued that in the increasingly complex process of coastal management, where evidence may come from different project teams or even different organisations, a decision-maker may find it very difficult to keep track of the dependability of the processes leading up to the decision. Under conditions of increasing complexity, and increasing demands for efficiency and transparency, intuition may not be an appropriate guide.

The approach to uncertainty management in this thesis therefore focuses on assembly and manipulation of ancillary evidence relating to the dependability of models and primary evidence. The need for appropriate expression of uncertainty in the primary evidence is recognised, but is both case-specific and is more readily recognisable, so is being increasingly addressed in coastal engineering, particularly from the frequentist and subjective probability points of view (Hall and Meadowcroft, 1995, Meadowcroft *et al.*, 1997).

The representation of dependability has received much less attention, perhaps because its importance is not immediately obvious and because it represents a rather more complex challenge. However, the foregoing discussion of information, models and decision-making forms the basis of a method of explicitly representing the dependability of the information used in a decision. The ancillary evidence of dependability may contain quantitative data, for example from information model verification studies. However, the bulk of this information will be the beliefs of the experts involved. To understand the dependability of the primary evidence requires an analysis of the process by which that evidence was obtained and manipulated. By modelling the process of gathering and manipulating the information and, on the basis of available evidence, attaching measures of dependability to each process it is possible to generate a coherent overview of the dependability of the evidence upon which a decision is based.

4.11.1 Hierarchical modelling of the process of assembling and manipulating evidence

The process of assembling evidence in the lead up to a decision and to some extent condensing that information to give a succinct indication of the way each option is expected to perform can be mapped on to a hierarchical structure. Figure 4.15 shows how a hierarchical model can be used to trace the assembly and manipulation of evidence. The hierarchical model is in fact an excerpt from one of the case studies that will be discussed in Chapter 8. The excerpt shows the process of assessing one of the attributes (the hydrodynamic impact of a flood defence scheme on the estuary regime) of one of the options. The dependability of the high level evidence, upon which the decision is based, is a function of the dependability of all of the lower level processes. The hierarchical model is can be used to track the dependability of all of these contributing processes. The hierarchical model works back from the evidence required for the decision and forward from the information available to the decision-maker. The assembly and manipulation should be driven

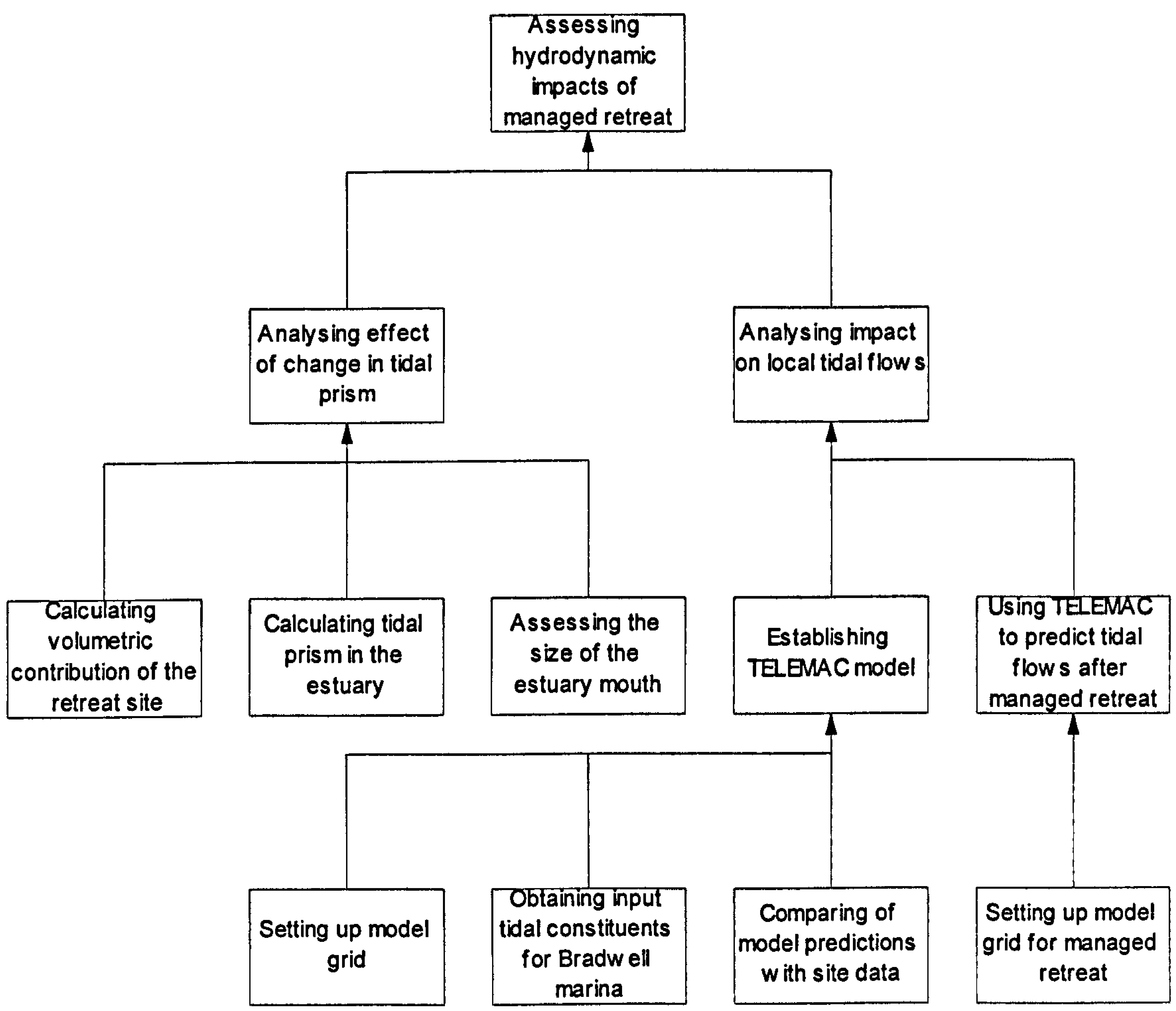

Figure 4.15 Example of a simple hierarchical model of the process of assembling evidence

by the needs of the decision-maker. Engineering modelling, and in particular coastal modelling, are fascinating activities in their own right, sometimes so much so that the process of assembling and analysing evidence sometimes seems to be driven by the needs and interest of the modellers rather than by the ultimate decision. On the other hand, decision-makers cannot plan the assembly of evidence in abstract terms. They should be clear about what evidence and analysis is available, the costs associated with it and the benefit in terms of uncertainty management. Hierarchical modelling of this type helps decision-makers to work forwards from the available, or potentially available, evidence to form a balanced view of the uncertainty in the decision.

Hierarchical structures allow ordering of information according to the granularity of uncertainty. At the top of the hierarchy are vague high level concepts like "Assessing hydrodynamic impact of managed retreat", which are progressively decomposed to more precise concepts lower in the hierarchy, which can extend down as far as is appropriate.

The information transformations that take place at each stage in the process hierarchy are of the type shown in Figure 4.13. Consider, for example the process of identifying a preferred design option on the basis of benefit-cost ratio. This requires some estimate of cost and some estimate of benefit for each of the options. These are the inputs. The output is the name of the preferred option (Figure 4.16). Without estimates of benefit and cost it is impossible to identify the preferred option on the basis of benefit-cost ratio. So the process of "Identifying preferred option" shown in Figure 4.16 has two sub-processes both of which are logically necessary. The data requirements of the decision process dictate which sub-processes are relevant *i.e.* the flow of information betrays what sub-processes are necessary for the transformation process to be successful.

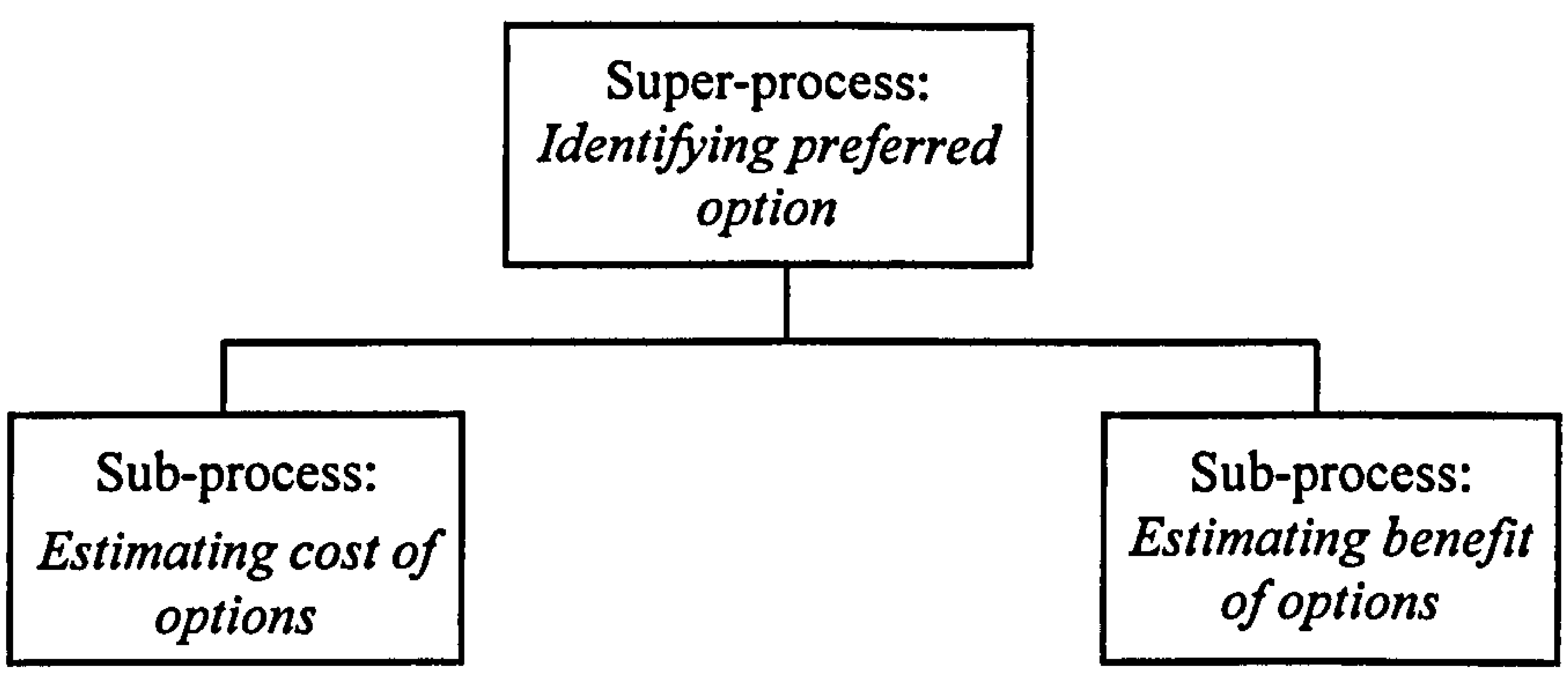

Figure 4.16 Process hierarchy for calculating benefit-cost ratio

With reference to Figure 4.13, at each stage in the process hierarchy:

- Each process represents some relation, which is used to manipulate uncertain evidence.

- The evidence that is input to the process comes from lower process in the hierarchy. The hierarchy is constructed to an appropriate level of detail. As discussed at the start of this chapter, at the deepest level information originates as sensory signals.
- The relation may preserve the type of uncertainty in the input into it (for example in a one-to-one mapping of probabilistic information) or it may change the type of uncertainty (for example in one-to-many mapping of precise information)
- The evidence input into the process will have some dependability associated with it.
- The process itself will have some dependability associated with it.
- The dependability of the output evidence will be a function of the dependability of the input evidence and the dependability of the process itself.

A process model of the type shown in Figure 4.15 forms the basis for mapping uncertainty in the processes leading up to a decision. It provides an overview of the sources of evidence and the modelling and manipulations that the evidence has been through. It therefore fills some of the objectives that were set for uncertainty management and decision support in Chapter 1 and expanded upon in Chapter 2. Establishing an appropriate model structure is a most important first step towards decision support. However, whilst uncertainty has been a central theme in this chapter, an appropriate way of representing the uncertain evidence about process dependability has only been hinted at. Uncertainty representation is the topic of the next chapter.

4.12 Conclusions

1. A philosophical discussion has led to the sceptical view that the external world is unknowable. This view is fundamentally different to a treatment of uncertainty based on the Laplacean ideal of a well-ordered, and even deterministic, world.

2. Humans make sense of the signals they seem to be receiving from their senses by constructing models. However, a discussion of Popper's philosophy of science has demonstrated that no model, theory or hypothesis can be proved to be true, it can only be refuted. A theory with high information content is highly testable. It is also more likely to be falsified than one with low information content. Theories with high information content are useful because a large number of propositions can be deduced from them.

3. Some, if not many, of the theories used by engineers have been falsified. Engineers should be more concerned with the dependability of their models in the context of a specific decision. Truth is a sufficient but necessary condition for dependability. A dependable model must be well tested and the test results must indicate that, in the context of the decision, the theory is not refuted.

4. Incompleteness is an inevitable consequence of constructing synthetic models from empirical information. Models that explicitly recognise incompleteness are referred to as 'open world' models. Measures of incompleteness will, at best, be judgements based on partial evidence. Dependability is a measure of the incompleteness of a theory in the context of a specific decision.

5. An analysis of uncertainty has indicated that a single item of information may be uncertain in an ambiguous (possibilistic and/or probabilistic) or vague/fuzzy sense. Consideration of a body of evidence may reveal conflict. Meta-judgements of uncertainty in the evidence are needed to identify incompleteness and irrelevance.

6. The 'pedigree' of evidence is a combined measure of its information content and dependability. Both information content and dependability need to be addressed in order to generate a balanced view of uncertainty.

7. Models and relations may be thought of as mappings. The most general type of mapping is a many-to-many mapping. Deterministic, probabilistic and fuzzy relations have been introduced. The uncertainty in the information output from a model will be a function of uncertainty in the input information and the uncertainty introduced by the relation.

8. Decision-making can be idealised as a cyclic process, which comprises problem definition; generating and analysing options; choice of a preferred option; implementation and monitoring. In practice it often involves looping between sub-processes which proceed in parallel until the moment of choice. The dependability of a decision will be a function of the dependability of each of the sub-processes. This research is focussed on uncertainty in the process of analysing options.

9. Uncertainty in a decision is implicit in the evidence about options and objectives, but is also a consequence of the dependability of the process of assembling and manipulating evidence. Engineers intuitively take account of dependability, but in complex, multi-disciplinary situations, with increasing demands for efficiency and transparency, intuition may not be a good guide. There is therefore a need for an explicit way of modelling the process of obtaining and manipulating evidence in the lead-up to a decision in order to give an overview of dependability issues. Hierarchical models of process dependability form the basis of uncertainty management. Hierarchical models enable ordering of information at appropriate granularity of uncertainty.

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CHAPTER 5

Developments in the representation of uncertain knowledge using numerical structures

5.1 Objectives of Chapter 5

- to establish criteria against which competing theories of uncertainty representation can be judged;
- to introduce the main approaches to uncertainty representation that use numerical structures;
- to evaluate various methods to uncertainty representation against the criteria established at the start of the chapter;
- to set out new developments of Interval Probability Theory for uncertain inference;
- to implement the new developments in Interval Probability Theory for modelling uncertainty in process hierarchies.

5.2 Introduction

Key concepts of process modelling and uncertainty were developed in the Chapters 3 and 4 respectively. In this Chapter approaches to uncertainty representation for uncertainty management in coastal engineering are identified and developed. Specifically, a methodology for uncertainty handling in process hierarchies is developed. This leads to the development of a software tool for representing the uncertain dependability of the process of assembling evidence in the lead-up to a decision. The tool is demonstrated in Chapter 7 by application to UK coastal engineering.

The introduction to uncertainty in the previous chapter indicated that uncertainty was a rich and complex phenomenon. Uncertainty was described as being a characteristic of information, and implicit in all of the models that are used to manipulate evidence. It seems strange therefore that the precise theoretical constructs of mathematics should be applied to an open world problem like uncertainty. Therein lies one of the fundamental problems associated with all of the mathematizations of uncertainty introduced in this chapter: mapping complex real world situations onto precise mathematical vocabulary. The problem can be thought of as one of mapping the semantics of an uncertain problem onto a mathematical syntax.

There is a long history of using mathematics to represent uncertainty, beginning with the early mathematizations of games of chance by Cardano, Galileo, Pascal and others (Hacking 1975, Shafer, 1978, Bernstein, 1996). Probability proved to be a powerful tool in the context of games of chance, as it subsequently did in other situations that are well modelled as random processes. The success of probability theory in these rather specific contexts has led to expectation that it will be equally powerful in more complex domains. These are expectations that probability theory has to some extent failed to live up to (see Lindley (1982), Zadeh (1986*a*) and Shafer (1987) for arguments for and against the primacy of probability). Indeed it is the failure of probability theory to cope with complex, vague, open world evidential situations that has been one motive for the development of generalisations and extensions of probability theory, which are described in this chapter. The problem being tackled here is representation of uncertain evidence or beliefs, a problem which differs from betting on a game of chance, even though some modern researchers (Raiffa, 1968, Lindley, 1971) still argue that these two situations can be treated in the same way.

Uncertainty representation in coastal engineering has to date focussed almost entirely on conventional probability and statistics. The more recent uncertainty representations introduced in this chapter have emerged outside coastal engineering, most notably in Artificial Intelligence. Whilst the underlying uncertainty issues are in many senses generic, the development of these methods has inevitably been driven by needs which are distinct from the needs of coastal defence systems. This chapter therefore examines these methods in the context of coastal defence. In particular the aim is to address the specific problem, which has been identified in the last Chapter, of representing the dependability of the process of assembling and manipulating evidence in the lead-up to a coastal engineering decision. In order to achieve that aim this chapter is structured as follows.

- Section 5.3: The requirements for a calculus for modelling uncertainty in process hierarchies are identified.
- Sections 5.4 to 5.9 and 5.11: The main approaches to representing uncertainty with numerical structures are critically examined. The approaches are introduced as a family of probability calculi, paying particular attention to the similarities and differences between the various approaches.
- Section 5.10: Interval Probability Theory (IPT) is introduced. New developments to IPT for logical inference under uncertainty are proposed and explained.
- Section 5.12: The various mathematizations of uncertainty, including IPT are evaluated against the criteria set out in Section 5.3. On the basis of this evaluation, IPT is identified as an appropriate calculus for modelling uncertainty in process hierarchies.

- Section 5.13: The implementation of IPT for uncertainty modelling in process hierarchies is described.
- Section 5.14: A theoretical approach to combining probabilistic data with uncertain evidence about process dependability expressed as interval numbers is introduced. This is a tentative solution to an outstanding theoretical problem and is introduced as a pointer towards further work.

5.3 Requirements for an uncertainty calculus

In the context of this research the requirements for an uncertainty calculus are that it should provide a useful support to reasoning with uncertainty in the complex situations identified in Chapters 1 and 2. In particular it should be capable of handling uncertain evidence about process dependability. This is a challenging task. At issue here is the complex problem of uncertain reasoning with vague and often incomplete evidence. It can by no means be assumed that classical theories of probability and statistics, which currently hold sway in coastal engineering, will provide an appropriate solution. A broad range of theories has therefore been examined in order to develop a mathematization that is suitable for the task.

Lemmer and Kanal (1988) contend that "there continues to be no consensus concerning the best approach to uncertainty... Moreover there is no agreement on how to measure 'best'." The criteria proposed below are a suggestion of how 'best' could be measured in the context of hierarchical process modelling of the dependability of evidence. As Krause and Clark (1993) suggest, there is no single best approach. Any solution will be a compromise based on judgement of how well the approach fulfils a set of requirements that are considered to be desirable.

The following desiderata distil the issues raised in the preceding chapters into a set of criteria for judging representations of uncertainty with numerical structures. Some of the desiderata have been drawn from Bonissone (1992). Others have emerged during the course of this research. The list has the fundamental problem that it is expressed in natural language, so in order to judge a mathematical syntax one has to attach some semantic meaning to that syntax. There is scope for a range of interpretations of the syntax. The extent to which a given syntax meets the stated criteria will depend on those interpretations. Nonetheless, producing a coherent list does seem to be a useful first step towards developing an appropriate uncertainty representation.

1. *The syntax should be capable of modelling uncertainty in hierarchical process models.* A hierarchical structure has been identified as an appropriate representation of the process of assembling evidence in the lead-up to a decision. The mathematical syntax should conveniently map onto such a structure. It should be able to propagate uncertainty through the hierarchy in order to identify sources and sensitivities to uncertainty.

2. *The syntax should reflect the types of uncertainty in the available evidence.* The range of different types of uncertain information was explained in Chapter 4. The syntax should reflect the nature of the uncertain information in the available evidence. Evidence about process dependability will on the whole be in the form of expert judgements, so the syntax should be appropriate for manipulation of vague expressions and there should be an explicit recognition of ignorance to allow non-committal statements. It is important that the syntax should represent an open world view, recognising that expert knowledge of any situation is inevitably incomplete. There are some events, which may be of great importance, which are not foreseen at the time of a decision. In mathematical terms, a syntax must be able to admit the existence of events which are outside a preconceived population of possible events, or knowledge which is outside a preconceived knowledge base. Conflict and inconsistency can be very revealing characteristics of uncertain information. It should be possible to identify and represent these characteristics. In summary the features of uncertainty that characterise expert judgements of dependability are vagueness, ambiguity, conflict and incompleteness, and it should be possible to reflect these in the mathematization.

3. *The axioms should not be so weak as to provide inferences that are of limited practical use, yet on the other hand it should not artificially constrain the problem, implying less uncertainty than is in fact the case.* The mathematizations introduced in this chapter on the whole are based on various generalisations or alterations to the axioms of probability theory. In all of these axiomatisations there is an inevitable compromise between having strong axioms, which are an inappropriate idealisation, and having weak axioms, which result in meaninglessly uncertain results.

4. *The syntax should be able to represent varied dependency relationships between evidence.* Dependency is an important issue in complex evidential situations, so it is important that a mathematical syntax can easily represent difference levels of dependency between evidence. The issue of dependency is explained in more detail in the discussion of IPT in Section 5.10.1.

5. *The syntax should reflect a range of inferential relationships.* The evidential support for a hypothesis depends not only on the weight of evidence but also on the relevance of the evidence to the hypothesis. The syntax should be able to represent a range of possible relevance relationships.

6. *The syntax should be capable of being implemented in a reasonably straightforward manner that will make it accessible to coastal engineering practitioners.* Although the underlying mathematization of uncertainty may be complex it is important that having been implemented in a decision support tool it is a practical and accessible support to decision-makers. This is

function of the user-friendliness of the tool, but is also a consequence of the mathematization itself.

7. *The syntax should enable straightforward elicitation of reasonably bias-free judgements.* The structure of a syntax influences the way knowledge is elicited from experts. The experts should be asked to articulate their knowledge in a way that maps conveniently onto the syntax. They should externalise their judgements in as full a way as possible without suppressing relevant issues.

5.4 Probability theory

Of all the methods for handling uncertainty, probability theory has by far the longest tradition and it is the best understood. That of course does not imply that it should be beyond criticism as a method of handling uncertainty. It does, however, mean that it is relatively well tested and well developed and can act as a standard against which other more recent approaches may be measured.

The concept of probability may be defined and interpreted in several different ways, the chief ones arising from the four approaches discussed in Sections 5.4.1 to 5.4.4 as follows.

5.4.1 The classical approach

A game of chance has a finite number of different possible outcomes, which are assumed to be equally likely. The probability of an event (*i.e.* particular outcome of interest) is then defined as the proportion of the total possible outcomes for which that event does occur. Evaluating probabilities in this framework involves counting methods (*e.g.* permutations and combinations).

Pioneering work on mathematizing games of chance was undertaken in the late sixteenth and early seventeenth century by Cardano, Galileo, Pascal and others (Hacking, 1975). However, these early researchers did not mention the word 'probability'. For medieval and Renaissance thinkers, probability belonged to the realm of opinion and argument, where the random was quite out of place (Bernstein, 1996).

5.4.2 The frequency approach

The frequency approach relates to the situation where an experiment can be repeated indefinitely under essentially identical conditions, but the observed outcome is random (not the same every time). Empirical evidence suggests that the proportion of times any particular event has occurred *i.e.* its relative frequency, converges to a limit as the number of repetitions increases. This limit is called the 'probability' of the event. Proponents of the frequency approach, R.A. Fisher having been perhaps the most notable, take the view that probability is about countable events.

5.4.3 The subjective approach

In this approach probability is used as a belief. An event is a statement, and the (subjective) probability of the event is a measure of the degree of belief that the subject has in the truth of the statement. Suppose that a 'prize' is available if the statement does turn out to be true, the subjective probability can be thought of as the proportion of the prize money that the subject is prepared to gamble in the hope of winning the prize. It is often by analogy to games of chance that subjective probabilities are constructed (de Finetti, 1937, von Neumann and Morgenstern, 1947). Any assignment of subjective probabilities can be permitted in principle, provided they satisfy the requirements of coherence. To be coherent it should not be possible to construct a 'Dutch Book' against the individual assigning the probabilities.

Subjective probabilities may be referred to as *epistemic* probabilities. The epistemic probability of a proposition is simply a measure of belief in the proposition. It was not until the late seventeenth century that the idea of chance or randomness was associated with probability, which was an attribute of opinion (Shafer 1978). Jacob Bernoulli made the link between games of chance and probability. In his *Ars Conjectandi,* completed by his nephew Nicolaus and published posthumously in 1713, Bernoulli explains that probability is a degree of subjective certainty – a measure of our knowledge. In calculating probabilities, Bernoulli used the methods of the theory of games of chance. However, Bernoulli did not insist that the probability of a thing and the probability of its opposite add up to one, an interpretation which, nonetheless, has since become axiomatic in probability theory.

Subjective probabilities are a function of our current state of knowledge and should be updated in the light of new information. The conventional procedure for updating prior probabilities in the light of new information is attributed to Thomas Bayes, hence the customary label of 'Bayesian' probabilities.

It is the subjective interpretation of probability that is of the most relevance to the theoretical developments in this chapter.

5.4.4 The logical approach

Formal logic depends on relationships of the kind $A \rightarrow B$ ('A implies B') between propositions. The logical approach to probability generalises the concept of implication to *partial* implication; the conditional probability of B given A measures the extent to which A implies B (Keynes, 1921, Jeffreys, 1948). Carnap (1950) constructed a systematic version of this analysis, but found that in effect there is an infinite number of ways of assigning probabilities (Cohen, 1989).

The logical approach to probability is of relevance to the theoretical developments in this chapter, the aim being to develop a mathematical vocabulary for reasoning under conditions of uncertainty.

However, the syntax developed in this chapter will be in terms of set theory rather than propositional logic. Every set theoretic statement has its counterpart in a logical statement (Klir and Folger, 1988). Similarly there is an intimate link between fuzzy logic and fuzzy set theory. Therefore it is to some extent a matter of taste whether syntax is introduced in set theoretic or logical terms. The set theoretic approach has been adopted here.

A common proposition will be of the type "process X is dependable," or "evidence Y is dependable." Evidence will be accumulated to refute or support the proposition, which may also be referred to as a 'conjecture' or 'hypothesis'. These propositions and the evidence relating to them will be mapped onto a set-based syntax.

5.4.5 Mathematical lattices

It is assumed that the reader is familiar with set theory. At this stage it is worth revising the concepts of power sets and mathematical lattices. The power set is the set of all the subsets. For example the power set of the set of integers $\{1,2,3\}$ is $\{\{1,2,3\},\{1,2\},\{1,3\},\{2,3\},\{1\},\{2\},\{3\},\varnothing\}$. There are 8 elements to this subset. A set with n elements has 2^n elements in its power set. The subsets with only one element are referred to as singletons.

The power set can be conveniently represented in a Hasse diagram (Figure 5.1). The top element is the universal set $\{1,2,3\}$ and the bottom element is the null set $\varnothing$. Elements lower in the diagram are joined to elements higher in the diagram if lower elements are subsets of the higher elements. The elements of the power set are therefore partially ordered by a criterion of set inclusion.

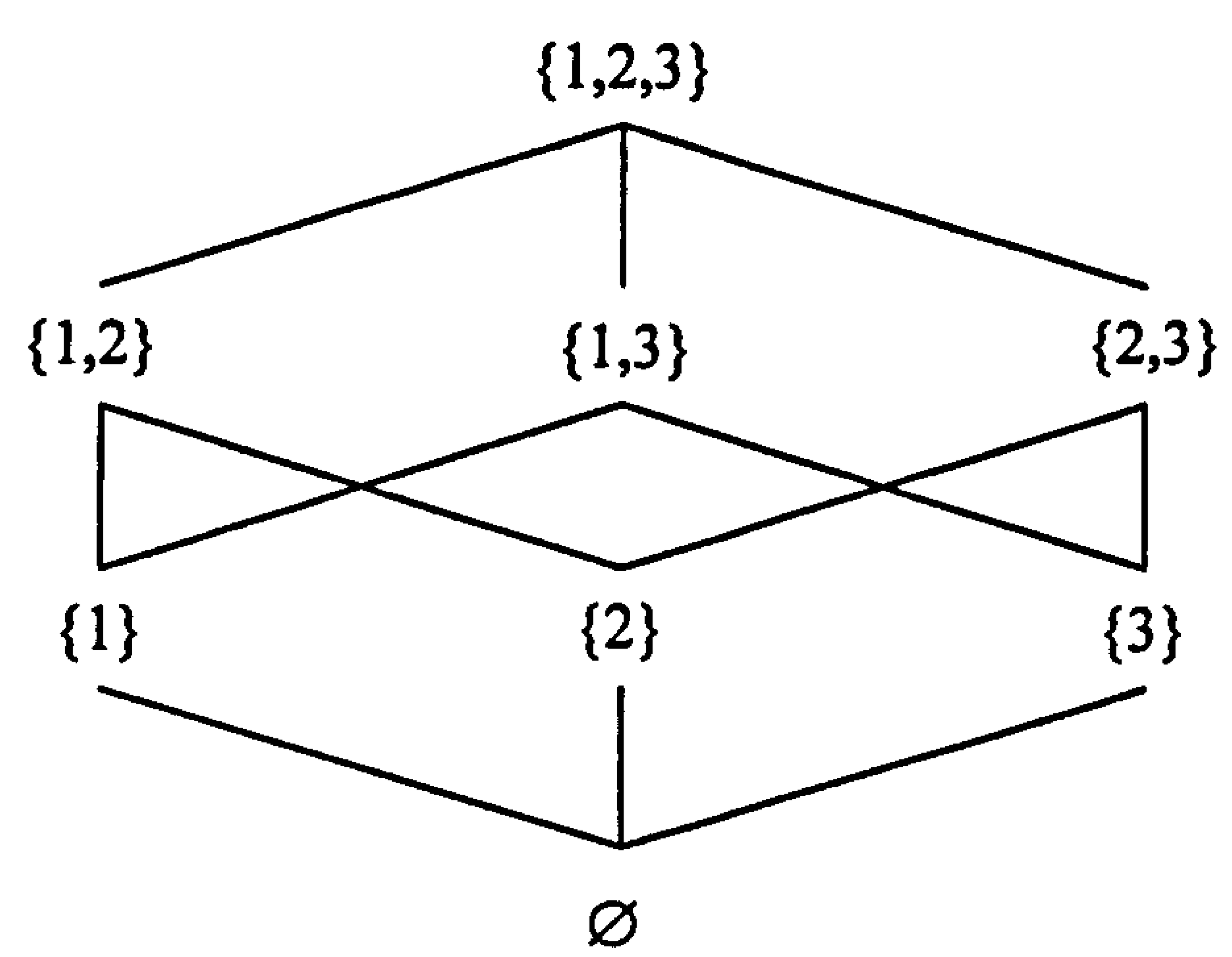

Figure 5.1 Hasse diagram of the power set of the set {1,2,3}

5.4.6 Axiomatisation of probability theory

The discussion of probability theory is developed using the abstract mathematical concept of a set rather than the more familiar probability concept of events. Conceptualising probability exclusively in terms of events can cause difficulties when the same mathematical syntax is then developed for use as a measure of belief.

In the following axiomatisation the universal set X contains all possible elements of concern. Probability can then be defined as a function

$$p : X \to [0, 1].$$

The axioms governing the behaviour of this function for a set A are

Axiom 1: $p(A) \geq 0$

Axiom 2: $p(X) = 1$

Axiom 3: *For any infinite sequence of disjoint sets*

$$p(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} p(A_i)$$

The third axiom is the additivity axiom (which is expressed in terms of an infinite series of sets for the sake of generality) from which it is deduced that

$$p(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} p(A_i)$$

and specifically

$$p(A) + p(\overline{A}) = 1.$$

The additivity axiom is also known as the law of excluded middle. It requires that if $p(A)$ is known then $p(\overline{A})$ is also known. Whilst in games of chance this is to be expected, in the context of uncertain beliefs human subjects may object that the additivity axiom implies more knowledge than they would otherwise be prepared to admit.

5.4.7 Conditional probability and Bayes theorem

The conditional probability of a set A given membership of set B is defined as

$$p(A \mid B) = \frac{p(A \cap B)}{p(B)}.$$

This is an expression of the dependency relationship between two sets. The concept of conditional probability is an important one in the context of uncertain inference. It suggests that probabilities are not absolute but always depend on the context and the information that is available at the time of the statement.

Bayes theorem provides a mechanism for updating probabilistic statements in the light of new information. It is central to the subjective concept of probability. Expressed in mathematical terms it states that

$$p(A_i \mid B) = \frac{p(B \mid A_i)p(A_i)}{p(B)},$$

which is a simple consequence of the definition of conditional probability. The customary interpretation placed on this equation is that $p(A_i)$ is some prior information, which was known before some new information, expressed as $p(B|A_i)$, becomes available. $p(A_i|B)$ is the updated belief in the light of new information referred to as the posterior probability. $p(B)$, the prior probability of the evidence, acts as a normalisation, and is obtained by evaluating the exhaustive and exclusive set of evidence scenarios:

$$p(B) = \sum_{i=1}^{n} p(B \mid A_i)p(A_i).$$

Bayesian inference has been applied to complex evidential situations, notably medical diagnosis (Lauritzen and Spiegelhalter, 1988, Pearl, 1988). To do so requires directed acyclic graphs of relationships between evidence (see for example Figure 5.2). Each node represents a proposition and an arc represents some relationship between those propositions (*i.e.* some conditionality). The absence of an arc indicates that the concepts are considered to be conditionally independent.

Figure 5.2 indicates the relationships between symptoms and diseases in a medical diagnostic situation. Metastatic cancer is a possible cause of brain tumour and may also increase the total serum count. Both a brain tumour and an increased serum count can cause the patient to fall into a coma. Sever headache can also be associated with brain tumour. Evidence may be received by any of the nodes in the graph and is propagated through the graph using the chain rule of conditional probability to update the probabilities throughout the graph.

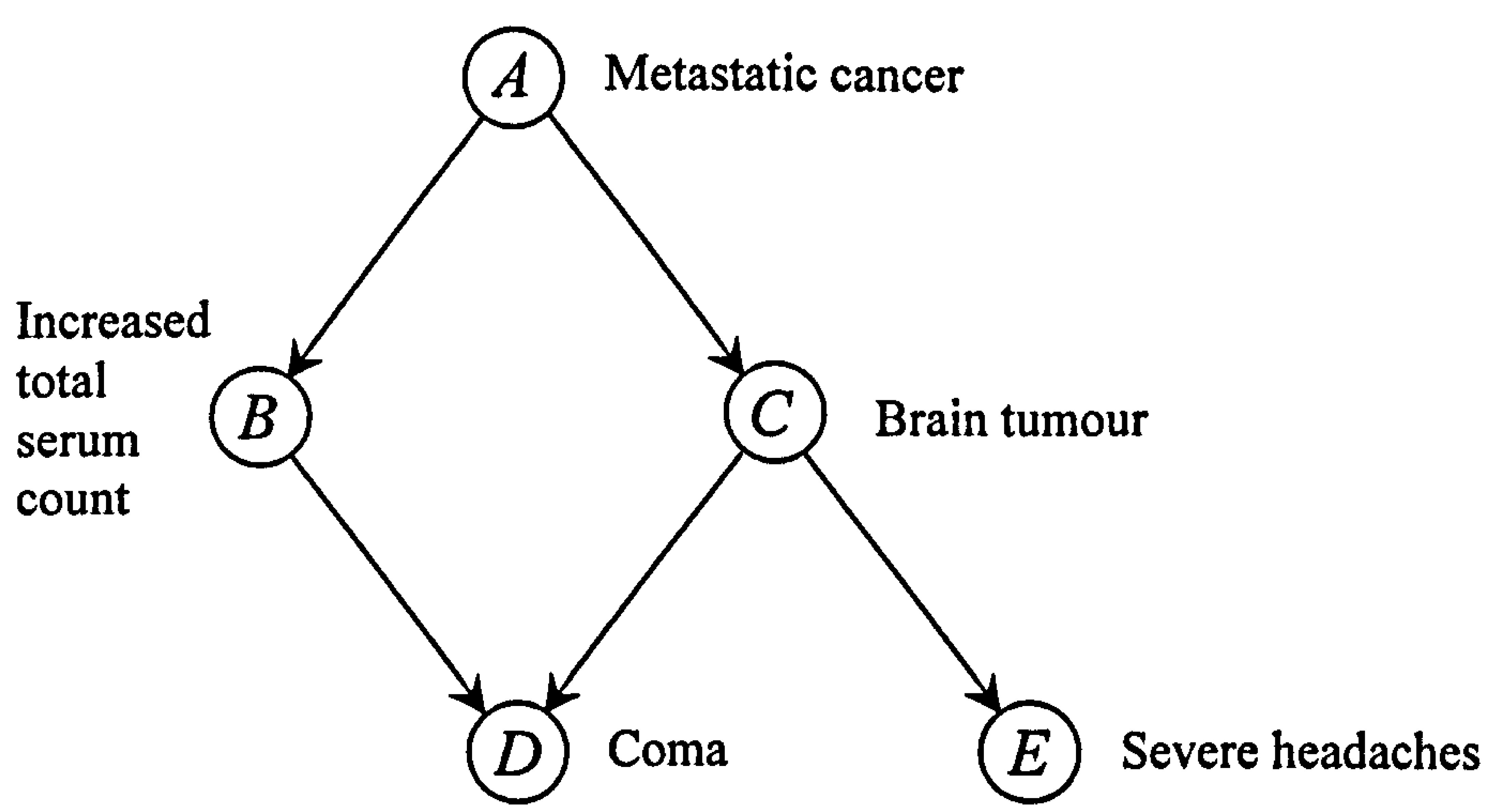

Figure 5.2 Belief network for metastatic cancer (after Henrion, 1988)

5.4.8 Probability bounds

In the absence of complete information it will not be possible to completely specify a probabilistic relationship between propositions. The most that can be deduced are the bounds on a probability. One way of illustrating this is by means of the voting model, which was first suggested by Gaines (1978).

Consider in the first instance the situation in which a population is asked to respond to two questions (Blockley *et al.*, 1983). Each member is obliged to respond with a 'yes' or 'no' reply. For some question A assume that the proportion voting 'yes' $p(A)$ is p and for some question B the proportion voting 'yes' $p(B)$ is q. As p and q are proportions they are measures on the interval [0,1] and do not necessarily sum to one. Figure 5.3 shows how p and q could be distributed over the power set of this space $A \times B$.

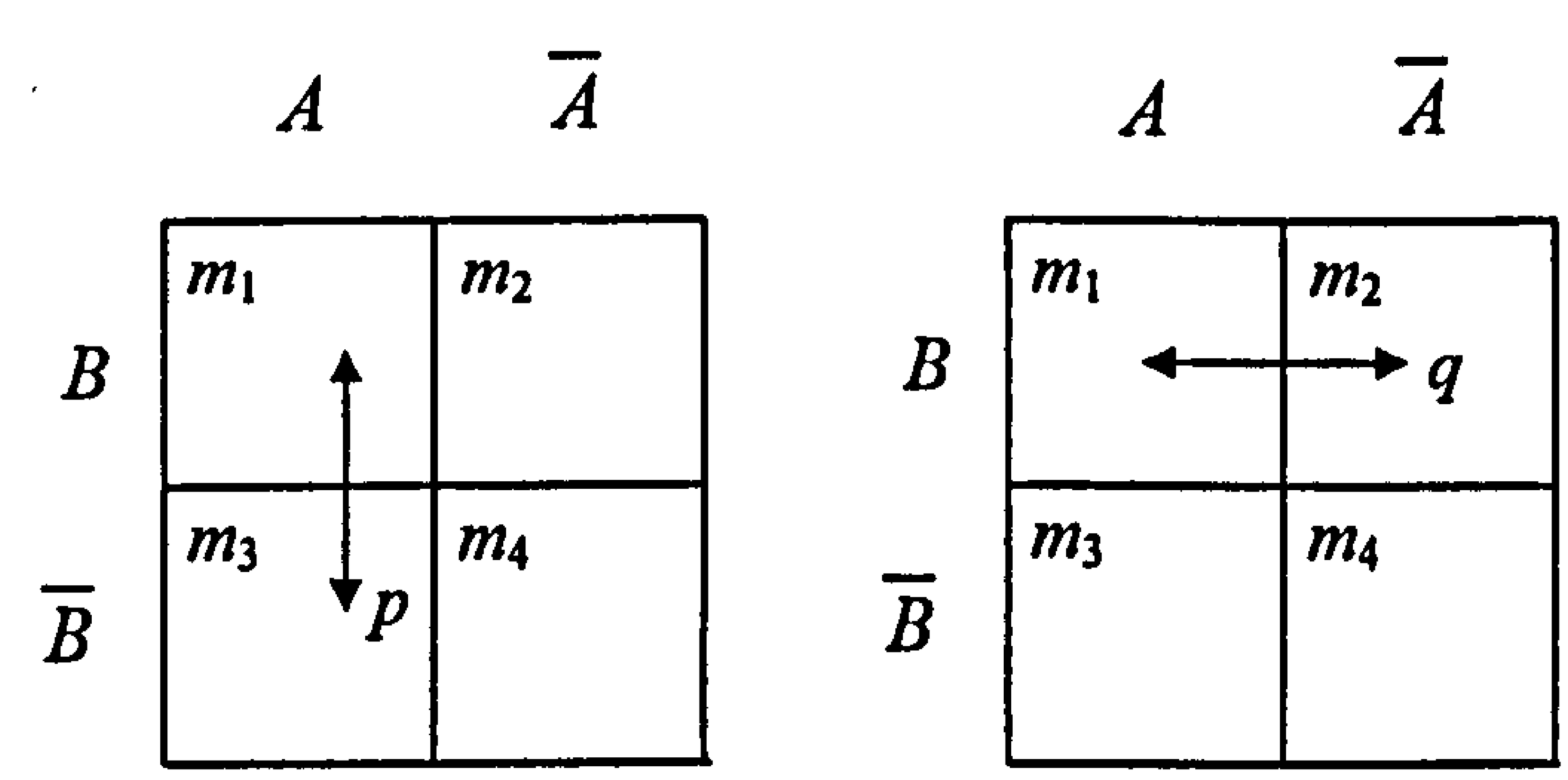

Figure 5.3 Voting model for compound propositions

If the way in which the votes are distributed over the power set is denoted as m_1, m_2, m_3, m_4 in Figure 5.3 then:

$$p = m_1 + m_3 = p(A)$$

$$q = m_1 + m_2 = p(B)$$

$$m_1 + m_2 + m_3 + m_4 = 1$$

and the individual values of m_i cannot be determined without one further item of information which says something about the relationship between A and B such as $p(A \cap B)$, $p(A \cup B)$ or $p(A|B)$. The situation therefore has one degree of freedom. If this degree of freedom is not constrained then it is not possible to uniquely specify m_1, m_2, m_3, m_4 though it is possible to define permissible ranges of values which are constrained by the information which is available.

Suppose for example that $p = 0.4$ and $q = 0.2$. Then the bounds on the values m_1, m_2, m_3, m_4 are as shown in Figure 5.4.

		A	$\overline{A}$
		0.4	0.6
B	0.2	m_1 [0.0,0.2]	m_2 [0.0,0.2]
$\overline{B}$	0.8	m_3 [0.2,0.4]	m_4 [0.4,0.6]

Figure 5.4 Bounds on probabilities for additive case with one unconstrained degree of freedom

Another way of looking at the situation is to draw a Venn diagram as shown in Figure 5.5. Given $p(A)$ and $p(B)$, the various areas in the Venn diagram are uniquely specified if the probability of one of the elements of $A \cap B, A \cap \overline{B}, \overline{A} \cap B$, $\overline{A} \cap \overline{B}$ is known.

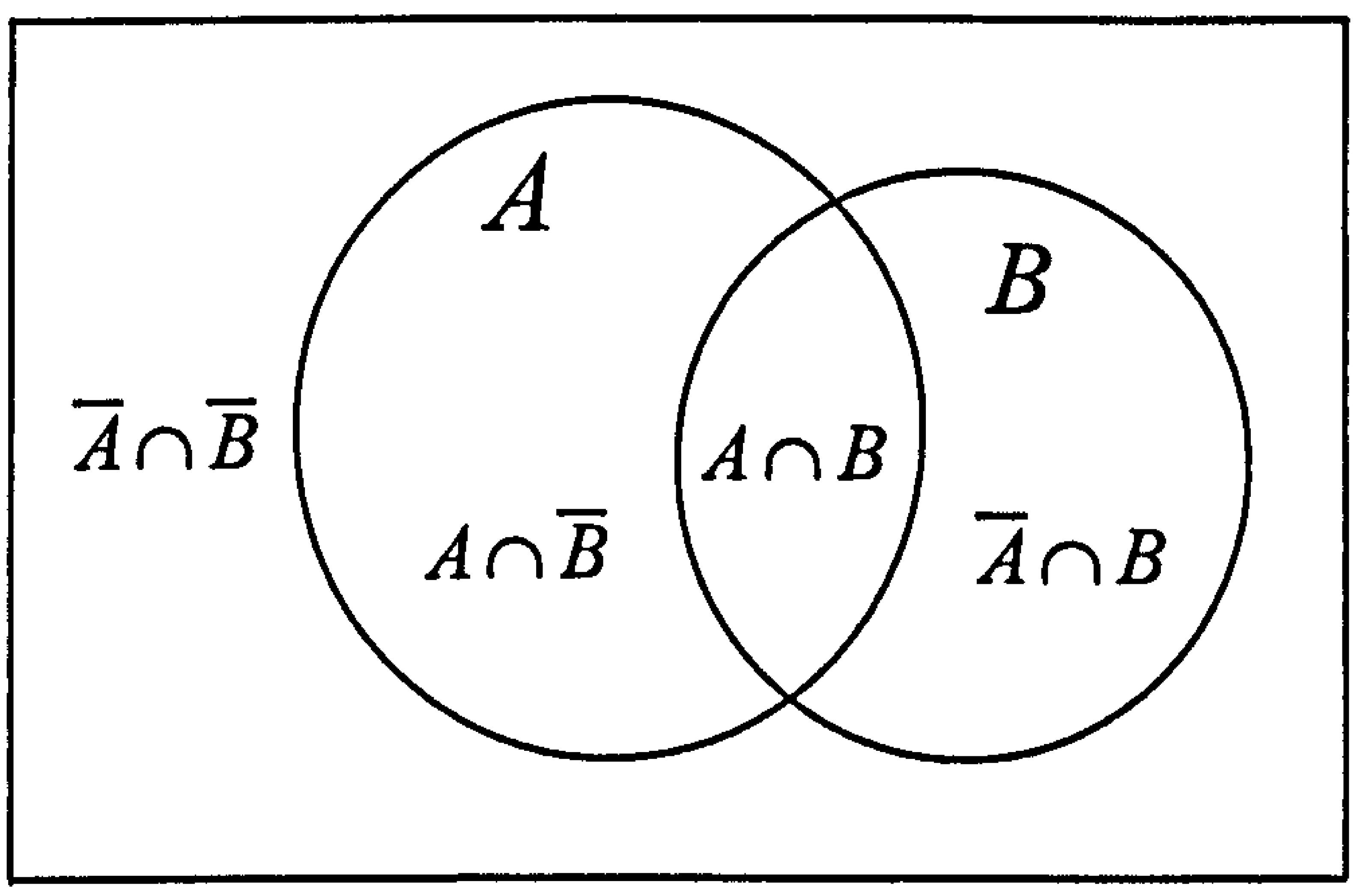

Figure 5.5 Venn diagram for compound propositions

Now if there are n questions, $E_1, E_2, \ldots, E_n$, in the voting model or n sets in the Venn diagram then the power set contains 2^n elements, so $2^n - 1$ constraints are needed to uniquely divide up the universal set (the final constraint being that all the probabilities should sum to unity). Knowing $p(E_1), p(E_2), \ldots, p(E_n)$ provides n independent constraints, so the problem has $2^n - n - 1$ degrees of freedom (Table 5.1), a result which will be of interest in the context of Interval Probability Theory.

Table 5.1 Degrees of freedom for compound propositions

n	1	2	3	4	5	n
Degrees of freedom	0	1	4	11	26	$2^n - n - 1$

5.5 Fuzzy set theory

In conventional, or crisp, set theory any element x of the universal set X can be classified as being either an element of some sub-set A,

$x \in A$,

or an element of its complement $\overline{A}$,

$x \in \overline{A}$

in which case

$x \notin A$.

This is workable if A is a precisely defined set such as "the set of integers greater than zero". However, if A is a vaguely defined concept such as "the set of tall men" then it can be difficult to specify whether a given individual falls into the set of not. In order to overcome this problem Lotfi Zadeh (1965) suggested that the boundaries of a set should be fuzzified so that an element need not simply be a member of a set or a member of its complement but can possess a degree of membership. For example (Baldwin, 1995), most Europeans would agree that the set of "tall men" should include men who are taller than 6ft 2ins and will exclude men shorter than 5ft 6ins. Between these extremes it is rather difficult to define the vague concept of tallness. This problem can be overcome in fuzzy set theory by saying that a man who is 5ft 10ins tall is a partial member of the set of tall men and a partial member of the set of not tall men.

In mathematical terms the fuzzy set A (which is a sub-set of the universal set X) is defined by the membership function

$\mu_A: X \rightarrow [0,1]$

where μ_A may be a continuous or a discrete function. Note that crisp set theory is a special case of fuzzy set theory in which any element x may have only one of two discrete membership values, 0 or 1.

By way of example, the membership function for the set of tall men may be as shown in Figure 5.6, which is a continuous function.

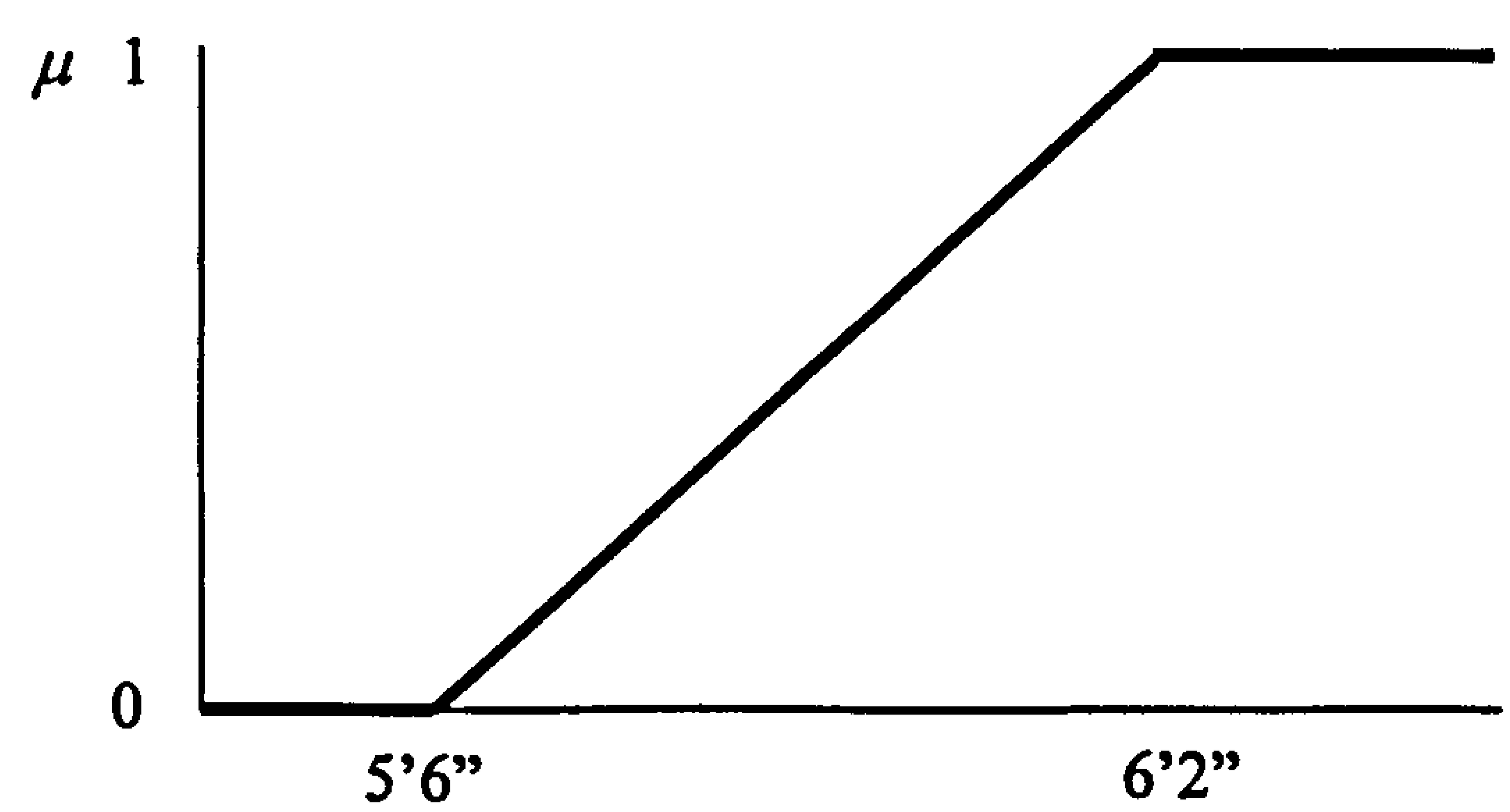

Figure 5.6 The fuzzy set of tall men

A discrete membership function of a fuzzy set A is usually written in the form

$$A = \sum_{i=1}^{n} \mu_i / x_i \, .$$

The fuzzy set of tall men might be written as

$A = 0 / 5'6" + 0.25 / 5'8" + 0.5 / 5'10" + 0.75 / 6'0" +1 / 6'2"$.

The fuzzy set "tall" is said to induce a *possibility* distribution on height space. Zadeh (1978) argued that humans reason with imprecise information in a possibilistic rather than probabilistic mode. Possibility is a much looser concept than probability, as was illustrated in Zadeh's example of Hans' breakfast (Table 4.1). The relationship between possibility and probability is explored in more detail in Section 5.9.1.

5.5.1 Operators on fuzzy sets

The union of two fuzzy sets A and B is a fuzzy set $A \cup B$ such that

$$\mu_{A \cup B}(x) = \max[\mu_A (x), \mu_B(x)]$$

for every $x \in X$. Thus the membership grade of each element of $A \cup B$ is either its membership grade in A or its membership grade in B, whichever is the larger value.

The intersection of fuzzy sets A and B is a fuzzy set $A \cap B$ such that

$$\mu_{A \cup B}(x) = \min[\mu_A (x), \mu_B(x)]$$

for every $x \in X$. Thus the membership grade of each element of $A \cap B$ is the smaller of its membership grades in set A or set B.

However, these original formulations for fuzzy union and intersection are special cases of the possible set relationships (Blockley and Baldwin, 1987, Bier, 1990, 1992). In the general case illustrated in the Venn diagram in Figure 5.5, by inspection

$$p(A \cap B) \leq \min(p(A), p(B)).$$

Only in the special case when $A \subseteq B$ or $B \subseteq A$ is it true that

$$p(A \cap B) = \min(p(A), p(B)).$$

Thus the conventional intersection operator in fuzzy set theory can therefore be interpreted as assuming complete dependence between sets. The same is true of the union operator. However, Dubois and Prade (1990) argue that this connotation misinterprets the fuzzy membership function as a set function. The fuzzy membership function is a measure of the degree of truth, whilst Blockley and Bier's arguments are based on concepts of degrees of probabilistic uncertainty.

Alternative definitions of union and intersection are discussed by Bellman and Zadeh (1970), Yager (1980), Dubois and Prade (1980) and Klir and Folger (1988). A more general set of relationships referred to as triangular norms (T-norms) have been developed (Schweizer, 1983, Schweizer and Sklar, 1983) in which case the conjunction operators T are given by Bonissone (1987). The cases listed below are instances from a continuous range of different operators.

$T_0(a, b)$	$\begin{cases} = \min(a, b) \text{ if } \max(a, b) = 1 \\ = 0 \text{ otherwise} \end{cases}$	$\begin{cases} \text{Maximum perversity,} \\ \text{Disjoint propositions} \end{cases}$
$T_1(a, b)$	$= \max(0, a + b - 1)$	
$T_{1.5}(a, b)$	$= (ab) / [2 - (a + b - ab)]$	
$T_2(a, b)$	$= ab$	Independent
$T_{2.5}(a, b)$	$= (ab) / (a + b - ab)$	
$T_3(a, b)$	$= \min(a, b)$	$\begin{cases} \text{Total dependence,} \\ \text{Nested propositions} \end{cases}$

Similarly the set of disjunction operators S are given by

$S_0(a, b)$	$\begin{cases} = \max(a, b) \text{ if } \min(a, b) = 0 \\ = 1 \text{ otherwise} \end{cases}$	$\begin{cases} \text{Maximum perversity,} \\ \text{Disjoint propositions} \end{cases}$
$S_1(a, b)$	$= \min(1, a + b)$	
$S_{1.5}(a, b)$	$= (a + b) / (1 + ab)$	
$S_2(a, b)$	$= a + b - ab$	Independent
$S_{2.5}(a, b)$	$= (a + b - 2ab) / (1 - ab)$	
$S_3(a, b)$	$= \max(a, b)$	$\begin{cases} \text{Total dependence,} \\ \text{Nested propositions} \end{cases}$

5.6 Fuzzy measures

In the previous section it has been explained how fuzzy sets can be used to mathematize vague concepts, by fuzzifying the boundaries of classical set theory. Fuzzy measures relate to the rather different situation where the set A is precisely defined, but the extent to which a given element x of the universal set X is a member of A is uncertain in a vague sense. Klir and Folger (1988) illustrate the distinction between fuzzy sets and fuzzy measures with the example of a criminal trial a defendant must be found either guilty or innocent.

> *The set of people who are guilty of the crime and the set of innocent people are assumed to have very distinct boundaries. The concern, therefore, is not with the degree to which the defendant is guilty but the degree to which the evidence proves his membership in either the crisp set of guilty people or in the crisp set of innocent people. We assume that perfect evidence would point to full membership in one and only one of these sets. Our evidence, however, is rarely, if ever perfect, and some*

uncertainty usually prevails. In order to represent this type of uncertainty, we could assign a value to each possible crisp set to which the element in question might belong. This value would indicate the degree of evidence or certainty of the element's membership in the set. Such a representation of uncertainty is known as a fuzzy measure.

Summarising:

- in *fuzzy set theory* a value is assigned to each element of the universal set to signify its degree of membership in a particular sub-set with unsharp boundaries;
- a *fuzzy measure* assigns a value to each crisp sub-set in the universal set to signify the degree of available evidence or belief that a given element of X belongs to that that particular sub-set.

In other words fuzzy set theory is applicable in circumstances where membership of a subset A of X is uncertain due to the vague definition of A. A fuzzy measure, meanwhile, is applicable in circumstances where membership of a well-defined subset A is nonetheless uncertain due to the incomplete or ambiguous evidence relating to the membership of A. The concepts of fuzzy measures are more relevant to this thesis than those of fuzzy sets.

A fuzzy measure is defined by a function

$g : P(X) \rightarrow [0,1]$

where $P(X)$ is the power set of X. The fuzzy measure function g therefore assigns to each crisp subset of X a number in the interval [0,1]. The function therefore maps to the same [0,1] range as probability theory, but the mapping is from the power set $P(X)$ rather than from the singletons of X. When a fuzzy measure is assigned to a subset $A \in P(X)$, $g(A)$ represents the degree of available evidence or belief that a given element of X belongs to the subset A.

The axiomatisation of fuzzy measures according to Klir and Folger (1988) is as follows

Axiom 1 (boundary conditions): $g(\varnothing) = 0$ and $g(X) = 1$.

Axiom 2 (monotonicity): For every $A, B \in P(X)$, if $A \subset B$, then $g(A) \leq g(B)$.

Axiom 3 (continuity): For every sequence $(A_i \in P(X) \mid i \in \mathbb{N})$ of subsets of X, if either $A_1 \subseteq A_2 \subseteq \ldots$ or $A_1 \supseteq A_2 \supseteq \ldots$ (i.e. the sequence is monotonic), then

$$\lim_{i \to \infty} g(A_i) = g(\lim_{i \to \infty} A_i).$$

Axiom 1 states that despite the degree of evidence, it is always known that a given element of X definitely does not belong to the empty set and definitely does belong to the universal set.

Axiom 2 requires that the evidence of membership of an element in a set must be at least as great as the evidence that the element belongs to any subset of that set.

Axiom 3 is applicable in the case of an infinite universal set. It requires that for every infinite sequence $A_1, A_2, \ldots$ of nested (monotonic) subsets of X that converge to the set

$$A = \lim_{i \to \infty} A_i,$$

the sequence of numbers $g(A_1)$, $g(A_2)$,... must converge to a number $g(A)$. This axiom can also be seen as a requirement of consistency: calculation of $g(A)$ in two different ways, either as the limit of $g(A_i)$ for $i \to \infty$ or by application of the function g to the limit of A_i for $i \to \infty$ is required to yield the same value.

5.7 Basic probability assignments

Basic probability assignments were introduced by Glen Shafer in the context of his *Mathematical Theory of Evidence* (1976). In the *Mathematical Theory of Evidence* the basic probability assignment is introduced as a derivative of Shafer's belief and plausibility measures. However, as will become clear, the basic probability assignment can be thought of as a more fundamental type of measure than belief and plausibility measures, which are introduced in the following section.

A basic probability assignment is a probability distribution over the power set of the universal set X (though in the context of basic probability assignments the universal set is more usually referred to as the *universe of discourse*). A basic probability assignment is represented by the function

$$m : P(X) \to [0,1]$$

such that $m(\varnothing) = 0$ and

$$\sum_{A \in P(X)} m(A) = 1$$

where $m(A)$ is interpreted as the *degree of evidence* supporting the claim *that a specific element of X belongs to the set A but not to any special subset of A* (Klir and Folger, 1988).

Every set $A \in P(X)$ for which $m(A) > 0$ is called a *focal element* of m. Total ignorance can be represented within the framework by the assignment $m(X) = 1$. This establishes that an element forms part of the universe of discourse X, but there is no information about which subset of X it forms part. So, thinking back to the Hasse diagram in Figure 5.1, a basic probability assignment could be written

$$m = \{1\} : 0.1,\ \{2\} : 0.2,\ \{1,2\} : 0.7,$$

which has three focal elements, the sets $\{1\}$, $\{2\}$ (which happen to be singletons) and $\{1,2\}$. All of the other elements of the power set illustrated in the Hasse diagram have zero probability assigned to them. This basic probability assignment thereby implies an ambiguous state of knowledge, since most of the probability measure is assigned to $\{1,2\}$.

The set (F, m), where F and m denote a set of focal elements and the associated basic assignments respectively, is often called the 'body of evidence'. The same structure is also referred to as a 'random set' (Robbins, 1944, 1945, Dubois and Prade, 1991, Bernardini, 1999).

A basic probability assignment is distinct from a probability distribution inasmuch as it is defined on $P(X)$ rather than on the singletons in X. A basic probability assignment reduces to a probability distribution when the focal elements are all singletons. In other words if $m(A) = 0$ for all of the subset of X that are not singletons, then $m(\{x\}) = p(x)$. Recalling the treatment of probability bounds in Section 5.4.8, a basic probability assignment can in general be considered to be a family of probability distributions. Some families of probability distributions cannot be represented by a single basic probability assignment, in which case they are represented by a family of basic probability assignments. This is an issue that is returned to in the context of mass assignments and Interval Probability Theory.

A basic probability assignment does not require that $m(X) = 1$ or that $m(A) \leq m(B)$ when $A \subset B$, which are two of the axioms of fuzzy measures. Basic probability assignments therefore are not fuzzy measures but in the following section it will be illustrated how there is a close relationship between the basic probability assignment and two special forms of fuzzy measures, belief and plausibility measures.

5.8 Belief and plausibility measures

Belief and plausibility measures, as introduced by Shafer (1976), in fact have their origins on Dempster's (1969) work on upper and lower probabilities. The theory that has now been developed around the concepts of belief and plausibility measures is therefore usually referred to as the Dempster-Shafer theory of evidence.

A belief measure is a function

$Bel: P(X) \rightarrow [0,1]$

that satisfies the three axioms of fuzzy measures and the following additional axiom:

$$Bel(A_1 \cup A_2 \cup \ldots \cup A_n) \geq \sum_i Bel(A_i) - \sum_{i<j} Bel(A_i \cap A_j) + \ldots + (-1)^{n+1} Bel(A_1 \cap A_2 \cap \ldots \cap A_n)$$

for every $n \in \mathbb{N}$ and every collection of subsets of X. For $n = 2$, for example, this axiom has the form

$$Bel(A_1 \cup A_2) \geq Bel(A_1) + Bel(A_2) - Bel(A_1 \cap A_2),$$

i.e. the belief in the disjunction of two propositions should be at least as great as the belief in the two individual propositions minus the belief in their conjunction. If $A_1 = A$ and $A_2 = \overline{A}$ then the following fundamental property of belief measures is obtained:

$$Bel(A) + Bel(\overline{A}) \leq 1.$$

Thus, unlike probability measures, the belief in a conjecture and the belief in its negation need not sum to unity.

For each $A \in P(X)$, $Bel(A)$ is interpreted as the *degree of belief* (based on available evidence) that a given element of X belongs to the set A. The total belief in A is the sum of the basic probability assignments in all of the subsets of A,

$$Bel(A) = \sum_{B \subseteq A} m(B).$$

Whilst $m(A)$ characterises the degree of evidence or belief that the element in question belongs to set A alone, $Bel(A)$ represents the total evidence or belief that the element belongs to A or any subset of A.

A plausibility measure is a function Pl defined by the equation

$$Pl(A) = 1 - Bel(\overline{A}).$$

Similarly

$$Bel(A) = 1 - Pl(\overline{A}).$$

The plausibility measure is the sum of all of the basic probability assignments that could plausibly support A:

$$Pl(A) = \sum_{B \cap A \neq \varnothing} m(B).$$

It therefore represents not only the total belief that the element in question belongs to set A or any of its subsets, but also the additional evidence or belief associated with sets that overlap with A. Hence,

$$Pl(A) \geq Bel(A)$$

for all $A \in P(X)$.

A belief measure reduces to a probability measure by strengthening Axiom 3, introduced above, so that

$$Bel(A \cup B) = Bel(A) + Bel(B) \text{ whenever } A \cap B = \varnothing.$$

The following theorem expresses the relationship between belief measures and probability distributions by reference to basic probability assignments (Klir and Folger, 1988).

Theorem. A belief measure Bel on a finite power set $P(X)$ is a probability measure if and only if its basic assignment m is given by $m(\{x\}) = Bel(\{x\})$, and $m(A) = 0$ for all subsets of X which are not singletons.

The main differences between the Dempster-Shafer theory of evidence and conventional probability are that (Krause and Clark, 1993):

1. *beliefs need not be additive,*

2. *beliefs can be assigned to sets of propositions rather than of necessity to each individual proposition, and*

3. *as a consequence, it is possible to pool evidence with respect to hierarchically nested hypothesis sets.*

Krause and Clark, therefore conclude that the Dempster-Shafer theory is a richer formalism than Bayesian probability. It is attractive as a basis for hierarchical systems modelling being founded on a hierarchical ordering of concepts.

5.8.1 Dempster's rule of combination

It is an essential aspect of any uncertainty calculus that there should be some mechanism for combining two items of (uncertain) evidence. In fuzzy set theory this is achieved by fuzzy union and fuzzy intersection. Dempster's rule is a method introduced for combining upper and lower probabilities, which was subsequently adopted by Shafer (1976) and expressed in terms of basic probability assignments.

Consider the situation in which there are two items of evidence from two independent sources, B and C, expressed by two basic assignments m_1 and m_2 on some power set $P(X)$. Then Dempster's rule combines these two items of evidence according to the formula

$$m_{1,2}(A) = \frac{\sum_{B \cap C = A} m_1(B).m_2(C)}{1-K}$$

for $A \neq \varnothing$, where $K = \sum_{B \cap C = \varnothing} m_1(B).m_2(C)$ and $m_{1,2}(\varnothing) = 0$.

Consider the very simple example of the set $X = \{a, b\}$ then the power set of X is $P(X) = \{\{a, b\}, \{a\}, \{b\}, \varnothing\}$. Consider two basic assignments of, m_1 and m_2,

$m_1 = \{a\} : 0.1, \{b\} : 0.2, \{a, b\} : 0.7$

$m_2 = \{a\} : 0.3, \{b\} : 0.7.$

Note that m_2 happens to be a probability distribution as all of the focal elements are singletons, though this has no bearing on the application of Dempster's rule of combination. The calculations of Dempster's rule first involve identifying the common subsets between the focal elements from the two bodies of evidence and then taking the product (Table 5.2).

Table 5.2 First step of Dempster's rule of combination (before renormalization)

	$\{a\}: 0.3$	$\{b\}: 0.7$
$\{a\}: 0.1$	$\{a\}: 0.03$	$\varnothing: 0.07$
$\{b\}: 0.2$	$\varnothing: 0.06$	$\{b\}: 0.14$
$\{a, b\}: 0.7$	$\{a\}: 0.21$	$\{b\}: 0.49$

The renormalization phase is then used to enforce the constraint that $m_{1,2}(\varnothing) = 0$. In the example $K = 0.13$ and the final basic probability assignment becomes

$m_{1,2} = \{a\} : 0.276, \{b\} : 0.724.$

Dempster's rule is based on multiplying common focal elements and then renormalizing. Both of these stages are questionable as an approach to combining evidence. Taking the product of common focal elements is done in exactly the same way in which the joint distribution is calculated from two independent marginal distributions in probability theory and is justified on the same grounds (Dempster, 1969). However, the need to represent dependency relationships between evidence other than independence has already been identified as a requirement for a mathematical theory of uncertainty. Many evidential situations are characterised by evidence that at some deep level is related or comes from a common source so cannot be assumed to be independent.

Renormalization occurs due to inconsistencies between items of knowledge. However, inconsistency or conflict may be a legitimate characteristic of the knowledge base. The effect of renormalization is to remove the inconsistency and distribute it amongst the consistent focal elements. The effect of this can be to generate counter-intuitive results (Zadeh, 1986*b*, Walley, 1991).

5.9 Mass Assignments

Mass assignment theory is a generalisation of the Dempster-Shafer theory and fuzzy set theory that has been developed at the University of Bristol by Baldwin (Baldwin, 1991*a*, 1991*b*, Baldwin *et al.*, 1995*a*) to overcome the limitations which have been identified with the Dempster-Shafer theory of evidence and generate a general theory of uncertainty which embraces probability and possibility theory.

Formally a mass assignment may be defined over the power set $P(X)$ as

$m : P(X) \rightarrow [0,1]$

where

$\sum_{A \in P(X)} m(A) = 1$ and $m(\varnothing) \geq 0$.

The distinction between mass assignments and basic probability assignments is clear, recalling that the definition of a basic probability assignment is the same but $m(\varnothing) = 0$. A mass assignment is said to be complete and consistent if $m(\varnothing) = 0$ in which case the relationship with probability distributions is the same as for the corresponding basic probability assignment and the mass assignment represents a family of probability distributions.

By relaxing the constraint that $m(\varnothing) = 0$ the theory of mass assignments recognises that there can be inconsistency in the knowledge base, and indeed that there can be relevant evidence that is not contained in the knowledge base. In this sense it can be considered to be an open world theory.

The equivalent in mass assignment theory to Dempster's rule is the so-called multiplication meet (Baldwin *et al.*, 1995*a*), but by eliminating the need for renormalization the anomalies associated with Dempster's rule are overcome. In cases where the evidence is entirely consistent the multiplication meet and Dempster's rule provide the same solution. However, it is recognised in mass assignment theory that the multiplication meet is a special case corresponding to an assumption of independent evidence. The general assignment method (Baldwin, 1991*d*) provides a general framework for combining evidences, making no assumptions of the dependency relationship between evidence.

The *meet* (or intersection) of two mass assignments combines two mass assignments by identifying the family of mass assignments that are consistent with the mass assignments being combined. Some of these common mass assignments will be more general than others. The requirement is that the meet is the most general of all of the possible solutions, in other words it is not a restriction on any of the other possible solutions.

The situation is illustrated by an example given by Baldwin *at al.* (1995).

$m_1 = \{a\} : 0.2,\ \{a, b\} : 0.5,\ \{a, b, c\} : 0.3$

$m_2 = \{a\} : 0.5,\ \{a, b\} : 0.1,\ \{a, b, c\} : 0.4.$

Each cell in Table 5.3 contains the set label that is common to the row set and the column set. Table 5.3 illustrates the most general solution (since the maximum mass is assigned to the superset $\{a, b, c\}$) which satisfies the row and column constraints. The meet $m_{1,2}$ is therefore

$m_{1,2} = \{a\} : 0.5,\ \{a, b\} : 0.2,\ \{a, b, c\} : 0.3$

Any other cell allocation that satisfies the row and column constraints is a restriction of this solution.

Table 5.3 Example of the meet of two mass assignments

	$\{a\} : 0.5$	$\{a, b\} : 0.1$	$\{a, b, c\} : 0.4$
$\{a\} : 0.2$	$\{a\} : 0.2$ √	$\{a\} : 0$	$\{a\} : 0$?
$\{a, b\} : 0.5$	$\{a\} : 0.3$	$\{a, b\} : 0.1$ √	$\{a, b\} : 0.1$
$\{a, b, c\} : 0.3$	$\{a\} : 0$	$\{a, b\} : 0$	$\{a, b, c\} : 0.3$ √

As far as possible a consistent solution is achieved so that $\varnothing = 0$. However, in some situations it will be necessary to assign mass to $\varnothing$ in order to achieve a solution.

It will not always be possible for a unique allocation to be found, in which case the solution becomes a family of mass assignments where each member of the family is not a restriction of any other member of the family, and a linear combination is a complete representation of the family of probability distributions required. This situation will also be encountered in the context of Interval Probability Theory when it will be examined in more detail. It is tackled in mass assignment theory by application of the *split algorithm*, which generates a family of mass assignments. However, the approach has been found by Coyne (1993) to be computationally impractical. In order to clearly distinguish information and uncertainty relating to individual elements of the domain Coyne developed the artificial split algorithm, which generates a single mass assignment, though the mass assignment generated by the split algorithm is a restriction on the general solution.

Another practical issue of note at this stage, which will be revisited in the evaluation of uncertainty methods, is that to achieve the most general solution involves maximising the mass assigned to the least specific super-sets. The consequence of this operation is that, particularly after multiple meets, the distribution of masses can imply very little knowledge. Whilst in mathematical terms this is the most general solution, it leaves the knowledge engineer in a position of being able to say very little specific about the inference in hand.

5.9.1 Relationship between mass assignments, probability theory and possibility theory

The relationship between mass assignments, probability theory and possibility theory is demonstrated in an example given by Baldwin *et al.* (1995*a*). Suppose the universe of discourse $X = \{a, b, c, d\}$ and A is a fuzzy set such that

$A = 1 / a + 0.7 / b + 0.5 / c + 0.1 / d.$

The fuzzy set induces a possibility distribution on X which is conventionally denoted by the function Π, such that

$\Pi(a) = 1, \Pi(b) = 0.7, \Pi(c) = 0.5, \Pi(d) = 0.1.$

Let the probability distribution over X for A be

$p(a) = p_1, p(b) = p_2, p(c) = p_3, p(d) = p_4,$

then

$p_1 + p_2 + p_3 + p_4 = 1.$

Now, without going into a full axiomatisation of possibility measures it is intuitively plausible (and also axiomatically the case) that the union of two events is possible if either of the two events it possible,

i.e. $\Pi(A \cup B) = \max(\Pi(A), \Pi(B)),$

so

$\Pi(\{a, b, c, d\}) = 1, \Pi(\{b, c, d\}) = 0.7, \Pi(\{c, d\}) = 0.5, \Pi(\{d\}) = 0.1.$

Since possibility is a weaker measure than probability $p(A) \le \Pi(A)$ so that

$$
\begin{aligned}
p_1 + p_2 + p_3 + p_4 &= 1 \\
p_2 + p_3 + p_4 &\le 0.7 \\
p_3 + p_4 &\le 0.5 \\
p_4 &\le 0.1
\end{aligned}
$$

so

$0.3 \le p_1 \le 1; 0 \le p_2 \le 0.7; 0 \le p_3 \le 0.5; 0 \le p_4 \le 0.1$ → ?

which is a family of probability distributions given by the mass assignment

$m = a : 0.3, \{a, b\} : 0.2, \{a, b, c\} : 0.4, \{a, b, c, d\} : 0.1.$?

Thus the mass distribution can be determined directly from the membership values of the fuzzy set.

5.10 Interval Probability Theory

Cui and Blockley (1990) introduced Interval Probability Theory (IPT) as a measure of evidential support in knowledge-based systems. An interval representation is used in order to capture in a relatively simple manner, features of fuzziness and incompleteness. Interval representation is implicit in many of the approaches to uncertainty introduced above, notably interval bounding approaches to classical probability (Section 5.4.8) and the Dempster-Shafer theory of evidence (Section 5.8). An interval representation is also a natural consequence of dealing with families of probability distributions (Baldwin, 1986, Baldwin *et al.*, 1995*a*) and forms the basis of Support Logic Programming (Baldwin, 1986, Baldwin, 1987, Blockley, 1987, Baldwin, 1993, Baldwin *et al.*, 1995*a*), which is discussed in Section 5.11. Cui and Blockley (1990) developed previous work by introducing the parameter ρ, which represents the degree of dependence between evidence.

IPT is closely related to probability theory, but, in common with basic probability assignments, belief and plausibility measures and mass assignments, it is not necessary to exclusively allocate probability to a conjecture or its negation. Thus if E is a proposition

$$p(E) \in [S_n(E), S_p(E)]$$

where $S_n(E)$ is the lower bound, and $S_p(E)$ is the upper bound of the probability $p(E)$. The negation is

$$p(\overline{E}) \in [1 - S_p(E), 1 - S_n(E)].$$

This representation could therefore be thought of as an example of a basic probability assignment or mass assignment, with focal elements $\{E\}$, $\{\overline{E}\}$, $\{E, \overline{E}\}$. It can be interpreted as the bounds on a point probability, whilst a mass assignment represents the bounds on a family of probability distributions.

If interval probability is interpreted as a measure of belief, then $S_n(E)$ represents the extent to which it is certainly believed that E is true or dependable, $1 - S_p(E) = S_n(\overline{E})$ represents the extent to which it is certainly believed that E is false or not dependable, and the value $S_p(E) - S_n(E)$ represents the extent of uncertainty of belief in the truth or dependability of E. Three extreme cases illustrate the meaning of this interval measure of belief:

$p(E) \in [0,0]$ represents a belief that E is certainly false or not dependable,

$p(E) \in [1,1]$ represents a belief that E is certainly true or dependable, and

$p(E) \in [0,1]$ represents a belief that E is unknown.

5.10.1 Dependency

The degree of dependence between two propositions E_1 and E_2 is defined by the parameter ρ.

$$\rho = \frac{p(E_1 \cap E_2)}{\min(p(E_1), p(E_2))}.$$

Thus $\rho = 1$ indicates that $E_1 \subset E_2$ or $E_2 \subset E_1$ (*i.e.* they are nested propositions), whilst if E_1 and E_2 are independent

$$\rho = \max(p(E_1), p(E_2))$$

so that

$p(E_1 \cap E_2) = p(E_1).p(E_2)$.

The minimum value of ρ is given by

$$\rho = \max\left[\frac{p(E_1) + p(E_2) - 1}{\min(p(E_1), p(E_2))}, 0\right]$$

where $\rho = 0$ indicates that E_1 and E_2 are disjoint.

If ρ is defined as an interval $[\rho_l, \rho_u]$ then

$$S_n(E_1 \cap E_2) = \rho_l.\min(S_n(E_1), S_n(E_2)) \tag{1}$$

$$S_p(E_1 \cap E_2) = \rho_u.\min(S_p(E_1), S_p(E_2))$$

$$S_n(E_1 \cup E_2) = S_n(E_1) + S_n(E_2) - \rho_l.\min(S_n(E_1), S_n(E_2))$$

$$S_p(E_1 \cup E_2) = S_p(E_1) + S_p(E_2) - \rho_u.\min(S_p(E_1), S_p(E_2)). \tag{2}$$

The dependency parameter ρ is an additional item of information, which is elicited in order to explicitly address the issue of dependency between items of evidence. In terms of mass assignments ρ induces a restriction on the general solution that would, for example, be obtained from the meet of two mass assignments. The dependence parameter ρ is a convenient means of exploring different dependence relationships between evidence when the exact nature of dependence is uncertain. The dependence parameter generalises other inference rules that assume a specific dependence relationship between evidence. It can be interpreted in terms of triangular norms (*T*-norms) (see Section 5.5.1) in which case the minimum value of ρ corresponds to

$T_1(a, b) = \max(0, a + b - 1)$;

the independence value of ρ corresponds to

$T_2(a, b) = a.b$;

and $\rho = 1$ corresponds to

$T_3(a, b) = \min(a, b)$.

Intermediate values of ρ correspond to other *T*-norms.

There are two types of dependency relationships that are germane to hierarchical process models:

1. *Set-based relationships between the propositions.* These are evident without reference to any evidence supporting the proposition or its negation, and come from the definition of the sets. In this interpretation ρ is a measure of set inclusion *i.e.* the extent to which $E_1 \subset E_2$, or in logical terms $E_1 \rightarrow E_2$. For example "*A* is a budgerigar" and "*A* is a bird" are propositions with a high

level of dependency (one is a sub-set of the other). The dependency can be identified without reference to any evidence as to whether or not A is indeed a bird or a budgerigar.

2. *Dependency relationships between the evidence.* In this interpretation ρ is a measure of conditional probability $p(E_1|E_2)$ *i.e.* a measure of the extent to which the evidence that supports E_1 is dependent on the evidence that supports E_2. For example some evidence may have been collected to support the hypothesis "A is a budgerigar" and some to support the hypothesis "A is a bird", but it is unlikely that that evidence comes from independent sources (for example, the same ornithologist may have been consulted).

ρ tends to be treated as a measure of the latter type of dependency, implicitly assuming that E_1 and E_2 as disjoint propositions. In the examples being addressed in this thesis, this is a reasonable assumption to make, but clearly it is not generally the case. Generalisations of the approach should therefore be treated with caution. In circumstances where propositions are clearly not disjoint, for example if $E_1 \subset E_2$ then any evidence supporting E_1 will also support E_2.

5.10.2 Accumulating evidence from different sources

	$S_n(E_2)$ E_2	$1 - S_p(E_2)$ $\overline{E_2}$	$S_p(E_2) - S_n(E_2)$ E_{2U}
$S_n(E_1)$ E_1	m_{11}	m_{12}	m_{13}
$1 - S_p(E_1)$ $\overline{E_1}$	m_{21}	m_{22}	m_{23}
$S_p(E_1) - S_n(E_1)$ E_{1U}	m_{31}	m_{32}	m_{33}

Figure 5.7 Representation of compound propositions

Consider two propositions E_1 and E_2 with dependency between them $[\rho_l, \rho_u]$. The probability assignments to $E_1 \times E_2$ are illustrated in tabular form in Figure 5.7 so that, in terms of interval probabilities,

$$p(E_1 \cap E_2) = [m_{11},\ m_{11} + m_{13} + m_{31} + m_{33}] \tag{3}$$

$$p(E_1 \cap \overline{E_2}) = [m_{12},\ m_{12} + m_{13} + m_{32} + m_{33}] \tag{4}$$

$$p(\overline{E_1} \cap E_2) = [m_{21},\ m_{21} + m_{23} + m_{31} + m_{33}] \tag{5}$$

$$p(\overline{E_1} \cap \overline{E_2}) = [m_{22},\ m_{22} + m_{23} + m_{32} + m_{33}]. \tag{6}$$

The values of m_{ij} on the interval [0,1] are by convention constrained so that

$$m_{11} + m_{12} + m_{13} = S_n(E_1), \tag{7}$$

$$m_{21} + m_{22} + m_{23} = 1 - S_p(E_1), \tag{8}$$

$$m_{11} + m_{21} + m_{31} = S_n(E_2), \tag{9}$$

$$m_{12} + m_{22} + m_{32} = 1 - S_p(E_2) \tag{10}$$

$$m_{11} + m_{12} + m_{13} + ... + m_{33} = 1. \tag{11}$$

From Equations 1 and 3

$$m_{11} = \rho_l.\min(S_n(E_1), S_n(E_2)). \tag{12}$$

From Equation 6

$\rho \in [\rho_L, \rho_u]$

$m_{22} = S_n(\overline{E_1} \cap \overline{E_2}) = 1 - S_p(E_1 \cup E_2)$

$S_n \to \rho_L$

$S_p \to \rho_u$

so from Equation 2

$$m_{22} = 1 - S_p(E_1) - S_p(E_2) + \rho_u.\min(S_p(E_1), S_p(E_2)). \tag{13}$$

Whilst $p(E_1 \cap E_2)$ and $p(\overline{E_1} \cap \overline{E_2})$ are uniquely defined, the constraints of Equations 7 to 13 do not result in unique intervals for $p(E_1 \cap \overline{E_2})$ and $p(\overline{E_1} \cap E_2)$ under all values of $p(E_1)$, $p(E_2)$ and ρ. In other words the problem has two degrees of freedom. To obtain unique values of $p(E_1 \cap \overline{E_2})$ and $p(\overline{E_1} \cap E_2)$, thus fully specifying the problem, would require specific knowledge about the dependency between E_1 and $\overline{E_2}$ and between $\overline{E_1}$ and E_2. Because this knowledge can be difficult to articulate it is argued that it is preferable to calculate the family of permissible intervals for $p(E_1 \cap \overline{E_2})$ and $p(\overline{E_1} \cap E_2)$. An example of the procedure is shown in Figure 5.8.

If E_1 and E_2 are items of the same evidence derived from different sources then the sum $m_{12} + m_{21}$ is the conflict between the two items of evidence. This measure $m_{12} + m_{21}$ is of great use in locating areas of conflicting evidence so that it may, where possible, be reconciled. Conflict is sometimes an unavoidable characteristic of the evidence and if so will be reflected in the compound proposition. This is unlike Dempster's rule of combination where conflict is removed altogether by renormalization, as discussed in Section 5.8.1. The above procedure represents a restriction on the meet of two mass assignments (Baldwin, 1991). This restriction is induced by the additional information provided by ρ.

(*a*)

	E_2 0.2	$\overline{E_2}$ 0.5	E_{2U} 0.3
E_1 0.3	0.2	m_{12}	$0.1 - m_{12}$
$\overline{E_1}$ 0.3	0	0.3	0
E_{1U} 0.4	0	$0.2 - m_{12}$	$0.2 + m_{12}$

subject to $0 \le m_{12} \le 0.1$

(*b*)

	E_2 0.2	$\overline{E_2}$ 0.5	E_{2U} 0.3
E_1 0.3	0.06	m_{12}	$0.24 - m_{12}$
$\overline{E_1}$ 0.3	m_{21}	0.15	$0.15 - m_{21}$
E_{1U} 0.4	$0.14 - m_{21}$	$0.35 - m_{12}$	$m_{12} + m_{21} - 0.09$

subject to

$p \in [0.3, 0.7]$

$0 \le m_{12} \le 0.24$

$0 \le m_{21} \le 0.14$

$0.09 \le m_{12} + m_{21}$

$m_{22} = 1 - Sp(E_1) - Sp(E_2) + p \cdot \min(Sp(E_1), Sp(E_2))$.

(*c*)

	E_2 0.2	$\overline{E_2}$ 0.5	E_{2U} 0.3
E_1 0.3	0	m_{12}	$0.3 - m_{12}$
$\overline{E_1}$ 0.3	m_{21}	0	$0.3 - m_{21}$
E_{1U} 0.4	$0.2 - m_{21}$	$0.5 - m_{12}$	$m_{12} + m_{21} - 0.3$

subject to

$0 \le m_{12} \le 0.3$

$0 \le m_{21} \le 0.2$

$0.3 \le m_{12} + m_{21}$

	(*a*)	(*b*)	(*c*)
$p(E_1 \cap E_2)$:	[0.2, 0.5]	[0.06, 0.35]	[0.0, 0.2]
$p(\overline{E_1} \cap \overline{E_2})$:	[0.3, 0.7]	[0.15, 0.56]	[0.0, 0.5]
$p(E_1 \cap \overline{E_2})$:	$[m_{12}, 0.5]$	$[m_{12}, 0.5 + m_{21}]$	$[m_{12}, 0.5 + m_{21}]$
$p(\overline{E_1} \cap E_2)$:	$[0.0, 0.2 + m_{12}]$	$[m_{21}, 0.2 + m_{12}]$	$[m_{21}, 0.2 + m_{12}]$

Figure 5.8 Example of compound proposition $p(E_1) \in [0.3, 0.7]$, $p(E_2) \in [0.2, 0.5]$ (a) maximum dependence: $\rho \in [1.0, 1.0]$; (b) independence: $\rho \in [0.3, 0.7]$; (c) minimum dependence: $\rho \in [0.0, 0.4]$ (after Hall et al., 1998b).

The approach for establishing the assignments to the power set of the universe of discourse can be extended to apply to three or more propositions. For n propositions the tableau will occupy n-dimensional space. In the general case an interval probability problem with n items of evidence has 3^n degrees of freedom. If each item of evidence is ascribed an interval probability then these provide $2n$ constraints. Values of ρ, the dependency measure, are conventionally obtained as pair-wise relationships between items of evidence. So for example for three items of evidence, A, B and

C, it would be usual to obtain three dependency measures ρ_{AB}, ρ_{BC}, ρ_{AC}, each of which provides two constraints. For n items of evidence the dependency measures will provide

$$\frac{n!}{(n-2)!}$$

constraints. A final constraint is provided by the probabilities summing to unity. Thus for n items of evidence there will be

$$3^n - 2n - \frac{n!}{(n-2)!} - 1$$

degrees of freedom (Table 5.4). This can be compared with the situation for classical probability theory in Table 5.1.

Table 5.4 Degrees of freedom for compound propositions (after Hall et al., 1998a)

n	1	2	3	4	5	n
Degrees of freedom	*	2	14	60	212	$3^n - 2n - \frac{n!}{(n-2)!} - 1$

* ρ is undefined for $n = 1$.

A comparison with mass assignment theory

IPT is a special case of mass assignment theory in the sense that ☆

1. *the focal elements are always E, $\overline{E}$ and E_U.* This is a special case of the general situation where the focal elements need not be pre-specified.

2. *some additional information is provided by ρ.* The solution is therefore a restriction on the general assignment method.

In the same way as mass assignment theory, IPT is more general than Dempster's rule as it does imply any independence assumption and is more coherent as it does not rely on an *ad hoc* renormalization.

5.10.3 Logical inference

Having established a general method for combining interval probabilities, the situation is now addressed where there is some information that relates to the strength of the relationship between the body of evidence in hand and the hypothesis H of interest. A relatively straightforward solution to this problem is proposed based on the total probability theorem, which is given the interpretation of Jeffrey's rule of conditioning (Jeffrey, 1983, Pearl, 1988, Schum, 1994). The total probability theorem is axiomatic in probability theory:

$$p(B) = \sum_{i=1}^{n} p(B \mid A_i)p(A_i). \quad + P(\bar{A}_i)\cdot C P(B \mid \bar{A}_i)$$

Jeffrey's rule uses this structure as a method for updating a hypothesis in the light of new information. Beliefs before the arrival of the new information are represented as the conditional probabilities $p(B|A_i)$. The new information is then inserted as $p(A_i)$, to calculate the updated probability $p(B)$.

Single item of evidence

Consider a conjecture H to which pertains evidence E. To establish the support $p(H)$ on the basis of the available evidence we require p(E) and some knowledge of the relationship between E and H which is defined by the conditional measures $p(H \mid E)$ and $p(H \mid \overline{E})$. $p(H)$ is obtained from the theorem of total probability

$$p(H) = p(H \mid E)p(E) + p(H \mid \overline{E})p(\overline{E}) \tag{14}$$

which can be rewritten as

$$p(H) = p(H \mid E)p(E) + p(H \mid \overline{E})(1 - p(E)).$$

Dubois and Prade (1990) showed that when all the terms are expressed as intervals the bounds on $p(H)$ are

$$\left.\begin{array}{ll} S_n(H) = S_n(H \mid E)S_n(E) + S_n(H \mid \overline{E})(1 - S_n(E)); & S_n(H \mid E) \geq S_n(H \mid \overline{E}) \\ S_n(H) = S_n(H \mid E)S_p(E) + S_n(H \mid \overline{E})(1 - S_p(E)); & otherwise \end{array}\right\} \tag{15}$$

and

$$\left.\begin{array}{ll} S_p(H) = S_p(H \mid E)S_p(E) + S_p(H \mid \overline{E})(1 - S_p(E)); & S_p(H \mid E) \geq S_p(H \mid \overline{E}) \\ S_p(H) = S_p(H \mid E)S_n(E) + S_p(H \mid \overline{E})(1 - S_n(E)); & otherwise \end{array}\right\} \tag{16}$$

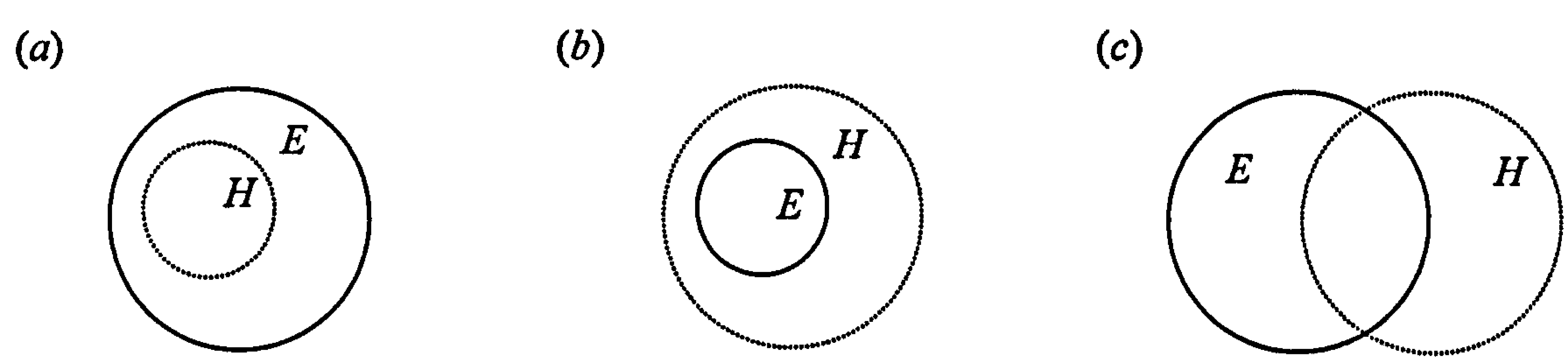

Figure 5.9 Venn diagrams of a) E necessary for H; b) E sufficient for H; c) E relevant to H.

The relationship between E and H is a feature of the structure of the inference problem. For example E may be a *necessary* condition for H (Figure 5.9*a*), in which case

$$p(H \mid E) \leq 1 \qquad p(H \mid \overline{E}) = 0,$$

or E may be a *sufficient* condition for H (Figure 5.9*b*), in which case

$$p(H \mid E) = 1 \qquad p(H \mid \overline{E}) \leq 1.$$

In the sufficient condition there may not be specific evidence relating to $p(H \mid \overline{E})$ so it can be set to any value in the 'unknown' interval of [0,1]. In the special case when E is a *necessary and sufficient* condition for H

$$p(H \mid E) = 1 \qquad\qquad p(H \mid \overline{E}) = 0 .$$

A weaker and more general condition is when E is *relevant* to H (Figure 5.9*c*), in which case

$$0 < p(H \mid E) \le 1 \qquad\qquad 0 \le p(H \mid \overline{E}) \le 1 .$$

Two items of evidence

Suppose now that there are two items of evidence E_1 and E_2 which are germane to H. H can now be partitioned into four mutually exclusive sub-sets, so

$$\begin{aligned} p(H) = {} & p(H \mid E_1 \cap \overline{E_2})p(E_1 \cap \overline{E_2}) + p(H \mid \overline{E_1} \cap E_2)p(\overline{E_1} \cap E_2) \\ & + p(H \mid E_1 \cap E_2)p(E_1 \cap E_2) + p(H \mid \overline{E_1} \cap \overline{E_2})p(\overline{E_1} \cap \overline{E_2}) \end{aligned} \tag{17}$$

where $p(H \mid E_1 \cap \overline{E_2})$, $p(H \mid \overline{E_1} \cap E_2)$, $p(H \mid E_1 \cap E_2)$, $p(H \mid \overline{E_1} \cap \overline{E_2})$ define the relationship between H and E_1 and E_2. The most general relationship is illustrated in Figure 5.10.

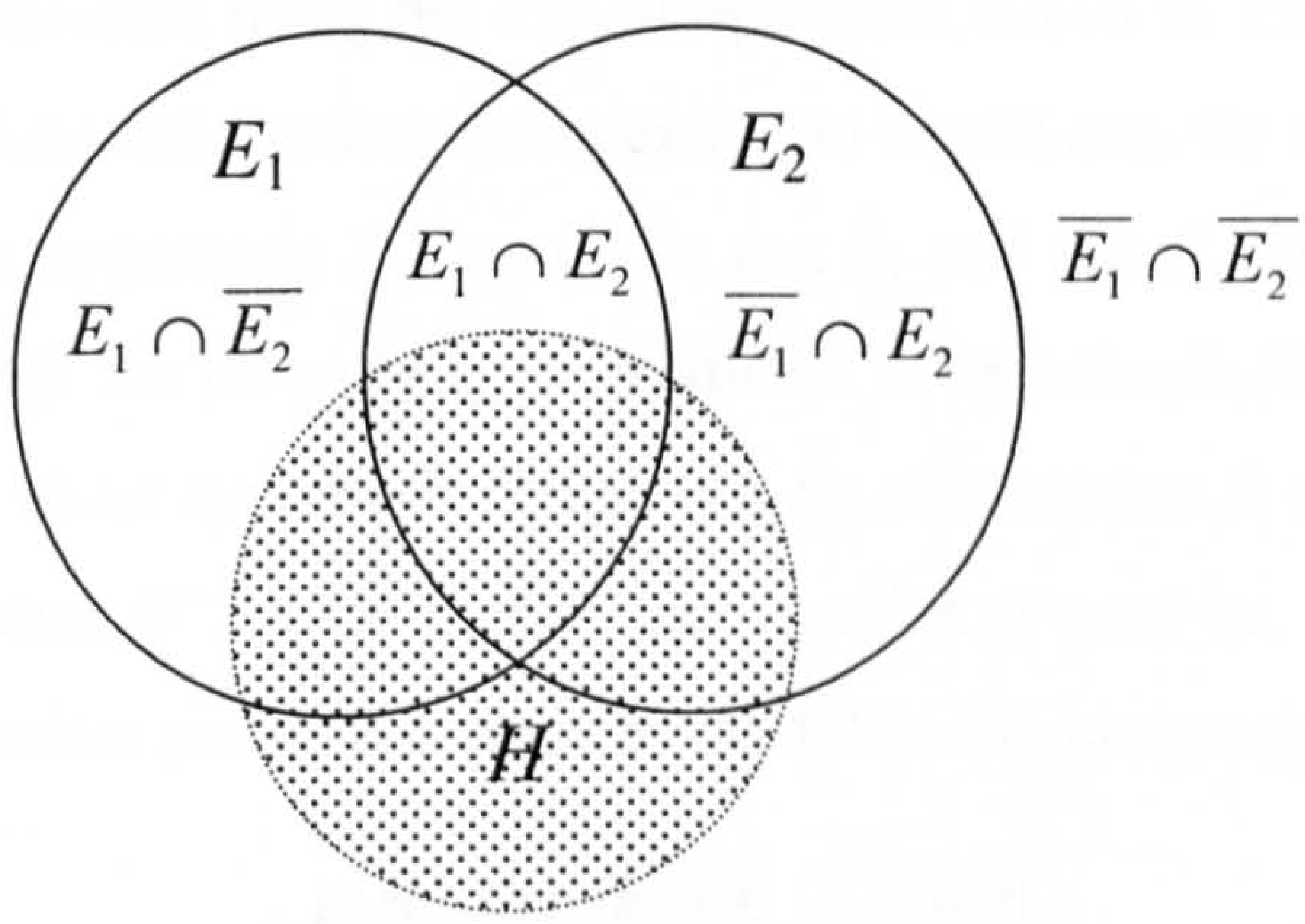

Figure 5.10 Illustration of the power set of H

So, for example, if E_1 and E_2 are both *necessary* conditions for H then

$$p(H \mid E_1 \cap \overline{E_2}) = p(H \mid \overline{E_1} \cap E_2) = p(H \mid \overline{E_1} \cap \overline{E_2}) = 0$$

so

$$p(H) = p(H \mid E_1 \cap E_2).p(E_1 \cap E_2),$$

and if E_1 and E_i are *necessary and sufficient* conditions for H then

$$p(H \mid E_1 \cap E_2) = 1$$

so

$$p(H) = p(E_1 \cap E_2)$$

i.e. the assignments reduce to a logical AND operator.

Table 5.5 Conditional assignments for logical operators

	$p(H \mid E_1 \cap \overline{E_2})$	$p(H \mid \overline{E_1} \cap E_2)$	$p(H \mid E_1 \cap E_2)$	$p(H \mid \overline{E_1} \cap \overline{E_2})$
E_1 AND E_2	0	0	1	0
E_1 OR E_2	1	1	1	0
E_1 XOR E_2	1	1	0	0
NOT E_1	0	1	0	1
NOT E_2	1	0	0	1
NOT (E_1 OR E_2)	0	0	0	1

The use of logical operators AND, OR, XOR, and NOT gives rise to special cases of the general Equation 17. In these special cases each element of the power set of the universe of discourse is either wholly included or wholly excluded from H (see Table 5.5). However, experts in practical evidential situations manipulate ideas of necessity, sufficiency and relevance in richer and more flexible ways than can be expressed using the logical operators shown in Table 5.5. The natural language used in these situations reflects their complexity. An expert may for example explain that "to convincingly demonstrate hypothesis H I would do test E_1 and test E_2, but carrying out only one of the tests may be enough for me to be quite confident in hypothesis H; of the two, test E_1 would probably tell me more about hypothesis H than test E_2; without test E_1 or test E_2 I wouldn't have any idea about hypothesis H". After further interrogation it may be established that the structure of the evidential situation can be described by the following assignments,

$p(H \mid E_1 \cap E_2) = 1.0$ $\qquad$ $p(H \mid E_1 \cap \overline{E_2}) \in [0.4, 0.9]$

$p(H \mid \overline{E_1} \cap E_2) \in [0.2, 0.6]$ $\qquad$ $p(H \mid \overline{E_1} \cap \overline{E_2}) \in [0.0, 1.0]$.

Establishing the relevance of evidence is a delicate empirical process. Great care is required in mapping from natural language to the mathematical structure of the problem. For example when in the above testimony the expert states that "...carrying out only one of the tests may be enough for me to be quite confident in hypothesis H; of the two, test E_1 would probably tell me more about hypothesis H than test E_2..." she would normally be referring to $p(H \mid E_1)$ and $p(H \mid E_2)$ and would require further interrogation in order to establish $p(H \mid E_1 \cap \overline{E_2})$ and $p(H \mid \overline{E_1} \cap E_2)$.

Bounds on $p(H)$ in Equation 17 can be found by testing each of the permissible factorisations and the family of values for $p(E_1 \cap \overline{E_2})$ and $p(\overline{E_1} \cap E_2)$ (recall that $p(E_1 \cap \overline{E_2})$ and $p(\overline{E_1} \cap E_2)$ will not generally be uniquely defined),

$$S_n(H) = \inf_{\substack{i \\ m_{12}, m_{21}}} \left\{ \begin{array}{l} S_n(H|E_1 \cap E_2)S_i(E_1 \cap E_2) + S_n(H|E_1 \cap \overline{E_2})S_i(E_1 \cap \overline{E_2}) \\ + S_n(H|\overline{E_1} \cap E_2)S_i(\overline{E_1} \cap E_2) + S_n(H|\overline{E_1} \cap \overline{E_2})S_i(\overline{E_1} \cap \overline{E_2}) \end{array} \right\} \tag{18}$$

and

$$S_p(H) = \sup_{\substack{i \\ m_{12}, m_{21}}} \left\{ \begin{array}{l} S_p(H|E_1 \cap E_2)S_i(E_1 \cap E_2) + S_p(H|E_1 \cap \overline{E_2})S_i(E_1 \cap \overline{E_2}) \\ + S_p(H|\overline{E_1} \cap E_2)S_i(\overline{E_1} \cap E_2) + S_p(H|\overline{E_1} \cap \overline{E_2})S_i(\overline{E_1} \cap \overline{E_2}) \end{array} \right\} \tag{19}$$

where $S_i = S_1 ... S_{16}$ are the permissible factorisations (see Table 5.6), each representing a different permutation of the nine assignments of the compound proposition, such that $S_n \le S_i \le S_p$ and $S_i(E_1 \cap E_2) + S_i(E_1 \cap \overline{E_2}) + S_i(\overline{E_1} \cap E_2) + S_i(\overline{E_1} \cap \overline{E_2}) = 1$.

Inspection of Equations 18 and 19 together with Table 5.6 demonstrates that the most general values of $S_n(H)$ and $S_p(H)$ will be found when m_{12} or m_{21} are at a minimum.

Table 5.6 Permissible combinations of E_1 and E_2

i	$S_i(E_1 \cap E_2)$	$S_i(E_1 \cap \overline{E_2})$	$S_i(\overline{E_1} \cap E_2)$	$S_i(\overline{E_1} \cap \overline{E_2})$
1	$m_{11} + m_{13} + m_{31} + m_{33}$	$m_{12} + m_{32}$	$m_{21} + m_{23}$	m_{22}
2	$m_{11} + m_{13} + m_{31} + m_{33}$	m_{12}	$m_{21} + m_{23}$	$m_{22} + m_{32}$
3	$m_{11} + m_{13} + m_{31} + m_{33}$	$m_{12} + m_{32}$	m_{21}	$m_{22} + m_{23}$
4	$m_{11} + m_{13} + m_{31} + m_{33}$	m_{12}	m_{21}	$m_{22} + m_{23} + m_{32}$
5	$m_{11} + m_{31}$	$m_{12} + m_{13} + m_{32} + m_{33}$	$m_{21} + m_{23}$	m_{22}
6	$m_{11} + m_{31}$	$m_{12} + m_{13} + m_{32} + m_{33}$	m_{21}	$m_{22} + m_{23}$
7	m_{11}	$m_{12} + m_{13} + m_{32} + m_{33}$	$m_{21} + m_{23} + m_{31}$	m_{22}
8	m_{11}	$m_{12} + m_{13} + m_{32} + m_{33}$	$m_{21} + m_{31}$	$m_{22} + m_{23}$
9	$m_{11} + m_{13}$	m_{12}	$m_{21} + m_{23} + m_{31} + m_{33}$	$m_{22} + m_{32}$
10	$m_{11} + m_{13}$	$m_{12} + m_{32}$	$m_{21} + m_{23} + m_{31} + m_{33}$	m_{22}
11	m_{11}	$m_{12} + m_{13}$	$m_{21} + m_{23} + m_{31} + m_{33}$	$m_{22} + m_{32}$
12	m_{11}	$m_{12} + m_{13} + m_{32}$	$m_{21} + m_{23} + m_{31} + m_{33}$	m_{22}
13	$m_{11} + m_{13}$	m_{12}	$m_{21} + m_{31}$	$m_{22} + m_{23} + m_{32} + m_{33}$
14	$m_{11} + m_{13} + m_{31}$	m_{12}	m_{21}	$m_{22} + m_{23} + m_{32} + m_{33}$
15	m_{11}	$m_{12} + m_{13}$	$m_{21} + m_{31}$	$m_{22} + m_{23} + m_{32} + m_{33}$
16	$m_{11} + m_{31}$	$m_{12} + m_{13}$	m_{21}	$m_{22} + m_{23} + m_{32} + m_{33}$

Table 5.7 Example of some permissible assignments to $m_{11} ... m_{33}$ using example shown in Figure 5.8b.

Case	m_{11}	m_{12}	m_{13}	m_{21}	m_{22}	m_{23}	m_{31}	m_{32}	m_{33}
1	0.06	0.00	0.24	0.09	0.15	0.06	0.05	0.35	0.00
2	0.06	0.00	0.24	0.14	0.15	0.01	0.00	0.35	0.05
3	0.06	0.09	0.15	0.00	0.15	0.15	0.14	0.26	0.00
4	0.06	0.24	0.00	0.00	0.15	0.15	0.14	0.11	0.15
5	0.06	0.24	0.00	0.14	0.15	0.01	0.00	0.11	0.29

For example suppose that $p(E_1) \in [0.3, 0.7]$ and $p(E_2) \in [0.2, 0.5]$ and $\rho \in [0.3, 0.7]$ as shown in Figure 5.8*b*. A range of permissible values for m_{12} and m_{21} and the corresponding assignments $m_{11} \ldots m_{33}$ are listed in Table 5.7.

Now suppose that the inference problem has a rather general structure so that $E_1 \cap E_2$, $E_1 \cap \overline{E_2}$, $\overline{E_1} \cap E_2$ and $\overline{E_1} \cap \overline{E_2}$ are all relevant to H and

$p(H \mid E_1 \cap E_2) \in [0.5, 0.9]$ $\qquad$ $p(H \mid E_1 \cap \overline{E_2}) \in [0.7, 0.9]$

$p(H \mid \overline{E_1} \cap E_2) \in [0.2, 0.6]$ $\qquad$ $p(H \mid \overline{E_1} \cap \overline{E_2}) \in [0.0, 1.0]$.

The calculation of the values of $S_n(H)$ and $S_p(H)$ according to Table 5.6 using the assignments in Table 5.7 is illustrated in Table 5.8. In this case the lower bound on $S_n(H) = 0.18$, which is found when m_{12} is at its minimum, and the upper bound on $S_p(H) = 0.96$, which is found when m_{21} is at its minimum. Thus the most general inference is that $p(H) \in [0.18, 0.96]$.

Table 5.8 Example of calculation of bounds on $S_n(H)$ and $S_p(H)$ using assignments shown in Table 5.6

i	Case 1		Case 2		Case 3		Case 4		Case 5	
	$S_n(H)$	$S_p(H)$	$S_n(H)$	$S_p(H)$	$S_n(H)$	$S_p(H)$	$S_n(H)$	$S_p(H)$	$S_n(H)$	$S_p(H)$
1	0.45	0.87	0.45	0.87	0.45	0.87	0.45	0.87	0.45	0.87
2	0.21	0.91	0.21	0.91	0.27	0.90	0.37	0.88	0.37	0.88
3	0.44	0.89	0.45	0.87	0.42	0.93	0.42	0.93	0.45	0.87
4	0.19	0.93	0.20	0.91	0.24	**0.96**	0.34	0.94	0.37	0.89
5	0.19	0.89	0.19	0.89	0.23	0.85	0.29	0.79	0.29	0.79
6	0.44	0.86	0.44	0.86	0.41	0.83	0.36	0.78	0.36	0.78
7	0.24	0.89	0.24	0.89	0.26	0.85	0.29	0.79	0.29	0.79
8	0.48	0.86	0.48	0.86	0.44	0.83	0.36	0.78	0.36	0.78
9	0.50	0.87	0.51	0.87	0.48	0.87	0.48	0.87	0.51	0.87
10	0.49	0.89	0.51	0.87	0.45	0.93	0.45	0.93	0.51	0.87
11	0.48	0.86	0.51	0.87	0.44	0.83	0.44	0.83	0.51	0.87
12	0.47	0.88	0.51	0.87	0.41	0.89	0.41	0.89	0.51	0.87
13	**0.18**	0.91	**0.18**	0.91	0.20	0.91	0.23	0.91	0.23	0.91
14	0.19	0.93	**0.18**	0.91	0.24	**0.96**	0.27	**0.96**	0.23	0.91
15	0.23	0.91	0.23	0.91	0.23	0.91	0.23	0.91	0.23	0.91
16	0.24	0.93	0.23	0.91	0.27	**0.96**	0.27	**0.96**	0.23	0.91

<u>*n* items of evidence</u>

It there are n items of evidence $E_1 \ldots E_n$ then the potential sample space of H can be partitioned into a power set with j elements $H|\theta_1 \ldots H|\theta_j$ and

$$p(H) = \sum_{i=1}^{j} p(H \mid \theta_i) p(\theta_i) \qquad \text{where } j = 2^n \tag{20}$$

The first term in the summation in Equation 20 determines the relationship between the body of evidence and H and is referred to as the *relevance*. The second term is calculated from the evidence $E_1 \ldots E_n$ and the dependencies between the various items of evidence. For two items of evidence it is practical, albeit inelegant, to generate all of the possible solutions and identify the least conservative bounds, which is the method illustrated in Table 5.7 and Table 5.8. For more than two items of evidence the problem rapidly becomes computationally intractable so it is necessary to establish a method for finding the required bounds. To do so rewrite Equation 20 as

$$p(H) = \sum_{i=1}^{j} C_i P_i$$

where C_i is the conditional probability $p(H|\theta_i)$, and P_i is the probability $p(\theta_i)$ in Equation 20.

Since $0 \leq C_i \leq 1$ for all i, and

$$\sum_{i=1}^{j} P_i = 1,$$

then $0 \leq p(H) \leq 1$ for all C_i and P_i, as expected. $S_n(H)$ is found when the right hand side of Equation 20 is minimised and the $S_p(H)$ is found when it is maximised. The following discussion is all in terms of $S_n(H)$, from which the method for finding $S_p(H)$ follows without need for further explanation. The probabilities P_i can each vary on $[S_n(P_i), S_p(P_i)]$ subject to

$$\sum_{i=1}^{j} P_i = 1.$$

Suppose that the conditional statements are ordered $C_1 \geq C_2 \geq \ldots \geq C_n$. In that case the least bound on $S_n(H)$ will be found when P_1 is minimised, then P_2 is minimised and so on. Consider the case with two items of evidence as illustrated in Table 5.6. The minimisation process has two stages:

1. Find the minimum permissible set of subsets m allocated to P_i.

2. Minimise, in order, the assignments which vary, to find the required bound.

Consider the example introduced above in which

$S_n(C_1) = 0.5$; $S_n(C_2) = 0.7$; $S_n(C_3) = 0.2$; $S_n(C_4) = 0.0$

i.e. $S_n(C_2) > S_n(C_1) > S_n(C_3) > S_n(C_4)$.

In the first stage, the subsets should therefore allocated as follows

$P_1 = m_{11} + m_{13}$

$P_2 = m_{12}$

$P_3 = m_{21} + m_{31}$

$P_4 = m_{22} + m_{22} + m_{32} + m_{33}$

There are two degrees of freedom, m_{12} and m_{21}. In the second stage m_{12}, which is lower in the ordering than m_{21}, should be minimised first, *i.e.* set to a value of 0.

The problem is essentially one of linear optimisation subject to the row and column constraints that define a convex space. The problem can therefore be tackled with the established tools of linear programming. In Section 5.10.2 it was established that in IPT a compound proposition of n items of evidence will have

$$3^n - 2n - \frac{n!}{(n-2)!} - 1$$

degrees of freedom. In other words there will be 3^n elements m and

$$2n + \frac{n!}{(n-2)!} + 1$$

constraints. A method for conducting the optimisation has been coded in C (using a simplex algorithm from Press *et al.* (1992)) for up to four items of evidence and tested for consistency against a (considerably slower) macro developed in a spreadsheet.

<u>Comparison of the total probability theorem method with other set-based belief formulations</u>

A method based on the total probability theorem is mathematically coherent and, being based on a fundamental theorem of probability, is quite straightforward to understand in principle. It is essentially an interval mapping from E space to H space. This is rather different to the approach that would be adopted in Dempster-Shafer or mass assignment theory where another set, representing the restriction on the E space that is occupied by H, would be introduced. In that way the evidence relating to a system could be modelled as a single coherent mathematical lattice, rather than the present approach which uses a power set at each level in the hierarchy connected to the next level by a set of weights in the total probability theorem. In practice the total probability approach has proved to be straightforward to explain, as well as being robust and effective in the context of this research.

5.11 Support Logic Programming

IPT has close links, and indeed common origins (Blockley *at al.*, 1983, Blockley and Baldwin, 1987), with Support Logic Programming (SLP) (Baldwin, 1986, Baldwin, 1987, Blockley, 1987, Baldwin, 1993). SLP forms the basis of manipulation of probabilistic uncertainty in the logic programming language Fril (Baldwin and Martin, 1994, Baldwin *et al.*, 1995*a*).

In common with IPT, the uncertainty measure in SLP is represented as a support pair $[S_n(E), S_p(E)]$, where

$$S_n(E) + (1 - S_p(E)) \leq 1,$$

so that the necessary support for a proposition and the necessary support for its negation do not necessarily sum to unity. Thus in SLP, as in IPT, support pairs represent a basic probability assignment or mass assignment, with focal elements $\{E\}$, $\{\overline{E}\}$, $\{E, \overline{E}\}$.

Support pairs for compound propositions are addressed in the same tableau format as Figure 5.7. Baldwin (1986, 1987) identifies a range of models for combination of evidence.

1. *Multiplication model*: In the multiplication rule the probabilities assigned to each focal element are multiplied, so

   $m_{11} = S_n(E_1)\, S_n(E_2)$

   $m_{12} = S_n(E_1).(1 - S_p(E_2))$

   ...

   $m_{33} = (S_p(E_1) - S_n(E_1)).(S_p(E_2) - S_n(E_2)).$

   This corresponds to the situation when the two assignments are independent, and is one of the solutions that is obtained in IPT when ρ is set to its independence value *i.e.* $\rho = \max(p(E_1), p(E_2))$. Note, however, that in IPT only m_{11} and m_{22} are fixed and the remainder of the cells can vary, subject to the row and column constraints, so the independence case in IPT may result in a family of distributions, rather than a unique solution, one of which will correspond to the multiplication model in SLP.

2. *Min model*: In the Min model, the Min rule of fuzzy logic is assumed for the main diagonal cells of the assignment tableau, so

   $m_{11} = \min(S_n(E_1), S_n(E_2))$

   $m_{22} = \min((1 - S_p(E_1)), (1 - S_p(E_2)))$

   $m_{33} = \min((S_p(E_1) - S_n(E_1)).(S_p(E_2) - S_n(E_2)),$

   which is sufficient to obtain a unique mass assignment. This situation corresponds to the maximum dependence combination in IPT *i.e.* $\rho = 1$, with the additional constraint that as well as fixing cells m_{11} and m_{22}, cell m_{33} is also fixed. Thus the SLP Min rule will be one of the possible solutions in IPT when $\rho = 1$.

3. *Mutual exclusion model*: In SLP's mutual exclusion model case $m_{11} = 0$. This usually corresponds to the situation in IPT when ρ is set to its minimum value *i.e.*

   $$\rho = \max\left[\frac{p(E_1) + p(E_2) - 1}{\min(p(E_1), p(E_2))}, 0\right].$$

   In this case both m_{11} and m_{22} are defined in IPT.

In general the multiplication model is used in Fril. Where there is a known dependency between items of evidence this has to be included by rewriting the conjunction in terms of a logical inference problem and using conditional supports to provide the support pair for the conjunction (Baldwin, 1987).

In common with IPT, the inference rule in SLP is based on the total probability theorem (Baldwin, 1992), which is given the interpretation of Jeffrey's rule (Baldwin, 1991*d*, Baldwin *et al.*, 1995*a*). Baldwin (1987) initially proposed the bounds on the inference

$$p(H) = p(H \mid E)p(E) + p(H \mid \overline{E})p(\overline{E})$$

as

$$S_n(H) = S_n(H \mid E)S_n(E) + S_n(H \mid \overline{E})(1 - S_p(E))$$

$$S_p(H) = 1 - (1 - S_p(H \mid E))S_n(E) - (1 - S_p(H \mid \overline{E}))(1 - S_p(E)).$$

However, the less conservative bounds identified by Dubois and Prade (1990) and given in Equations 15 and 16 have been implemented in subsequent versions (Baldwin, 1993, Baldwin *et al.*, 1995*a*). Baldwin *et al.* (1995*a*) provide an algorithm analogous to the optimisation developed for IPT in Section 5.10.3. The starting point for the SLP algorithm is that the probability bounds on all of the mutually exclusive sub-sets of the universe of discourse have been calculated, presumably using the multiplication meet, which generates a unique probability distribution across these mutually exclusive sub-sets. Finding the bounds on the inference $p(H)$ is then a deterministic process. This is distinct from the approach developed in Section 5.10.3, which starts with the bounds on each (generally not mutually exclusive) sub-set representing an item of evidence, together with the dependencies between those sub-sets. In this more general case there is no unique probability distribution over the space of mutually exclusive sub-sets. Instead there is a family of permissible distributions. In the IPT algorithm the full space of permissible assignments is then used in the optimisation. It would be possible to generate bounds on each of the mutually exclusive sub-sets and then use the SLP algorithm. However, unlike in the multiplication meet in SLP, in IPT the bounds are not a consequence of a unique distribution, so do not necessarily co-exist. The inference mechanism would therefore generate rather conservative bounds. A more precise inference has been achieved by searching the whole space of permissible solutions to find the greatest lower bound and the least upper bound.

In Fril it is recognised that a proposition may be supported by different paths of evidence. The existence of multiple proof paths will serve to reduce the uncertainty in a proposition. This is achieved by identifying the support pairs that are consistent with the support pairs from the different proof paths. The following example is provided by Baldwin (1991*d*), in which the support for proposition A from proof paths 1 and 2 respectively are

$p(A_1) \in [0.3, 0.6]$ and $p(A_2) \in [0.2, 0.5]$.

The problem is then to find a support pair $[p, q]$ that is consistent with the two assignments *i.e.* from $p(A_1)$: $0.3 \le p \le 0.6$ and from $p(A_2)$: $0.2 \le p \le 0.5$, and

from $p(A_1)$: $0.4 \le q \le 0.7$ and from $p(A_2)$: $0.5 \le q \le 0.8$

subject to $p + q = 1$.

In this case the support from the different proof paths is $p(A) \in [0.3, 0.5]$ and $p(\overline{A}) \in [0.5, 0.7]$. Note how the existence of evidence from different proof paths has decreased the uncertainty in the evidence, in this case from 0.3 to 0.2. There may be circumstances when the evidence from $p(A_1)$ and $p(A_2)$ is inconsistent, so it is not possible to identify a support pair that is consistent with both items of evidence, in which case the Fril user is alerted to the inconsistency. Alternatively the user can combine the evidences using Dempter's rule of combination.

5.12 Evaluation of uncertainty calculi

The various mathematical treatments of uncertainty introduced in Sections 5.4 to 5.11 can now be evaluated against the criteria established in Section 5.3. The evaluation involves a combination of syntactic, semantic and procedural issues. Ultimately the debate resolves to the issue of which axioms are more appropriate for a particular evidential situation. In other words it relates to the appropriateness of a particular syntax to convey the required semantic meaning.

1. *The syntax should be capable of modelling uncertainty in hierarchical process models.*

All of the approaches examined have been used in inference networks so should in principle lend themselves to hierarchically structured models. Recent developments in IPT have taken place with hierarchical process modelling in mind, making it well suited to the purpose. SLP has also been successfully applied to hierarchical structures (Baldwin *et al.*, 1994, Baldwin *et al.*, 1995*b*).

2. *The syntax should reflect the types of uncertainty in the available evidence.*

In Section 5.3 it was suggested that expert judgements of process dependability could be characterised by vagueness, ambiguity, conflict, incompleteness, or a combination of the four. Each of these types of uncertainty will be addressed.

Vagueness. Zadeh (1978) argued that this impression is possibilistic rather than probabilistic in nature and developed fuzzy set theory and possibility theory in an attempt to represent this imprecision. Mass assignment theory is a generalisation of fuzzy set theory. Fuzziness can also be addressed by recourse to the fundamental systems concept of hierarchy, with precisely defined low level concepts being nested within more vaguely defined high level concepts. The hierarchical implementation of IPT enables this hierarchical representation of fuzziness. In Figures 3.2 and 3.3 (Chapter 3) the vague high level concepts, like "proneness to failure of the project in an earthquake" or "developing Dunlin Northwest" are progressively decomposed into more and more precise processes and items of evidence. Figures 3.3. and 3.4, and hierarchical models later in this thesis, illustrate the tension between hierarchical ordering of concepts according to their fuzziness, and faithful representation of the flow of evidence. This tension is revisited in the discussion of the case studies in Section 7.5.

Ambiguity. Numerical structures that assign probability to the power set rather than of necessity to specific singleton sub-sets allow non-committal statements. This is true of basic probability assignments, Dempster-Shafer structures and mass assignment theory. It is also true of SLP and IPT, which have been shown to be a special case of these structures, with three focal elements.

Conflict. It has been demonstrated how Bayesian probability and Dempster's rule of combination artificially suppress conflict in the inference situation, sometimes with undesirable consequences. Mass assignment theory and IPT endeavour to minimise conflict, but, where it is an inevitable aspect of the inference, reflect it in the numerical structure. This enables areas of conflict and its sources to be identified and explored.

Incompleteness. The use of the total probability theorem for logical inference in SLP and IPT allows the strength of inference to be weighted by a judgement of the completeness of the evidence in the context of the hypothesis in question. It therefore requires explicit reflection on issues of incompleteness. This, combined with the capacity to make non-committal statements, enables an open world approach in which phenomena that are unknown are nonetheless acknowledged and regarded as being uncertain.

3. *The axioms should not be so weak as to provide inferences that are of limited practical use, yet on the other hand it should not artificially constrain the problem, implying less uncertainty than is in fact the case.*

There is an inevitable compromise between forced idealisation and information content. Bayesian approaches are constrained by a strict set of axioms which severely limit their applicability but which enable them to make rather precise inferences. As axiom sets are weakened to embrace the complexity of the real world, inferences become correspondingly general. Probability assignments to the power set are the general case, which should be restricted to the special probabilistic case of assignments to singletons only when there is information to demonstrate the validity of the assumptions.

A general practical problem that is encountered with interval representations is that in large inference networks the inference which is derived from the network has wide bounds, *i.e.* it has a very high level of uncertainty associated with it. The uncertainty in the inference no coincidence. It is a logical consequence of the various judgements input in the network and the axioms used to manipulate the uncertainty measures. An expert's intuition that the uncertainty is too high is function of natural biases in human reasoning which under-estimate the level of uncertainty in a complex inferential situation (Royal Society, 1992), rather than a reflection of a fault in the mathematics. Nonetheless, mathematical outcomes must be of some practical use, and when confronted with very wide interval bounds a decision-maker may be at a loss about how to make a choice. It will be seen in Chapter 6 that Bayesian probabilities provide a normative basis for choice, whilst there is no normative theory of choice based on intervals. An interval-based inference with a very high level of uncertainty gives the decision-maker an important message about the absence of information and will provide some guidance as to where information-gathering activities should be directed to remedy the situation.

It has been demonstrated how SLP uses evidence from different proof paths to narrow the bounds on inference problems.

4. *The syntax should be able to represent varied dependency relationships between evidence.*

Bayesian methods require elicitation of joint distributions of related propositions and use these to take account of dependency. Dempster's rule is analogous to an independence assumption, while the operations of fuzzy set theory can be interpreted as corresponding to a total dependence assumption – their practical applicability is therefore severely limited. The dependence parameter ρ in ITP and the general set of conjunctive expressions described by T-norms enable a range of dependence relationships to be explored. In this way domain experts are forced to consider dependency relationships in the knowledge base. The dependence parameter may be unknown, *i.e.* any value in [0,1], in which case the axioms of the theory are correspondingly weak.

5. *The syntax should reflect a range of inferential relationships*

The new approach to uncertain inference using IPT and the total probability theorem provides a convenient way of representing a rich range of inferential relationships. This is an approach that has been proven in SLP and implemented in Fril.

6. *The syntax should be capable of being implemented in a reasonably straightforward manner that will make it accessible to coastal engineering practitioners*

The basis structure of IPT is straightforward and easy to explain. In common with SLP, IPT has only three focal elements so is a special case of more general evidential reasoning structures, which can represent more complex fuzzy and random sets. The flexibility of these more general structures is sacrificed for the simplicity of the basic interval representation. Inference based on the total probability theorem is also straightforward to explain in principle, though eliciting judgements can be delicate in multi-dimensional problems, an issue that is addressed below. The implementation of IPT is described in Section 5.13.

7. *The syntax should enable straightforward elicitation of reasonably bias-free judgements.*

Most of the psychological literature of behavioural decision theory (see for example Kahneman *et. al.*, 1982) is based around identification, comprehension and elimination of biases in subjective probabilities as they relate to Bayesian models of uncertainty. It is generally accepted that the Bayesian model is normative rather than descriptive (Bell *et al.*, 1988). There is evidence to suggest that this is also the case for methods based on fuzzy sets (Johnston and Palmer, 1988).

In the field of medical diagnosis, where much useful work on uncertainty has been carried out, it has been shown that it is cognitively easier to make probability assessments in the 'causal direction' (Buxton, 1989). For example, given that a patient has a symptom, it is difficult to make an assessment of the probability of a disease that would cause the symptom. It is easier cognitively to assess the probability of a symptom, given that the patient has a disease, which maps quite conveniently onto Bayesian likelihood. When, however, the situation is one of design rather than diagnosis, probability assessments in the 'causal direction' are much less meaningful. The need for Bayesian priors then becomes a significant handicap.

Shafer's theory has, however, been strongly criticised for its failure to give any meaning to the measures of belief and plausibility, or to show how someone might arrive at a particular numerical assessment (Buxton, 1989). In practice most theories, other than the Bayesian approach, which corresponds to a betting model, suffer from difficulty in interpreting results. Possibility distributions typically extend across a range of values, which means that the elicitation of possibilistic functions from human sources may be more complex than the elicitation of point probabilities (Krause and Clark, 1993).

In practice an interval representation has proved to be intuitively appealing. In principle it suffers from the same criticism of interpretation as Shafer's theory mentioned above, though voting model explanations can help to attach meaning to the numerical structures. More difficulty is encountered in practice in interpreting the conditional probability assignments that are needed in logical inference using the total probability theory. Although the idea of a probability weighting is intuitively appealing, the meaning of different elements of the power set, to which conditional probabilities have to assigned, is not always clear. However, these problems of interpretation seem to be no more severe than for any of the alternative formulations.

The balance of the foregoing discussion indicates that IPT satisfies, in a relatively simple manner, the desirable features for uncertainty representation. It has therefore been adopted as a basis for uncertainty representation in hierarchical process models. In general there is no single best approach to uncertainty representation, and no claims are being made about the general suitability of IPT. However, when assessed against a set of criteria that are desirable in the context of the current research IPT has proved to be appropriate. As with all uncertainty methods, the test of its appropriateness comes above all through use. Evidence from use in practice is presented in Chapter 7.

5.13 Application of Interval Probability Theory to uncertainty representation in hierarchical process models

Having identified IPT as an appropriate calculus for manipulating uncertainty in hierarchical process models, this section outlines how in practice this was achieved. A sound and appropriate uncertainty calculus is merely the first step in the development of a useful uncertainty management tool. The tool must be straightforward to use and should effectively communicate the structure of the evidential situation in hand.

The research was assisted by, as well as contributing to, concurrent work in the Department of Civil Engineering on uncertainty management in oil exploration. As part of the oil industry research a Windows-based software tool call Juniper had been developed in Visual C++ to support hierarchical process modelling. Juniper included an implementation of an early version of IPT, including various *ad hoc* rules to give inferential behaviour that was considered to be desirable by the oil industry participants.

The existence of the Juniper software meant that the new developments in IPT described in Section 5.10 could be implemented in a Windows environment without having to develop new software from scratch. The new developments required fundamental changes to the uncertainty algorithms and significant changes to the graphical user interface and programme structure. However, the existence of the established Juniper software meant that substantive issues of uncertainty handling became the focus of the programming work, and once these had been resolved the research could proceed to case studies (see Chapter 7).

5.13.1 The model structure

A number of important aspects of process models, for example process attributes, were introduced in the Chapter 3. Here the emphasis will be on how in practice uncertainty propagation with IPT has been integrated with hierarchical process models.

The basic entities in the software developed for hierarchical process modelling with IPT are *processes* and *nodes* (see Figure 5.11). Processes have an interval probability associated with them, which is either entered by the user or calculated from sub-processes. Nodes contain the information about the relationship between processes, in terms of conditional probabilities and dependency measures, which are also expressed as interval probabilities.

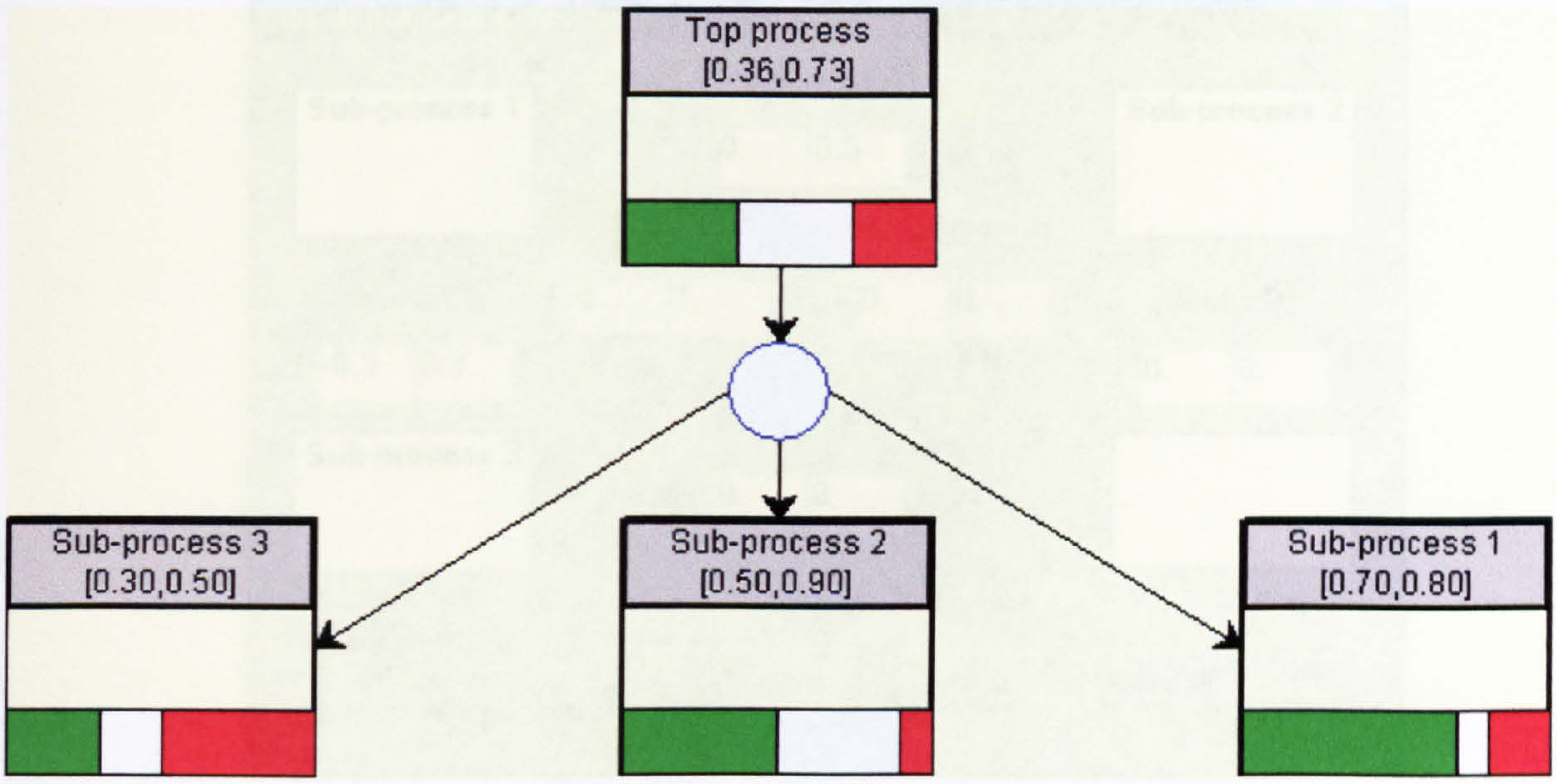

Figure 5.11 Simple process model

The three-shaded bar at the bottom of each process represents the interval probability associated with that process. The available evidence relating to the dependability of the process is mapped onto this interval probability. Criteria for dependability were discussed in Chapter 4. The green on the left side of the bar represents S_n, the necessary support for the process, in other words the evidence supporting the hypothesis that the process is dependable. The red represents 1 - S_p, and the white in the middle represents the uncertainty S_p - S_n.

The model is constructed in a top-down manner starting with the top process. The user has to enter:

1. Interval probabilities representing the evidential support for each 'leaf' process in the hierarchy. These are entered by double clicking on the process icon and either typing the interval numbers, or dragging the three coloured bar (Figure 5.12).

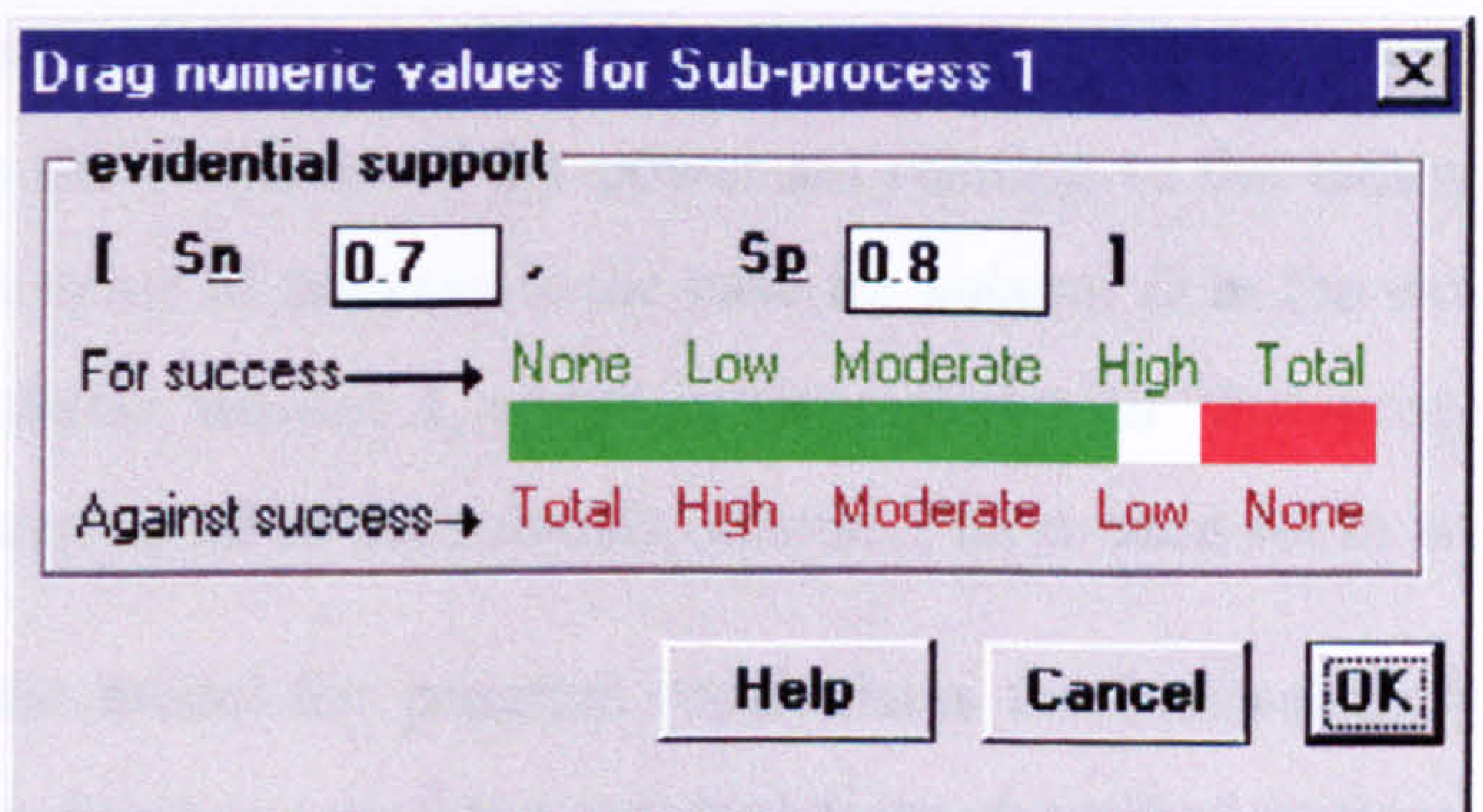

Figure 5.12 Dialog for entering interval numbers

2. Dependency measures, which represent the relationship between sub-processes. Interval measures of dependency are entered for each pair of sub-processes. In the example shown in

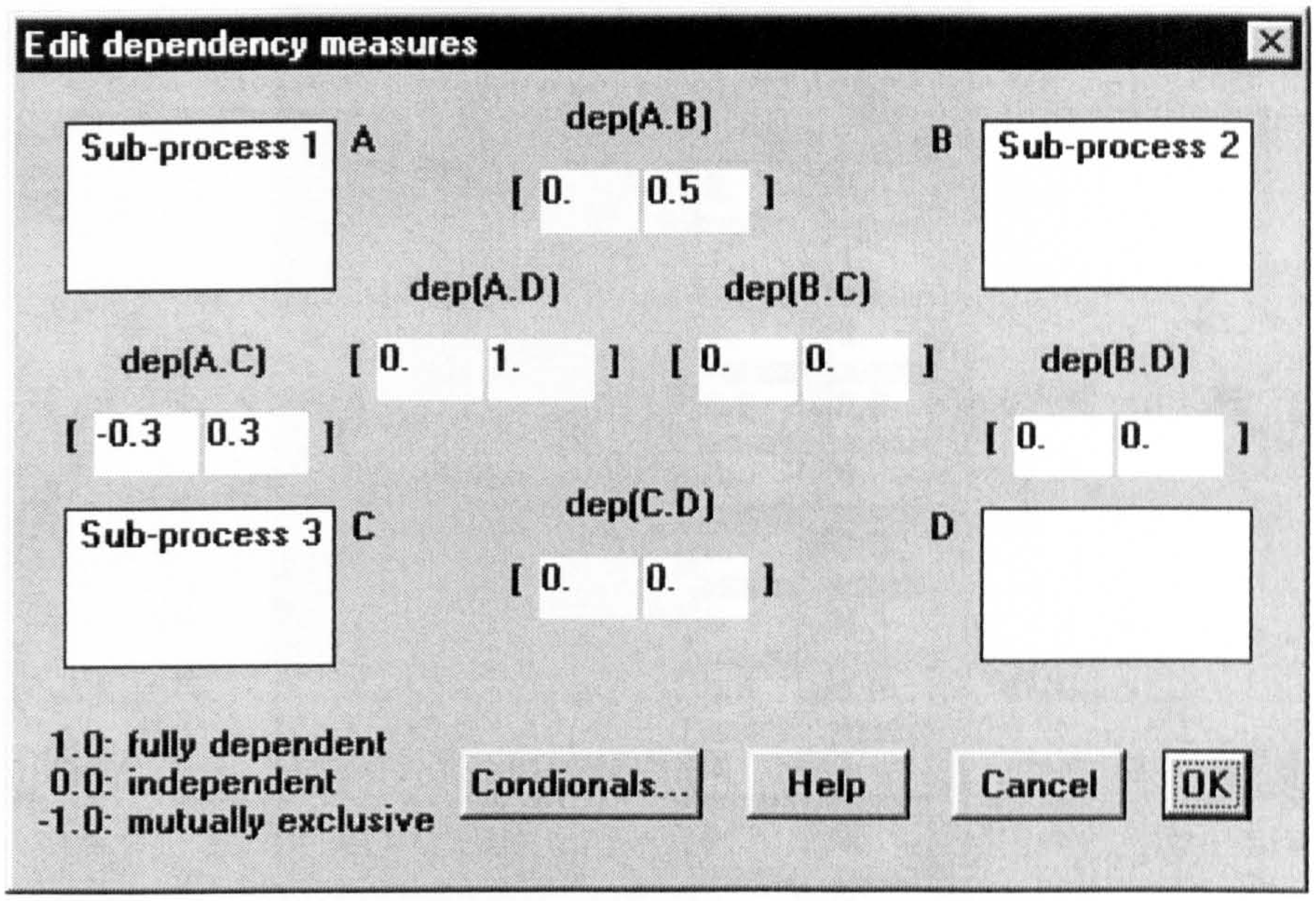

Figure 5.13 Dialog for entering dependency measures

In Figure 5.13 there are three sub-processes so the dependency values for relating to the fourth process are left at the default value of zero. Dependency measures are entered on a scale from [-1,1], with 0 representing independence. A linear transformation is then used to convert this to ρ on a scale [0,1]. This approach has been adopted because, from the definition of ρ, the value of ρ representing independence varies with the level of evidence assigned to or calculated for the relevant sub-processes. The use of the linear transformation means that the user can make a simple judgement of dependency on a three point scale from mutual exclusion to independence to full dependence without having to calculate the independence value of ρ.

3. Conditional probabilities, which represent the relationship between sub-processes and the super-process. The dialog box for entering conditional probabilities is accessed by clicking the "conditionals" button in the dependency dialog. As with the dependency dialog, the conditionals box is set up for up to four sub-processes. In situations where there are fewer than four sub-processes the elements of the power set relating to the redundant sub-processes are left at their default value of zero (as is the case for sub-set D in the example shown in Figure 5.14). In the case shown sub-set A, which is associated with "Sub-process 1" is considered to be logically necessary so all of the sub-sets outside A have been set to zero.

Upon any change to the model the program recalculates the interval probabilities in the whole hierarchy, up to the top process, using the manipulations described in Section 5.10. As explained in Section 5.10.3, this involves repeated optimisations using the simplex algorithm to find the least conservative bounds on the interval solution, so can be practice be quite time-consuming in large hierarchies. To overcome the inconvenience of having to wait for the model to recalculate it is possible to disable recalculation during a session of model development and re-enable recalculation at a convenient moment.

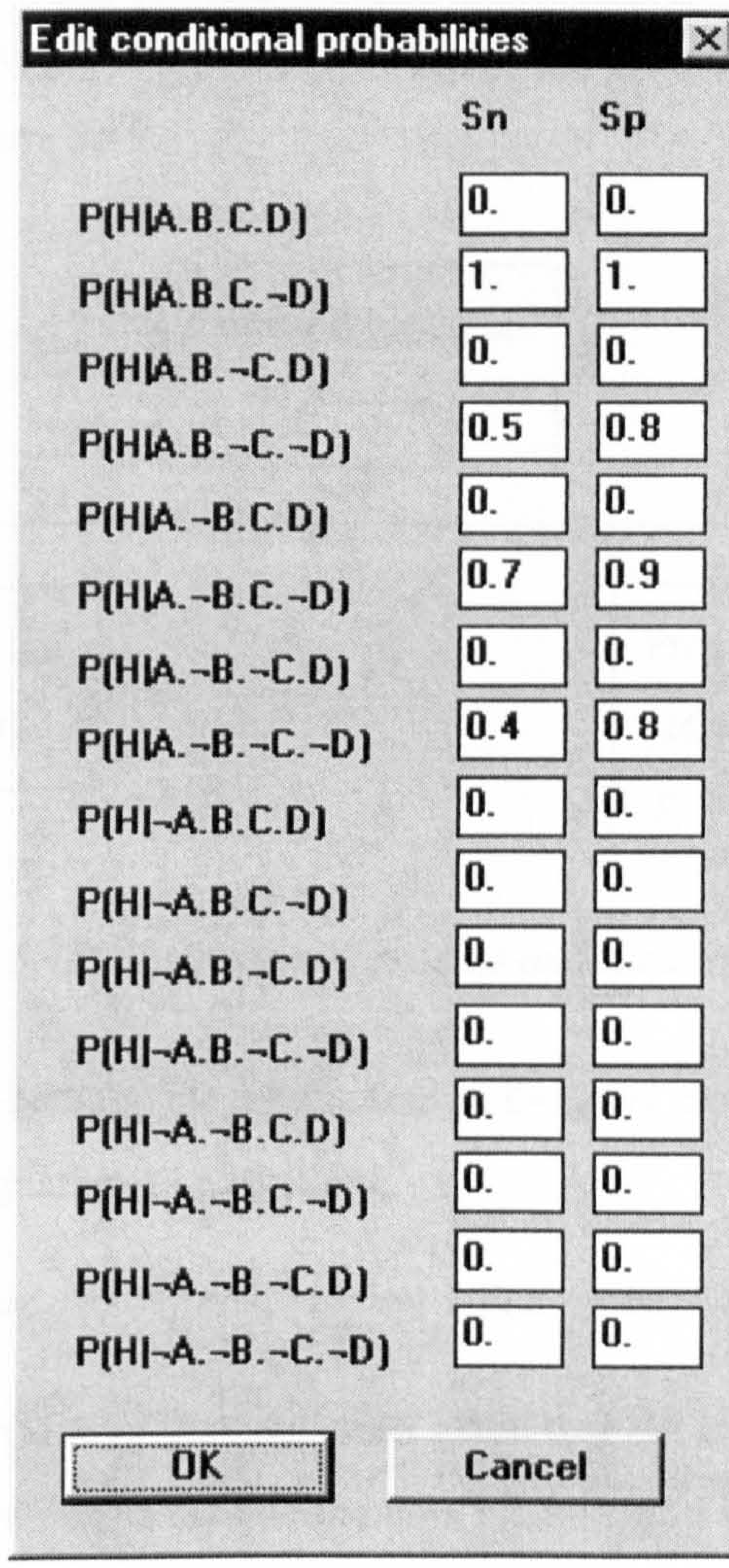

Figure 5.14 Dialog for entering conditional probability measures

Case studies of implementation of process models in coastal engineering and management are discussed in detail in Chapter 7. A number of important practical issues associate with model construction and interpretation are addressed in that chapter so, in keeping with the theoretical spirit of this chapter, will not be addressed here.

5.14 Combining Probabilistic data with evidence about process dependability expressed as interval probabilities

The total probability theorem (Equation 14) has been applied to the important problem of how evidence concerning the dependability of a process can be integrated with the data handled by that process. The process may, for example, involve use of some data model *dm* to manipulate some input parameters and obtain some prediction *x*, which may be expressed as a probability distribution $p(X{=}x|dm)$. Now the domain experts will recognise that, because the abstractions involved in its construction, model *dm* is not a completely dependable. Suppose that the domain experts therefore construct a process model *pm* to analyse the dependability of *dm*. The evidence concerning the dependability of *dm* may be vague, incomplete or contradictory so it would be appropriate to construct *pm* using IPT. Using the method outlined in the previous section a measure of the dependability of *dm*, $p(dm{=}True|pm)$, is obtained from the process model. The situation is illustrated in Figure 5.15.

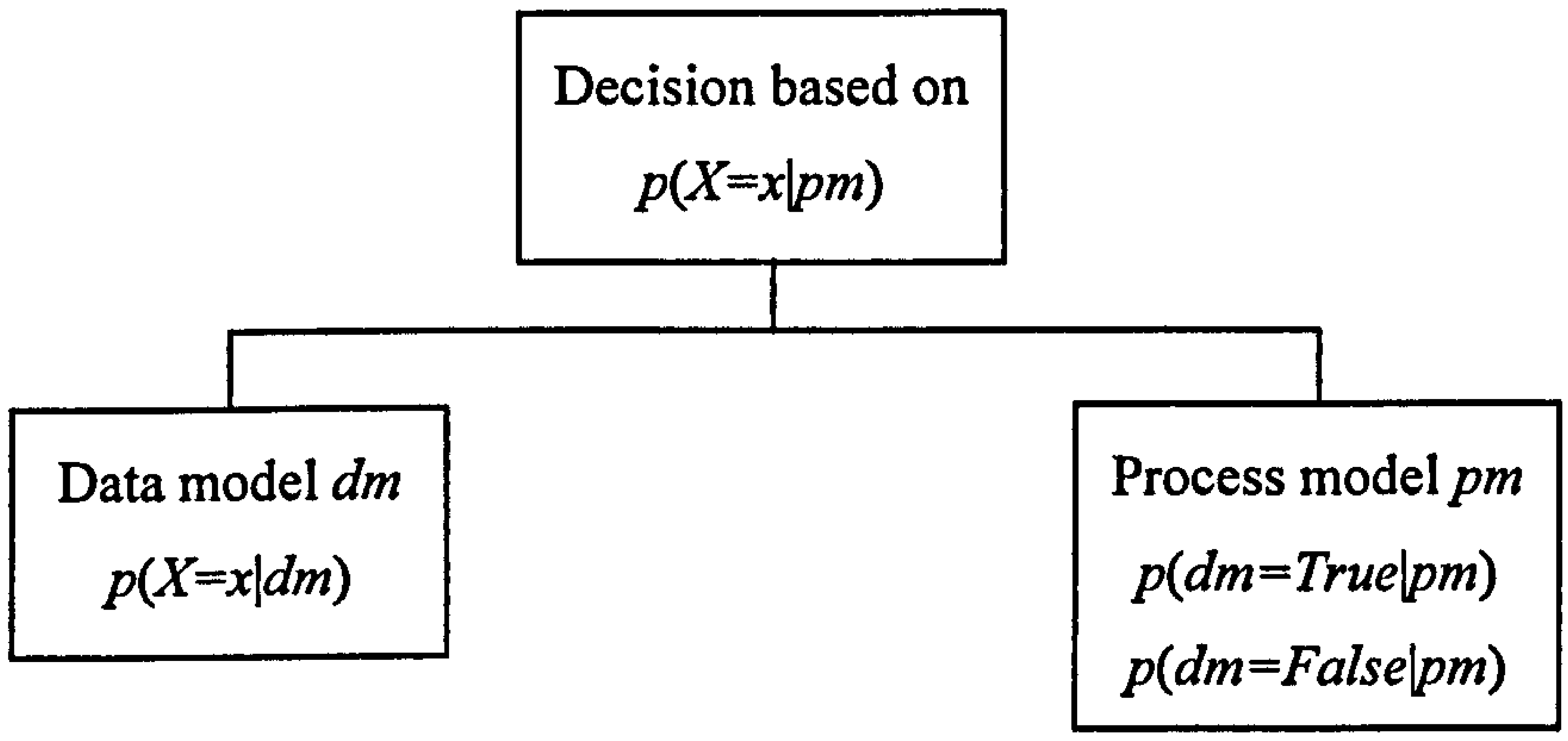

Figure 5.15 Integration of process and data information

Thus from the process modelling, the prediction $p(X{=}x|pm)$ can be expressed as an interval probability obtained from Equation 14 as follows

$$P(X{=}x|pm) = P(X{=}x|dm{=}True,pm).P(dm{=}True|pm) + P(X{=}x|dm{=}False,pm).P(dm{=}False|pm). \quad (21)$$

$p(X{=}x|dm{=}False,pm)$ is the value of the prediction given that *dm* is not true or dependable. Whilst the expert may recognise that events outside the data model could be as important as those inside, it is very difficult to make any estimate of $p(X{=}x|dm{=}False,pm)$. It may therefore in practice be assigned the [0,1] interval constraint.

It may be argued that the measure of process support $p(dm|pm)$ and the model result $p(X{=}x|dm)$ are essentially too distinct to be related in the same equation. If this is the case the decision-maker will be inclined to use the evidence from the process model in some heuristic way, perhaps adopting a satisficing strategy by which the model has to achieve some minimum dependability before a decision is taken. On the other hand there must be some boundary (albeit fuzzy) to possible deviations from model predictions. Equation 21 enables some limits to be put on that boundary in a way which is not possible with existing probabilistic approaches.

Suppose for example that an engineer is designing a harbour. The harbour operator has specified a maximum frequency with which a certain wave height may be exceeded within the harbour. The engineer has carried out some analysis of how effective the proposed design is at limiting wave heights and now wishes to decide whether the design dependably satisfies the harbour operator's specification. The situation is illustrated in Figure 5.16.

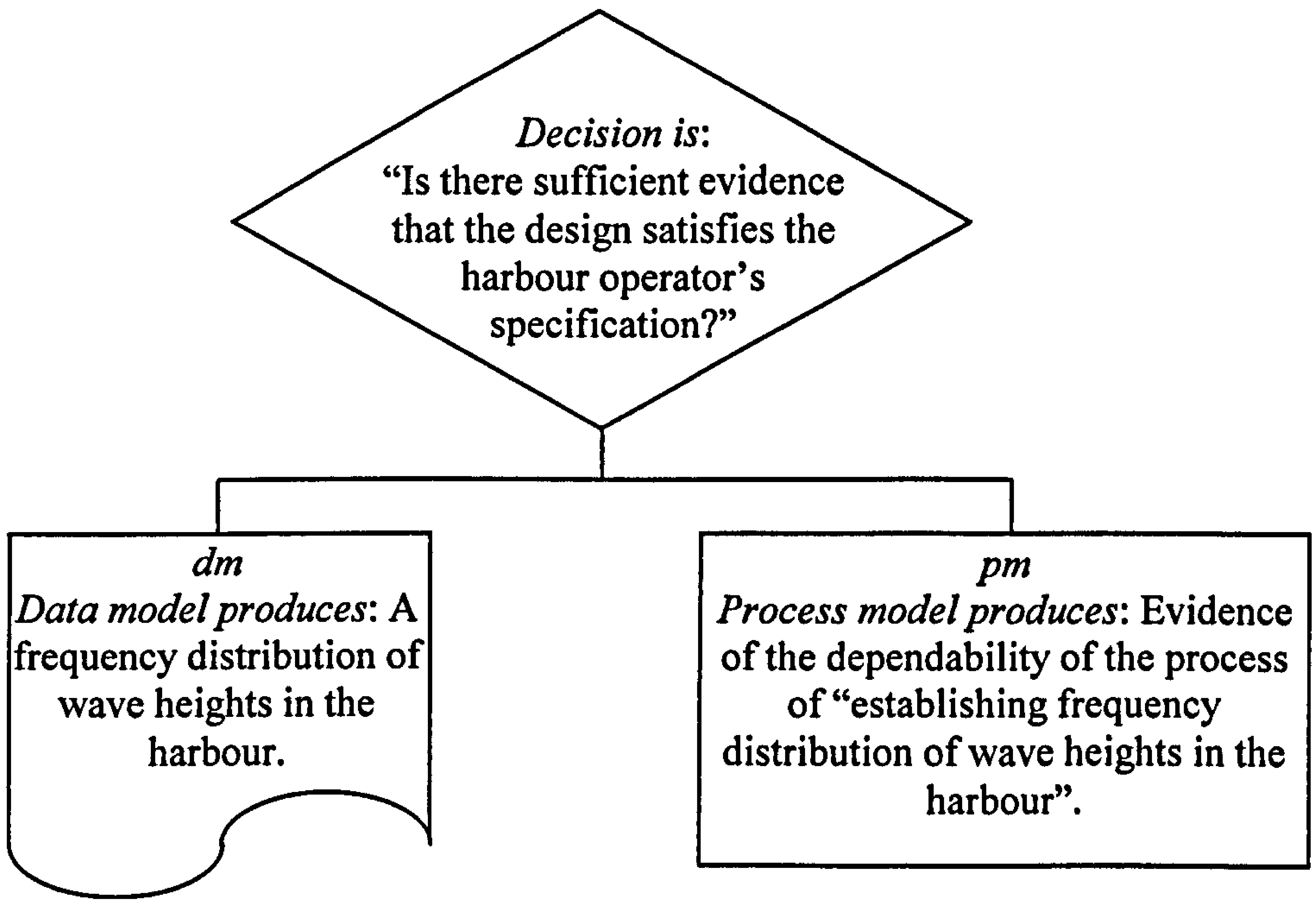

Figure 5.16 Example of integration of process and data information

A simplified version of a process model for establishing the frequency distribution of wave heights in the harbour is illustrated in Figure 5.17. The model consists of processes ordered according to their precision of definition. The dependability of the process of establishing wave heights in the harbour is estimated by evaluating the evidence for and the relevance of the sub-processes.

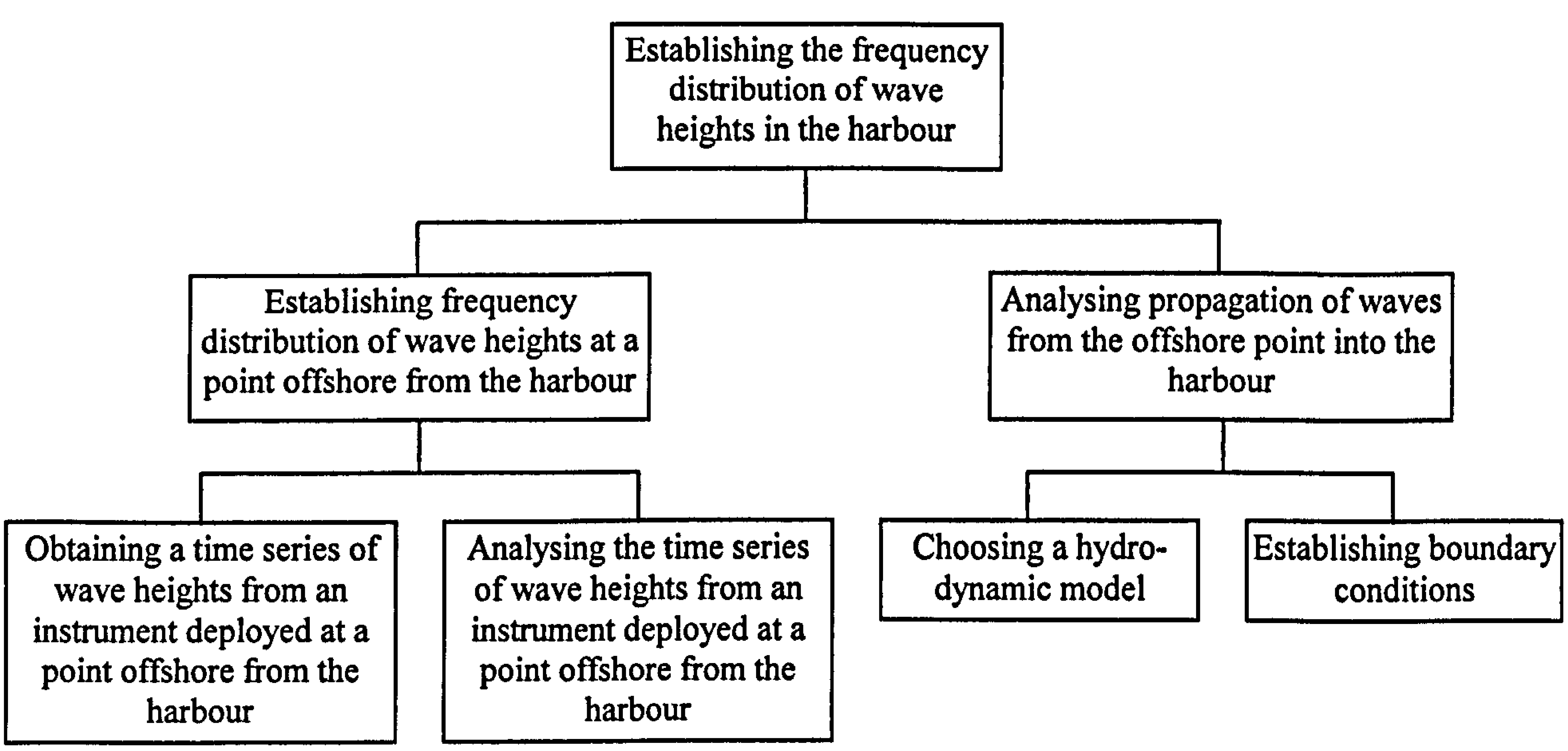

Figure 5.17 Hierarchical model for establishing the dependability of a design process

The engineer's beliefs in the dependability, relevance and inter-dependencies of the sub-processes can be propagated using the calculus described in Section 5.10 to obtain an overall measure of the dependability of the top process in Figure 5.17, that of establishing the frequency distribution of wave heights in the harbour. Suppose for example that this process has a support interval [0.78, 0.93] and the data model indicates that the specified wave height x in the harbour will be exceeded in any year with a probability of 0.36 and the engineer has no knowledge of the wave conditions in the harbour given that the model is not dependable, then it follows that:

$p(dm{=}True|pm) \in [0.78, 0.93]$ $p(X{\geq}x|dm{=}True{,}pm) \in [0.36, 0.36]$

$p(dm{=}False|pm) \in [0.07, 0.22]$ $p(X{\geq}x|dm{=}False{,}pm) \in [0.0, 1.0]$

and so, applying Equation 21 (expanded as in Equations 15 and 16)

$S_n(X{\geq}x|pm) = 0.36{\times}0.78 + 0.0{\times}0.22 = 0.28$

$S_p(X{\geq}x|pm) = 0.36{\times}0.78 + 1.0{\times}0.22 = 0.50.$

So the evidence that the specified wave height will be exceeded is in [0.28, 0.50].

Clearly the results obtained from this approach are entirely dependent on the values of $p(dm{=}True|pm)$ and $p(dm{=}False|pm)$. The situation is illustrated in Figure 5.18, which shows how any value of $p(X{=}x|pm)$ on [0,1] can be obtained by varying $p(dm{=}True|pm)$ and $p(dm{=}False|pm)$.

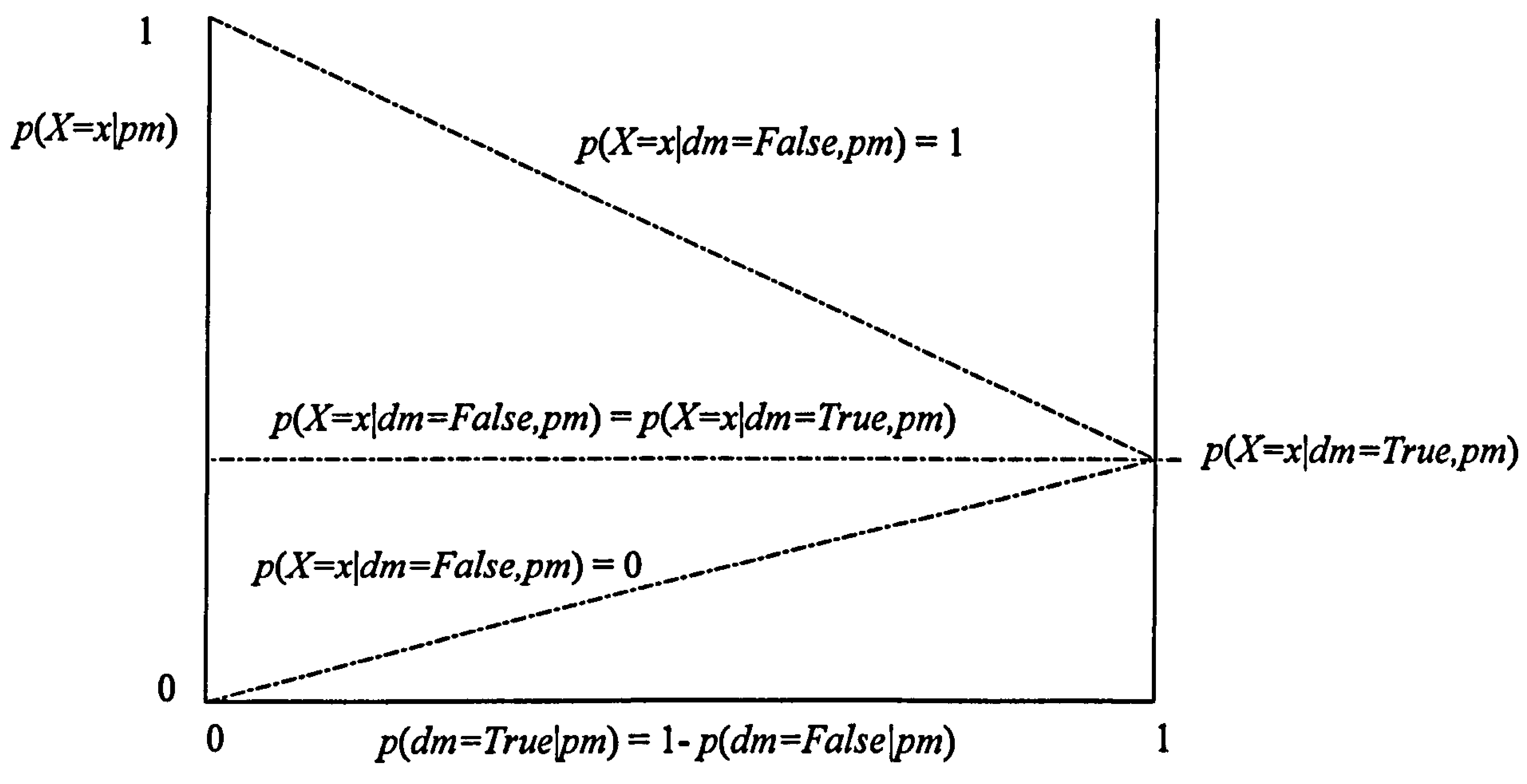

Figure 5.18 Results obtainable by combining probabilistic data with evidence about process dependability expressed as interval probabilities

The problem of combining probabilistic statements about events, with a set-based treatment of beliefs is an important challenge which this thesis has not fully managed to address. The need to admit (usually frequentist) data into a knowledge base, in addition to expert statements of belief in the evidence for or against a proposition, has become increasingly clear during the course of this research. This approach has been adopted in the Fril database browser (Baldwin, 1995*a*). The approach proposed above is another possible solution, but is not without its problems. Random set theory and mass assignment theory provide other potentially fruitful pointers towards a solution to this remaining challenge.

5.15 Conclusions

1. This chapter has addressed the problem of representing uncertainty with numerical structures. A set of criteria by which the appropriateness of an uncertainty calculus can be judged has been proposed. The aim has been to represent uncertainty in hierarchical process models. This requires an open world view and a syntax, which, on the one hand, is flexible enough to capture the complexities of the problem, but, on the other hand, does not have such weak axioms that it generates results so vague that it is hard to see how they can inform a decision.

2. Probability theory is the most established numerical representation of uncertainty but is inappropriate for representing vagueness, some types of ambiguity, incompleteness and conflict. When applied in these situations, probability theory can imply that more knowledge than exists in practice.

3. Theories of fuzzy measures govern how probability can be distributed between concepts in the absence of complete information. Beliefs can be assigned to sets of propositions rather than to each individual proposition. Belief and plausibility measures, and the more fundamental concept of basic probability assignments have been introduced in the context of the Dempster-Shafer theory of evidence. However, Dempster's rule has been rejected as a basis for uncertain inference. The rules of mass assignment theory are more general and coherent.

4. Adopting and interval representation is attractive because of the much-increased flexibility over point probabilities it provides for representation of uncertain knowledge. The number of degrees of freedom of an inference problem with n items of evidence increases from a function of 2^n to a function of 3^n. Whilst the knowledge engineer will rarely be able to elicit sufficient information to constrain all of these degrees of freedom, it is possible to identify bounds on the outcome to the inference problem, conditional only on the available information and without making any further assumptions.

5. In IPT intervals are use to represent the probability measure in order to capture in a relatively simple manner, features of fuzziness and incompleteness. The dependence parameter ρ generalises other inference rules that assume a specific dependence relationship between evidence. Conflict between items of evidence, which can be unavoidable but also informative, is measured and propagated by the calculus. The links between IPT and the other uncertainty methods introduced in this chapter have been demonstrated.

6. A new and general approach to logical inference in IPT based on the Jeffrey's rule interpretation of the total probability theorem has been presented. The proposed inference method partitions the available evidence amongst the power set of the universe of discourse and assigns a relevance to each member of the power set. The same equation can therefore embody the available evidence and the structural relationship between the items of evidence and the hypothesis of interest. This approach recognises that the structure of an inference problem is as important as the evidence itself. The structural situations of necessary or sufficient evidence are special cases of the proposed approach. Use of the total probability theorem requires a judgement of the incompleteness and relevance of the available evidence in the context of the hypothesis in question.

7. The fundamental problem of uncertainty handling with numerical structures is one of mapping messy real world situations onto precise mathematical syntax. There is no definitive solution to this problem, which ultimately depends on a judgement of the relative merits of different mathematical formulations for handling different types of uncertain information. There are few right or wrong answers in the uncertainty handling, so ultimately success is judged according to what proves to be a support to decision-makers. IPT provides decision-makers with information in a simple format, which at the same time reflects the complexity of the inference problem and the richness of available evidence. It has therefore been selected as an appropriate syntax for representing the uncertain dependability of coastal engineering processes.

8. Uncertainty handling with IPT has been implemented in Visual C++ and integrated in a Windows-based software package for hierarchical process modelling. The software has been tested in a range of conditions and found to be robust. Practical applications of IPT are recounted in detail in Chapter 7.

5.16 References

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CHAPTER 6

A contingency approach to choice

6.1 Objectives of Chapter 6

- to propose a contingency approach to choice, in which a choice mechanism is selected according to the nature of the information available and the meta-level constraints and objectives of the decision-maker;
- to describe the axiomatic bases, structural requirements and practical application of established theories of choice, and fit them within a generic framework;
- to assess the appropriateness of established theories of choice for complex open world decision-making situations;
- to introduce an open world decision theory, which uses evidence expressed as interval probabilities.

6.2 Decision-making and choice

Decision-making, which is a primary theme of this thesis, is revisited in this chapter. The decision-making process was discussed in Chapter 4. The critical moment identified in models of the decision-making process is the moment of choice when one option is identified as being preferred. Dependable decision-making requires an appropriate choice mechanism, be it optimising, satisficing or a combination of the two. It is the mechanism by which choice takes place that is the subject of this chapter.

To make a choice it is necessary to have some objectives and the decision options must have some attributes that make them distinguishable in terms of the objectives. In other words choice is based on information concerning objectives and option attributes, information that in practical situations will, to some extent, be uncertain. Choice therefore reduces to the problem of using uncertain information in order to achieve objectives.

In this chapter theories of choice are classified according to the format of information they manipulate. Normative theory states that uncertainty about future states of nature is expressed in probabilistic terms, and that the decision-maker's objectives are expressed in terms of utilities (Lindley, 1971). Fuzzy decision theory (Appendix 5) makes use of uncertain information expressed in terms of fuzzy membership functions. Incompleteness is represented in the Dempster-Shafer theory, mass assignment theory, Support Logic Programming and Interval Probability

Theory by using interval numbers (or support pairs) though there is disagreement about how uncertain information expressed in an interval format should be used as a basis for choice.

The aim of this chapter is to develop a generic framework for choice in the context of coastal defence systems. This contingency approach to choice involves

- reflecting the nature and extent of uncertainty in the information upon which the choice will be based;
- identifying a choice mechanism which satisfies the meta-objectives of the decision-maker.

To develop this contingency approach (which is introduced in Section 6.3) involves recasting normative (Section 6.4) theories of choice within the general context. Satisficing approaches are an important aspect of practical choice and are introduced in Section 6.5. In Section 6.7 a range of approaches to multi-attribute choice are critically reviewed. A new theory of choice based on intervals, which is suitable in situations of incomplete and possibly conflicting information, is proposed in Section 6.8. Supporting theory is included in Appendices 2, 3, 4 and 5.

6.3 A contingency approach to choice

A contingency approach to choice recognises the diversity of situations in which decision-makers find themselves and endeavours to adopt a choice mechanism which is contingent upon, or 'fits', the characteristics of the decision-making situation. This is fundamentally different to normative theories of choice which maintain that there is only one rational way to make a decision (Lindley, 1971). It adds value to non-normative approaches (such as fuzzy theories of choice) by identifying the situations in which these approaches will be useful and highlighting those situations when they will not. A classification of the choice mechanisms examined in this paper is given in Table 6.1.

Table 6.1 Summary of choice mechanisms

Single attribute	Multi-attribute
Choice under strict uncertainty (Section 6.4)	Multi-attribute value theory (Section 6.7 and Appendix 4)
Subjective expected utility theory (Section 6.4 and Appendix 2)	Multi-attribute utility theory (Section 6.7 and Appendix 4)
Satisficing strategies (Section 6.5)	Fuzzy decision theory (Appendix 5)
Various strategies for choice based on intervals (Section 6.8)	Open world decision theory (Section 6.8)

Identifying a choice mechanism depends on the cognitive style of the decision-maker (Kleindorfer *et al.*, 1993, Krause and Clark, 1993). Yet there are some general characteristics of the situation which, regardless of the preferences of the decision-maker, render some choice mechanisms more applicable to a given situation than others. These are characteristics of the information available to the decision-maker and the structure of the decision-making situation. Therefore, a choice mechanism should:

- reflect the nature of the information available;
- meet the meta-level objectives and constraints of decision-makers.

6.3.1 The nature of available information

In a given decision situation the choice mechanism should

- make the most of available information;
- use information in a form which is natural and comprehensible to the decision-maker;
- avoid distorting the information by changes of format;
- recognise that information exists in an open world.

Information used in choice will invariably be uncertain, often in several of the senses introduced in Chapter 4. If the information that is generated in the decision-making process naturally appears purely one format then there is little difficulty in selecting a choice mechanism. For example the information concerning the uncertain future outcomes of a game of dice is customarily stated in purely probabilistic terms. A choice mechanism based on probabilistic decision theory is implied. However, in practical coastal engineering decision-making situations information is seldom in a consistent format. Information about options and objectives often involves a potent cocktail of uncertainties. To make a choice under these circumstances involves some explicit or implicit mapping to generate information that can be used as basis for choice. Implicit mapping means that information is elicited in a particular format when an expert would not under normal circumstances express the information in that format. For example utilities are customarily elicited by reference to lotteries, when in practical engineering situations this is seldom the most natural way of expressing preferences or beliefs. Explicit mapping involves some traceable change of scale. An example of an explicit mapping from benefit-cost ratio to interval probabilities is given in Section 6.8.1.

A classification of theories of choice is summarised in Table 6.2. The classification encompasses that of Stone and Blockley (1993) but includes a more comprehensive set of choice mechanisms. Identifying a choice mechanism involves a compromise between two extremes. At the one extreme the choice mechanism is such a crude idealisation that it represents an unacceptable distortion of the available information. This may involve neglecting some or all types uncertainty, thereby assuming more information than is in fact available. These assumptions are not necessarily explicit, but can be implied by the axioms of the choice mechanism that is adopted. By making strong assumptions it is possible to use choice mechanisms which give the power of unique choice, in which case, once the idealised choice mechanism has been established, it will always identify a unique preferred option. Unique choice is so attractive that it may beguile decision-makers into adopting choice mechanisms that do scant justice to the complexities of the available information.

At the other extreme of the dilemma of choice is a situation where every attempt is made to represent the complexity and uncertainty in the choice situation without adopting distorting axioms. By acknowledging incompleteness the decision-maker looses the power of unique choice.

Table 6.2 Classification of choice mechanisms according to information requirements

Type of information	Theory
Deterministic	Choice under certainty
	Multi-attribute value theory
Probabilistic	Subjective expected utility theory
	Multi-attribute utility theory
Fuzzy	Fuzzy decision theory
Interval	Open world decision theory
Incomplete	Choice under strict uncertainty
	Open world decision theory

The contingency approach to choice recognises that there is no universally applicable solution to the dilemma of choice. A decision-maker will always have to weigh up the idealisations of abstraction with the simplicity and power of the choice mechanism. In complex situations choice mechanisms which recognise uncertainty in all its guises will be preferred to more rigid approaches even if they lack the power of unique choice that is enjoyed by normative approaches.

6.3.2 The meta-level objectives and constraint of decision-makers

A meta-level objective relates to the *procedural* requirements for a choice. It may, for example, be that a choice mechanism has to be easily understandable. Choice mechanisms can be classified according to the extent to which they meet common meta-level objectives, which are listed as follows.

<u>Transparency and repeatability</u>

Coastal defence infrastructure is on the whole a public good delivered in a business environment. Transparency, defendability and repeatability in decision-making are therefore important issues. In situations where transparency is a requirement, established choice mechanisms that use measurable and repeatable items of information are to be preferred. Cost-benefit analysis is a typical example of a choice mechanism that is preferred because of its supposed transparency and repeatability, though the implied values judgements are questionable (Cothern, 1996). Costs and benefits are obtainable from the market or by analogy to the market. Value judgements are implied in the adoption of the methodology (for example by assuming pre-determined discount rates) so need not be explored during each application.

On the other hand, methods which depend on the beliefs or values of the decision-maker, which may not even be directly articulated, are not particularly transparent and may not be repeatable even for the same decision-maker. Thus utility theory is not usually acceptable in economic decision-making in the public sector since it relies on subjective measures of attitudes to risk (HM Treasury, 1997). Fuzzy decision theory (Appendix 5) and open world decision theory (see Section 6.8) require judgements in a number of respects, which may not be repeatable:

- mappings from numerical scales of data values to membership functions or intervals;
- explicit or implicit value judgements of trade-offs between goals, constraints or attributes;
- identification of choice mechanism based on fuzzy membership or intervals.

Whilst decisions concerning coastal defence infrastructure are value-laden, the decision procedure is dominated by guidance set by central government. The values that are to be taken into account are therefore established *ex ante* at least in general terms. Nonetheless, the mechanism for including values relating to sustainability and remote impacts is not, and arguably cannot, be defined in precise terms.

Understandability

Very complex choice mechanisms may be repeatable and transparent to experts but can be criticised on grounds of understandability. Non-technical decision-makers and other stakeholders will tend to be hostile to choice mechanisms that they do not understand.

Legitimation

Legitimation is a part of the decision-making process in any situation where the decision-maker is not the sole stakeholder in the decision. Legitimation is the process by which the preferred choice is rationalised to other stakeholders. It involves assessing the impacts of the preferred choice and explaining the reasons for those impacts. The legitimation process has been increasingly recognised by political scientists and sociologists as an important aspect of the decision-making process (Kleindorfer *et al.*, 1993). The legitimation process will tend to be straightforward in those rare situations when all the stakeholders, including the decision-maker, share the same values and objectives. When this is not the case the need for a legitimation process will tend to influence the choice mechanism adopted. Transparency and understandability are characteristics that assist legitimation. Moreover, the capacity to explore conflicting values and develop value trade-offs will help the legitimation process.

Optimality

Optimisation involves searching a decision space to find the solution that scores most highly with respect to a utility function. Optimisation depends on closed world models that may not be dependable representations of complex situations. However, in many situations there is still a need for optimising choice mechanisms. Issues of transparency can be a powerful driver towards optimising mechanisms. An optimising mechanism in a convex space will usually identify a unique option which means that, provided the optimising model is specified in the same way, it will be a repeatable choice mechanism. Purely economic situations are customarily driven by optimising choice mechanisms.

Ease of application

There is a very wide variation in the ease of applicability of choice mechanisms, from satisficing strategies to, for example, multi-attribute utility theory. Some researchers confidently assert that multi-attribute utility theory with joint distribution functions of belief can be coped with by non-expert human subjects (Keeney and Raiffa, 1976). It seems, nonetheless, that this level of complexity is close to the limit of what is manageable for most decision-makers. There are other choice situations where, although the decision-maker is able to cope with a complex choice mechanism, the effort involved is not warranted.

Use of expert judgement

Technical data collection and analysis is customarily used to support decisions in coastal engineering. Basic quantitative decision analysis is routinely applied. Nonetheless, the complexity of many civil engineering infrastructures is such that expert judgement is an indispensable aspect of decision-making processes in which case the choice mechanism must be able to accommodate vague judgements of variable dependability.

The characteristics of decision-making for coastal defence suggest a conflict in the criteria for an appropriate choice mechanism. On the one hand there is the requirement for transparency and repeatability. In situations where transparency and repeatability are important criteria formalised decision analysis is to be recommended (Kleindorfer *et al.*, 1993). Indeed a decision analysis approach to choice is prescribed in various government guidelines and rules (HM Treasury, 1997, MAFF 1993*a*) which pre-specify aspects of the choice mechanism.

On the other hand decision relating to coastal infrastructure are recognised as being value-laden and government guidance on weighing up of values is, and can only be, incomplete. Values are only one of the aspects of coastal infrastructure that make decision-making situations complex and fraught with uncertainty. Problems are highly nested and interact with each other at a number of levels. Under these situations it will be argued that analytical optimising approaches to choice represent an unacceptably incomplete idealisation of the situation and are therefore unsuitable. This chapter therefore endeavours to identify appropriate approaches to choice contingent on the characteristics of the decision-making situation and the nature of the information that is at hand or could be obtained.

6.4 Normative theory of choice

Normative theory is a model of the behaviour of an ideal rational person who provides a standard or norm for the behaviour of ordinary people. In the field of decision-making it was Leonard Savage in his *Foundations of Statistics* (1954) who advocated the distinction between normative and descriptive theory which has been so influential since. The distinction has become steadily more important as psychologists have shown in more and more detail that human behaviour systematically departs from Savage's theory.

The normative approach is usually referred to as a decision theory. However, the process by which options and objectives are obtained and structured does not feature at all in the theory, which relates only to the choice between options given specific items of information. Normative theory assumes that all the information needed to make the choice is complete and in an appropriate format.

Normative theory is customarily subdivided into the following categories:

1. choice under certainty,
2. choice under risk, and
3. choice under strict uncertainty.

The third class of decision situations is, unfortunately, more normally referred to as 'decision-making under uncertainty'. The terminology is unfortunate because, as was demonstrated in Chapter 4, uncertainty is a rich and varied concept whilst at issue here is a mathematical situation which is uncertain in a very specific sense. To distinguish this mathematical situation from the generality of decisions, which are all to some extent uncertain, the label 'choice under strict uncertainty' has been adopted.

In Table 6.3 there is a set of decision options d_1, d_2,..., d_l and a set of future states of nature θ_1, θ_2,..., θ_m that may materialise after the choice. Depending on which state of nature in fact materialises, option d_i will yield one of m possible outcomes x_{i1}, x_{i2},..., x_{im}.

Suppose that and $v(x_{ij})$ is the value associated with a given decision outcome x_{ij} then the following scenarios apply.

Choice under certainty: The state of nature after the decision is known *i.e.* $m = 1$. The decision-maker chooses the option with the highest value $v(x_{i1})$. This is the situation usually addressed by benefit-cost analysis, in which case $v(x_{i1})$ is a benefit-cost ratio or net present value. If $v(x_{i1})$ is expressed against a number of criteria then the table extends into a third dimension, which is the situation addressed by multi-attribute value theory.

Choice under risk: Only the probabilities $p(\theta_1)$, $p(\theta_2)$,..., $p(\theta_m)$ of occurrence of set of states of nature θ_1, θ_2,..., θ_m are known. Provided the decision-maker accepts the axioms of rationality, which will be discussed presently, then he should chose the option that maximises $\sum_{j=1}^{m} v(x_{ij})p(\theta_j)$. This is the situation addressed by benefit-cost analysis that takes risk into account. The multi-attribute extension is multi-attribute utility theory.

Choice under strict uncertainty: There is no information about the probabilities of states of nature θ_1, θ_2,..., θ_m. Under these circumstances there are various decision strategies which are in some sense rational, for example maximin utility, minimax regret, Hurwicz α, and that based on Laplace's principle of insufficient reason.

Table 6.3 General representation of choice problems

States of nature		θ_1	θ_2	...	θ_m
	d_1	x_{11}	x_{12}	...	x_{1m}
	d_2	x_{21}	x_{22}	...	x_{2m}
Options	.	.	.	...	.
	.	.	.	...	.
	.	.	.	...	.
	d_l	x_{l1}	x_{l2}	...	x_{lm}

All possible strategies for choice under strict uncertainty will violate at least one criterion of rationality (Luce and Raiffa, 1957 who quote Milnor, 1954). It is not clear which criterion should be relaxed so a unique decision rule cannot be identified. Theories of choice under strict uncertainty are thereby denied the power of unique choice and have been rejected as a normative model.

Normative theorists have therefore resorted to the tractable problem of choice under risk. It is reasoned that the decision-maker usually has some vague partial information about the true state of nature. The school lead by Savage (1954), who drew on earlier ideas of Ramsey (1931) and de

Finetti (1937), holds the view that this partial information can be used to construct a probability distribution over the states of nature, which is appropriate for making decisions. A person confronted with uncertain events whose probabilities are not directly known should therefore decide by recourse to a personal probability measure for the uncertain events, hence the usual label of 'subjective expected utility' (SEU) theory. The theoretical background to SEU is explained in Appendix 2. The case is stated by Lindley (1971) with fundamentalist certainty,

> *...there is essentially only one way to reach a decision sensibly. First, the uncertainties present in the situation must be quantified in terms of values called probabilities. Second, the various consequences of courses of action must be similarly described in terms of utilities. Third, that decision must be taken which is expected - on the basis of calculated probabilities - to give the greatest utility. The force of 'must', used in three places here, is simply that any deviation from the precepts is liable to lead the decision-maker into procedures which are demonstrable absurd - or as we shall say, incoherent.*

Provided that the model of choice is complete and the relevant information is expressed in an appropriate format then Lindley's argument is compelling. He and others have convincingly shown that approaches other than the normative one lead to incoherent conclusions (Dyson, 1980, Lindley, 1982). Making behaviour equivalent to expected utility maximisation is a logical consequence of adopting a rational set of consistency and continuity axioms. Normative theory in closed world contexts like games of chance and lotteries therefore has a logical weight that is hard to challenge. Normative theory also provides an attractive means of evaluating the benefit of obtaining information about future states of nature, which is usually referred to as 'value of information theory' and is discussed in detail in Appendix 3.

However, the proviso that the model of choice is complete and the relevant information is expressed in an appropriate format is a very strong one. In the sorts of practical situations encountered in coastal engineering there are important practical and philosophical grounds on which to question the applicability of normative theory. These criticisms are rather closely linked but are addressed separately below.

The practical problem of constructing models of choice

There are practical problems with the assumption that probabilities and utilities are pre-existing or instantaneously constructed. In practice it can take considerable effort on the part of decision-makers to construct a problem in an appropriate format to apply normative decision theory. By resorting to personal measures of utility and probability the theory has extended from the moment of choice to the process of constructing beliefs and preferences.

It can be very difficult in practice to disentangle beliefs about future states of nature from values and attitudes to risk (Shafer, 1986). Utility functions are constructed in abstract terms, yet the decision-maker will normally be aware of the decision options available, which may well bias the utility function. Admittedly a skilled analyst will be able to identify inconsistencies and incoherencies in the decision-maker's testimony. However, by ironing out these inconsistencies the analyst may not be doing justice to the decision-maker's state of knowledge or confusion. The process of forcing a complex problem into the format required for normative theory may so distort the problem that the decision-maker does not feel that the outcome does justice to her beliefs or preferences.

By way of reaction, a 'prescriptive' (Bell *et al.*, 1988) or 'constructive' (Shafer, 1986) approach has been proposed which attempts to take explicit account of the process of using a normative model of choice. Lessons from empirical studies of decision-making are used to help decision-makers construct probabilities and beliefs that are consistent with the axioms of normative theory. These approaches therefore recognise that there is no unique way of constructing choice situations (for example based on betting analogies) and thus no normative decision process.

The probabilistic format does not express all relevant information and uncertainties

Normative theory supposes that there is one probabilistic format in which beliefs and preferences should be expressed. It cannot therefore accommodate vague, ambiguous, incomplete or other types of uncertain information introduced in Chapter 4. A decision-maker my justifiably object that probability is not an expressive enough language to articulate his objectives, values, preferences and beliefs.

Incompleteness

All models are inevitably incomplete, so models of choice are not unique in this respect. However, at issue here is whether the format of normative choice in any way exacerbates problems of incompleteness. The existence of such a well-formed model of choice can apparently be beguiling inducing what Herbert Simon (1957) referred to as "bounded rationality".

The coastal engineers interviewed as part of this study (see Chapter 2) were not satisfied that even risk-based benefit-cost analysis did justice to the complexity of the choices with which they were confronted. They recognised the incompleteness of the models upon which those analyses were based. Thus, whilst a good benefit-cost analysis provides very useful evidence, decision-makers were not convinced that it captured all of the evidence they considered to be important.

Normative models of choice are optimising models, so will under most circumstances identify a unique preferred solution. If the dimensions of decision space are attributes (which may be unitary or multiple) then the option which occupies the most advantageous point in decision space is chosen. Optimisation is possible because the decision problem is modelled as an exhaustive set of options and future states of nature. The optimum identified by the decision model will be a *local optimum* in an open world. Under some circumstances the decision model will be appropriate in relevant respects and so the optimum identified will represent a good decision. In situations of complexity where the idealisations of options, objectives and states of nature are outlandish, the local optimum identified in the decision model will not be a useful guide to the decision-maker.

> ... [Our] *ignorance of the future, and the extent to which our bounded rationality forces us to decompose systems in misleading ways and artificially to restrict agendas, means that we have no way of identifying an overall optimum even if we could find one....* [A] *rational model may well not be a model of rational behaviour: to rely entirely on a model which misrepresents the real situation, as all models must, though rational in the technical sense of following a logical programme, is liable to prove irrational in the original sense of being a reasonable response to that situation.*
>
> (Loasby, 1976)

6.5 Robustness in the face of uncertainty

The limitations of normative theories of choice are becoming more apparent in the more complex multi-disciplinary problems that prevail on the coast. For example, there is currently much interest in making the coast more resilient. To do so involves restoring natural systems and enhancing flexibility and diversity. Much of the value of a resilient coastline is its capacity to cope with the unforeseen and unpredictable, in particular climate change. These benefits cannot be fully evaluated in probabilistic terms. Moreover, a resilient coastline will be highly dependent on human systems of monitoring and management. Predicting the reliability of these systems represents a particular challenge in complex, dynamic situations because some behaviour is truly unforeseen and so by definition is not included in the reliability model. Unfortunately many engineering failures are the unforeseen consequences of human actions (Blockley, 1980). Collingridge (1980) suggests that unforeseen outcomes in socio-technical systems can be controlled by:

- monitoring,
- reducing the cost of error,
- reducing the corrective response time,
- reducing the cost of remedy,
- keeping one's options open (adopting flexible solutions, enhancing variety).

The intention of these strategies is to improve the robustness of a plan or design in the face of events which are not predictable by the models available to the planner or designer at the moment of decision making. It may be necessary to weigh the apparent loss of expected value involved in adopting a robust decision, against the apparent loss of flexibility in adopting the 'optimal' decision. The balance will tend to favour robustness in conditions of high uncertainty (Rosenhead *et al.*, 1972).

Some of Collingridge's characteristics of a system that is robust in the face of unforeseen conditions are realised in geotechnical engineering by the so-called observational method (Peck, 1969). In geotechnical engineering, in common with coastal engineering, the conditions actually encountered on a project often depart significantly from those predicted by site investigation and modelling. Peck suggested that rather then designing for the worst case it can be more economical to design for the most likely condition and then put monitoring in place and be ready to change the design as the actual conditions on site materialise. By so doing, it is possible to reduce reliance on inevitably incomplete predictive models.

Observational methods already form the basis of soft engineering techniques like beach nourishment. Renourishment operations are designed in response to feedback from monitoring. Clearly there will always be a need for predictive models when the conditions in which the system operates are changing significantly due to human intervention. Indeed the need for predictive models is not eliminated by observational methods. A model is still required in order to make decisions, but by actively monitoring, and designing a system that can rapidly respond to monitoring information, it is possible to cope when the system starts to depart from the model predictions.

6.6 Satisficing

Decision-making which is based on criteria being satisfied rather than optimised is referred to as satisficing. The decision options are assessed to see whether they meet a set of criteria. If all the criteria are met then the option is acceptable. When complexity or ignorance renders credible optimisation intractable, satisficing is often a practical choice mechanism. In conditions of partial ignorance the establishment of a set of criteria may well be better than the attempt to optimise any sub-objective. That is, satisficing may enable one to approach some nominal global optimum in open world decision space more nearly than is achieved by the optimum of a bounded decision space. The levels at which satisficing criteria are set are clearly a matter of judgement. Yet so too is the specification of an optimising model, in particular specification of the constraints, which defined the boundaries of the model (Loasby, 1976).

In the normative model of choice objectives are maximised. Constraints may be explicitly referred to, as is the case in mathematical programming, or may be implied by the limited number of available options and the values that the decision outcomes assume under different states of nature. In satisficing models, objectives and constraints are isomorphic with each other. An objective that merely needs to be satisfied is a constraint. The distinction is to some extent notional. Indeed in terms of subjective expected utility, satisficing criteria can be thought of as objectives with a step utility function. Table 6.3 therefore presumes that some pre-selection and pre-analysis has already taken place in order to identify at set of options which satisfy all the decision constraints and a set of predicted outcomes which are consistent with the constraints. The relationship between satisficing and optimising is revisited in the context of choice based on intervals (Section 6.8) and fuzzy decision theory (Appendix 5).

6.7 Multi-attribute choice

In the discussion of normative theories of choice it was assumed that the value of the outcome x_i of decision option d_i could be described by a single parameter function $v(x_i)$. For example in benefit-cost analysis it is assumed that all of the relevant attributes of the decision options can be described in monetary terms. In some situations a decision-maker may find it necessary to evaluate decision options against a number of attributes, so that its outcome is described by a vector of attributes rather than a scalar measure.

In the context of coastal engineering multi-attribute analysis is sometimes used when weighing up values and objectives during shoreline management planning, or for pre-feasibility screening of project options. The most common situation in which multi-attribute choice is encountered is when the costs or benefits of proposed coast defence options span a number of years. In this case the decision-maker has to make a comparison between values in different years, the value in each year representing a different attribute of the option. The conventional approach is to discount the costs and benefits to obtain a net present value (HM Treasury, 1997, MAFF, 1993*a*). Though discounting over time is one of the commonest forms of multi-attribute choice it is also one of the most controversial, being based on strong normative assumptions (see Appendix 2).

If there is only one state of nature (*i.e.* the situation is assumed to be one of choice under certainty) then the matrix shown in Table 6.4 can be used to represent the multi-attribute choice problem. Choice under risk is the situation addressed by multi-attribute utility theory. Introducing uncertainty extends Table 6.4 into a third dimension, which is occupied by the set of future states of nature θ_1, θ_2,..., θ_m. Strict uncertainty can only be tackled from a multi-attribute point of view in special cases or by introducing a proliferation of rules.

Table 6.4 Representation of choice problem with multiple attributes

	Attributes	A_1	A_2	...	A_n
	d_1	a_{11}	a_{12}	...	a_{1n}
	d_2	a_{21}	a_{22}	...	a_{2n}
Options	.	.	.	...	.
	.	.	.	...	.
	.	.	.	...	.
	d_l	a_{l1}	a_{l2}	...	a_{ln}

The first stage in multi-attribute choice is to develop a set of relevant attributes $A_1, A_2,..., A_n$ by which the options can be assessed, a processes that proceeds in parallel with development of options and identifying an appropriate choice mechanism. Recourse to the important systems concept of a hierarchy of objectives is useful in this context. Vague high level objectives can be decomposed into more specific low level objectives, which can be associated with measurable attributes. The aim is to obtain a set of objectives that as completely as possible express the high level objectives yet are precise enough to be measured and compared. Theories of fuzzy choice, which are addressed in the following section, treat objectives in fuzzy terms so can manipulate a smaller high level set of objectives and attributes.

Consider for example a choice relating to a coastal defence project (Figure 6.1). The high level objectives are set by government (MAFF, 1993*b*). These are then decomposed into more specific measurable objectives. Economic aspects are naturally measurable on a monetary scale. Even to agree an appropriate measure for some other objectives is extremely difficult, not to mention the difficulty of then deriving option attributes in those measurable terms. Some measures have been suggested in Figure 6.1 but their validity and the extent to which they represent the high level objectives is questionable, a situation which is typically in problems of multi-attribute choice.

Keeney and Raiffa (1976) suggest that the set of attributes that are used in a multi-attribute choice should, as far as possible, be:

- *complete* so that it covers all the important aspects of the problem;
- *operational* so that is can be meaningfully be used in the analysis;
- *decomposable* so that aspects of the evaluation process can be simplified by breaking it down into parts;
- *nonredundant* so that double counting of impacts can be avoided;
- *minimal* so that the problem dimension is kept as small as possible.

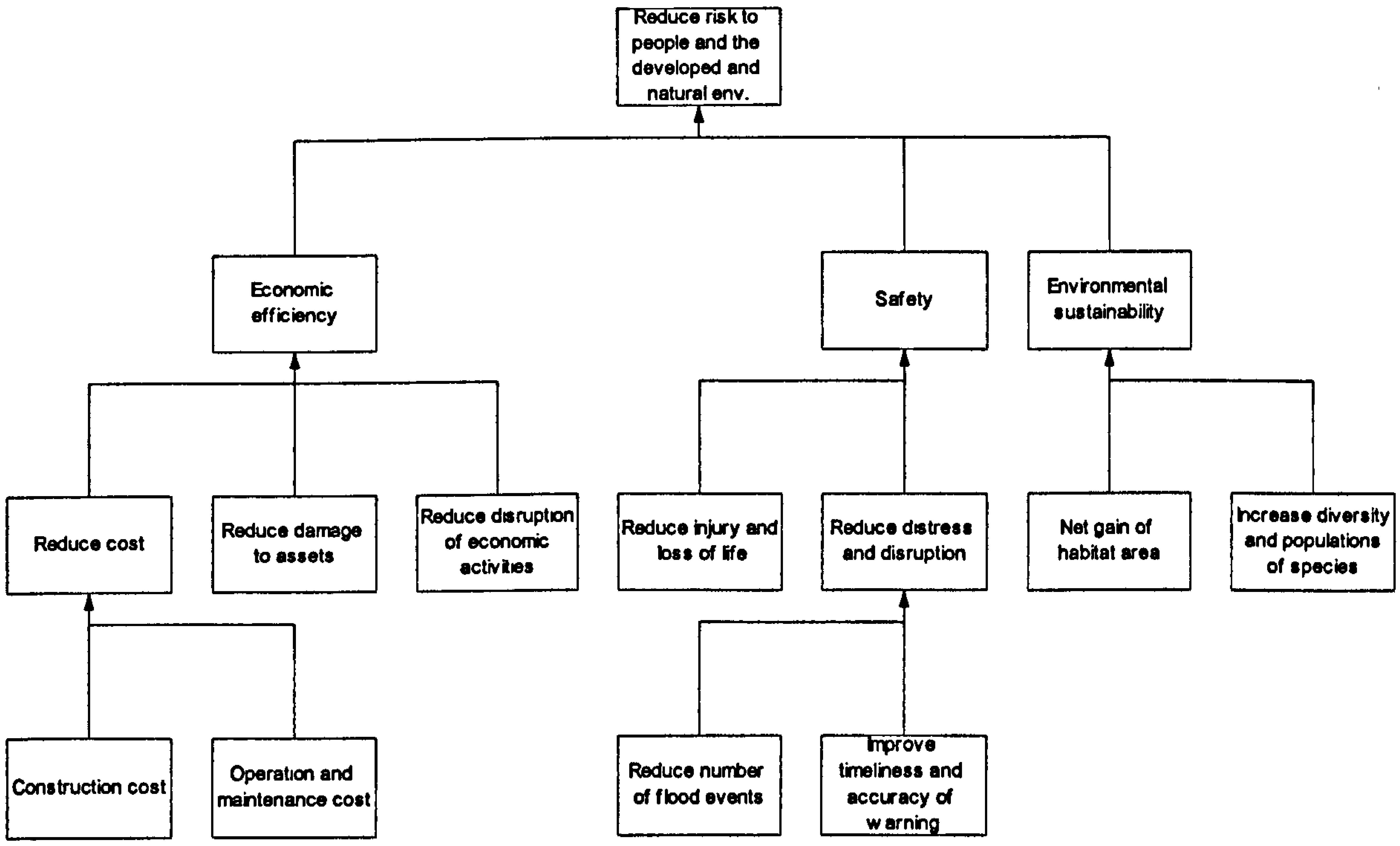

Figure 6.1 Hierarchy of objectives for a coastal defence scheme

All of these aspects are a matter of judgement and expertise on the part of the analyst. In any practical situation they will only be partially achieved. For example there will often be hidden dependencies between attributes, particularly as they are all derived from the same high level objectives. Fortunately the existence of dependencies between attributes will not normally render multi-attribute analysis invalid, though situations in which this may be the case are discussed in more detail in Appendix 4.

A variety of decision rules that can be applied to multi-attribute choice problems are summarised in Table 6.5. In essence these amount to various combinations of satisficing and optimising mechanisms. When a set of options is being assessed against only one attribute the choice mechanism is either to satisfice or optimise. When the number of attributes increases the possible decision rules proliferate.

Dominance is a powerful decision criterion, so when a unique option can be identified on the basis of dominance it cannot be rejected on other grounds. However, only in special cases will an option dominate the others on all attributes. Multi-attribute value theory and multi-attribute utility theories are considered to be the normative model of multi-attribute choice. They rely on trade-offs between various attributes. It is argued that the other approaches, which are all based on some form of satisficing, do not take good account of the performance of the options against the full range of attributes and so can result in counter-intuitive choices. So for example in lexicographic choice if one option does better than another on the most important attribute then it is the better option regardless of how disastrously it compared on the remaining attributes (French, 1988). The question reduces to the fundamental comparison of the merits of optimising and satisficing approaches, which were discussed previously.

Table 6.5 Rules for multi-attribute decision-making (after Kleindorfer et al., 1993)

Dominance procedures	Eliminate options which are equal or worse than some other option for every attribute
Conjunctive procedures	An option is accepted if each attribute meets a set of predetermined standards or thresholds.
Disjunctive procedures	An option is accepted is it scores sufficiently high on at least one dimension.
Elimination by aspects	Each attribute is examined in order against a predetermined standard or threshold. Options which fail to reach the standard are eliminated from further analysis.
Lexicographic rules	The options are assessed against the most important attribute. If one option is preferred on the basis of this single attribute then it is chosen. If there are ties on the first attribute the second most important attribute is considered, and so on, until a single option remains.
Multi-attribute value theory	The decision-maker evaluates the set of relevant attributes through a value function $v(a_{i1}, a_{i2},..., a_{in})$ and identifies a preferred option according to the value (see Appendix 4).
Multi-attribute utility	Multi-attribute value theory under risk (see Appendix 4).

Multi-attribute utility theory addresses two aspects of risk, the probabilities of various states on nature materialising and the decision-maker's attitude to risk. Decision analysts using multi-attribute utility theory go to sometime elaborate ends to develop multi-dimensional functions, which represent the decision-maker's attitude to risk (see Appendix 4). However, other important sources of uncertainty are neglected, including

- uncertainty about the dependability of probability estimates for future states of nature;
- incompleteness in the set of attributes;
- uncertainty as to whether value functions are a true representation of the decision-maker's beliefs and preferences.

Multi-attribute utility theory does not therefore address all of the uncertainties and items of information which a decision-maker may wish to make use of for choice in an open world. The alternative strategies for multi-attribute choice that have been introduced involve some degree of satisficing which does not rely on a complete model of the problem.

6.8 Open world decision theory: choice based on intervals

6.8.1 An interval development of multi-attribute choice

An approach to multi-attribute choice based on interval probabilities is proposed here, which uses an inference mechanism based on the total probability theorem. The approach is in some respects analogous to fuzzy decision theory (Appendix 5), inasmuch as objectives and constraints are treated in the same way. However, use of the dependency parameter ρ enables a richer set of interactions between the various attributes than is possible using the conventional *minmax* operators in fuzzy decision theory.

The performance of a set D of options $\{d_1, d_2,..., d_l\}$ is measured against a set of attributes (which may be thought of as goals and constraints) $A_1, A_2,..., A_n$. An interval probability $p(A_k|d_i)$ is used to express the evidence for and against the hypothesis that d_i satisfies a given goal or constraint A_k (Table 6.6).

Table 6.6 Multi-attribute choice based on intervals

	Attributes	A_1	A_2	...	A_n
	d_1	$p(A_1\|d_1)$	$p(A_2\|d_1)$	...	$p(A_n\|d_1)$
	d_2	$p(A_1\|d_2)$	$p(A_2\|d_2)$	...	$p(A_n\|d_2)$
Options	.	.	.	...	.
	.	.	.	...	.
	.	.	.	...	.
	d_l	$p(A_1\|d_l)$	$p(A_2\|d_l)$	...	$p(A_n\|d_l)$

Now $A_1, A_2,..., A_n$ can be thought of as an incomplete set of sub-sets of the super-objective X. The overall evidence that d_i satisfies X can be found by assessing the evidence from the power set of X which is partitioned according to $A_1, A_2,..., A_n$. The dependencies between $A_1, A_2,..., A_n$ enable the probability distribution across the power set to be calculated. The decision-maker then has to be interrogated to establish the trade-offs between the various attributes.

Consider the case when $n = 2$ *i.e.* there are only two attributes A_1 and A_2, in which case the overall evidence that d_i satisfies the super-objective X can be found from

$$\begin{aligned} p(X|d_i) = & \; p(X|A_1 \cap A_2)p(A_1 \cap A_2|d_i) + p(X|A_1 \cap \overline{A_2})p(A_1 \cap \overline{A_2}|d_i) \\ & + p(X|\overline{A_1} \cap A_2)p(\overline{A_1} \cap A_2|d_i) + p(X|\overline{A_1} \cap \overline{A_2})p(\overline{A_1} \cap \overline{A_2}|d_i) \end{aligned} \qquad (1)$$

The approach can be considered to be an open world one because it admits that evidence relevant to X need not necessarily only be supplied by the attributes A_1 and A_2. In other words, the approach admits that A_1 and A_2 may not be a complete reflection of X, as is often the case.

The nature of the trade-offs between A_1 and A_2 defines the values taken by the conditional probabilities. If A_1 and A_2 are both considered to be *necessary* for an option to achieve X then

$$p(X \mid A_1 \cap \overline{A_2}) = p(X \mid \overline{A_1} \cap A_2) = p(X \mid \overline{A_1} \cap \overline{A_2}) = [0,0]$$

so

$$p(X \mid d_i) = p(X \mid A_1 \cap A_2)p(A_1 \cap A_2 \mid d_i)$$

and if A_1 and A_2 are *necessary* and *sufficient* conditions for X then

$$p(X \mid A_1 \cap A_2) = [1,1].$$

This latter condition is somewhat analogous to the *min* condition in fuzzy decision theory, but note that in this case the dependencies between the sets are expressed as ρ_{A1A2}, as opposed to the condition of complete dependency implied by fuzzy set theory.

At the other extreme is the situation when any individual condition is *sufficient* to achieve X. In that case a sufficiency $p(X \mid ...)$ is assigned to each element of the power set. These sufficiencies express the weights applied to each element of the power set.

In the general case, if there are n attributes, the space of X can be partitioned into a power set with k elements $X|\beta_1,... X|\beta_k$ and

$$p(X \mid d_i) = \sum_{j=1}^{k} p(X \mid \beta_j)p(\beta_j \mid d_i) \quad \text{where } k = 2^n$$

The first term in the summation above determines the weight of evidence which a given sub-set of the attributes $A_1, A_2,..., A_n$ provides for achievement of X. The second term is calculated from the specific measures against the attributes for option d_i and the dependencies between those measures.

Some practical problems identified in Appendix 5 in connection with fuzzy decision theory also apply to this approach. There is no definitive approach to mapping from numerical measure of attributes (say cost measured in £) to interval measures of the extent to which a given attribute (say "low cost") is satisfied. The mapping has to be decided in the context of each case and needless to say is open to dispute. Similarly, assessment of dependencies and conditional probabilities is a delicate activity, which may initially may be confusing for decision-maker who are not versed in the technicalities of interval probabilities. This, however, is a problem encountered by every theory of choice. It is inevitable that some link has to be constructed between options, attributes and objectives. Interval probability does at least have the facility for expressing genuine ignorance or uncertainties on the part of the decision-maker as wide interval bounds. The process of developing trade-offs encourages decision-makers to explore their beliefs and preferences and has in practice proved to be valuable.

Consider an example from coastal engineering. Suppose that there are four options d_1, d_2, d_3, d_4 for a coastal defence project. Options will be chosen on the basis of benefit-cost ratio (BCR) subject to constraints of being environmentally and technically sound. The evidence for environmental and technical soundness has been analysed and expressed as interval numbers as shown in Table 6.7. The interval format enables the uncertainty associated with these measures (which will vary from option to option) to be expressed. Options d_1, d_2, d_3, d_4 have benefit-cost rations of 2.4, 1.8, 1.6 and 1.1 respectively, which need to be transformed into an interval measure of "economic efficiency". Suppose that a BCR less than 1.0 is considered to be unacceptable then a suitable mapping function might be

$$S_n = S_p = \begin{cases} 1 & for \quad d > 3 \\ 0.5(d-1) & for \quad d \leq x \leq 3 \\ 0 & for \quad d < 1 \end{cases}$$

giving the interval probabilities entered in Table 6.7.

Table 6.7 Interval measures in an example of multi-attribute choice

	Environmental soundness, A_1	Technical soundness, A_2	Economic efficiency, A_3
d_1	[0.31, 0.55]	[0.09, 0.65]	[0.70, 0.70]
d_2	[0.58, 0.69]	[0.55, 0.83]	[0.40, 0.40]
d_3	[0.60, 0.95]	[0.80, 1.00]	[0.30, 0.30]
d_4	[0.76, 0.85]	[0.90, 1.00]	[0.05, 0.05]

All of the attributes are considered to be necessary for an option to achieve the overall objective of the project, whilst together they are sufficient so

$$p(X \mid \overline{A_1} \cap A_2 \cap A_3) = p(X \mid A_1 \cap \overline{A_2} \cap A_3) = p(X \mid A_1 \cap A_2 \cap \overline{A_3}) = p(X \mid \overline{A_1} \cap \overline{A_2} \cap A_3) =$$
$$p(X \mid \overline{A_1} \cap A_2 \cap \overline{A_3}) = p(X \mid A_1 \cap \overline{A_2} \cap \overline{A_3}) = p(X \mid \overline{A_1} \cap \overline{A_2} \cap \overline{A_3}) = 0$$

and

$p(X \mid A_1 \cap A_2 \cap A_3) = 1$.

Using the method described in chapter 5 the interval measures shown in Table 6.8 are obtained.

Table 6.8 Interval measures for each option in an example of multi-attribute choice

	$p(X \mid d_i)$
d_1	[0.01, 0.36]
d_2	[0.14, 0.28]
d_3	[0.15, 0.29]
d_4	[0.04, 0.04]

6.8.2 Using process evidence in choice

It was argued in Chapter 4 that a balanced view of uncertainty requires consideration of the information content and dependability. In the context of choice, the decision-maker needs to consider the decision objectives and option attributes and also the dependability of the process of generating objective, options and attributes. The approach advocated here treats evidence relating to process dependability as an attribute A_{n+1} of each option. This evidence is integrated with other interval measures of the options using the method described in the previous section. It therefore requires the decision-maker to make a direct estimate of the importance (or value) of process dependability relative to the other attributes. This approach is therefore rather different to the one previously proposed in Section 5.14, in which process evidence was projected onto a probability scale of the dimensions of another attribute.

Consider a situation where a coastal engineer is choosing, on the basis of benefit-cost ratio (BCR), which coast protection scheme to implement. The engineer has carried out a probabilistic analysis of the three options d_1, d_2, d_3 and thereby generated a set of probability distributions of benefit-cost ratio as shown in Figure 6.2. The design and analysis processes to obtain the estimate of BCR associated with each of the options differed in some important respects and the engineer would therefore like to include relevant process evidence in the choice. Now the engineer is risk neutral so the distributions in Figure 6.2 can be replaced with their expected values. The transformation from data space to a common attribute space uses the approach described in the previous section. The evidence from process modelling indicates the following process dependabilities shown in Table 6.9.

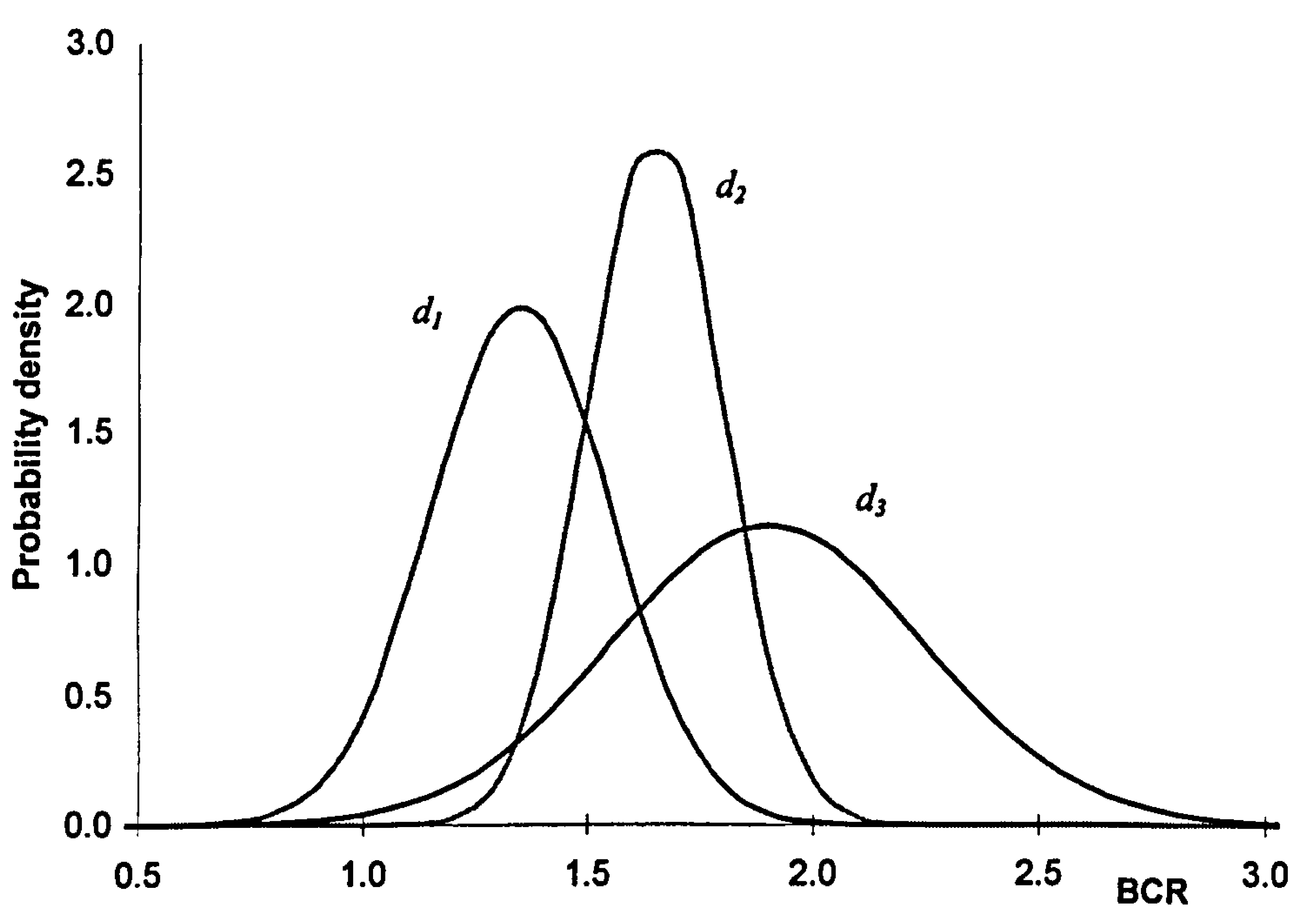

Figure 6.2 Probability density function of BCR of options

Table 6.9 Process dependabilities

Option	Expected value of BCR	Interval measure of economic efficiency, A_1	Dependability of design/analysis process, A_2
d_1	1.35	[0.18, 0.18]	[0.82, 0.93]
d_2	1.65	[0.33, 0.33]	[0.55, 0.90]
d_3	1.90	[0.45, 0.45]	[0.65, 0.79]

Now suppose that in this case there is no reason to believe that the data values of BCR and the process measures are dependent on each other. In general there will be some dependency, for example between cost estimates based on the same unit rates. Assuming independence for the time being, the power set of A_1 and A_2 for the three options has the probabilities in Table 6.10 associated with it.

Table 6.10 Power set of A_1 and A_2

Option	$p(A_1 \cap A_2)$	$p(A_1 \cap \overline{A_2})$	$p(\overline{A_1} \cap A_2)$	$p(\overline{A_1} \cap \overline{A_2})$
d_1	[0.15, 0.17]	[0.01, 0.03]	[0.67, 0.76]	[0.06, 0.15]
d_2	[0.18, 0.90]	[0.03, 0.15]	[0.37, 0.60]	[0.07, 0.30]
d_3	[0.29, 0.35]	[0.09, 0.15]	[0.36, 0.44]	[0.12, 0.20]

Now ideally the decision-maker wants both economic efficiency and process dependability so $p(X \mid A_1 \cap A_2) = 1$. Without economic efficiency the option is considered to be unacceptable so $p(X \mid \overline{A_1} \cap A_2) = P(X \mid \overline{A_1} \cap \overline{A_2}) = 0$.

The situation where a process that is not dependable suggests that the economic efficiency is acceptable is rather more uncertain. However, given low process dependability the value of economic efficiency could be as low as zero, so

$p(X \mid A_1 \cap \overline{A_2}) \in [0.00, 0.50]$.

Applying Equation 1 therefore obtains the interval measures given in Table 6.11.

Table 6.11 Support intervals including evidence of economic efficiency and process dependability

Option	$p(X \mid d_i)$
d_1	[0.15, 0.18]
d_2	[0.18, 0.97]
d_3	[0.29, 0.42]

To identify a preferred option requires a choice mechanism based on intervals.

6.8.3 Choice based on intervals

Table 6.12 summarises various approaches to choice based on intervals. Dominance is the most powerful condition, in which both the lower and upper interval measures for one option exceed those for all other options. Dominance is only satisfied in special cases but it is often possible to use dominance to eliminate some options from further consideration. In Table 6.8 d_3 dominates d_4, so d_4 can be eliminated from the choice problem.

The most straightforward general approach is to map the belief interval onto point value probabilities for all propositions (Krause and Clark, 1993). Thus the 'unknown' probability $S_p(A_k|d_i)$ - $S_n(A_k|d_i)$ should be distributed evenly between $S_n(A_k|d_i)$ and 1 - $S_p(A_k|d_i)$. Referring to the Dempster-Shafer (D-S) theory of evidence rather than to interval probabilities, Krause and Clark (1993) write:

Table 6.12 Rules for decision-making based on intervals

Dominance procedures	Eliminate options that are equal or worse than some other option for both S_n and S_p for every attribute.
Ordering based on midpoint of upper and lower bounds	Distributes uncertainty evenly between $S_n(A_k|d_i)$ and 1 - $S_p(A_k|d_i)$ according to the principle of insufficient reason.
Ordering based on lower or upper bound	Where outcomes are expressed in terms of benefit, ordering based on upper bounds represents an uncertainty-prone attitude and ordering based on lower bounds represents an uncertainty-averse attitude.
Linear combinations of bounds	Orders according to $\lambda S_n(A_k|d_i)$ + (1- $\lambda)S_p(A_k|d_i)$ where λ is a measure of the decision-makers attitude to uncertainty.
Satisficing strategies	Thresholds set for lower or upper bounds or for a maximum allowable uncertainty.

> *From the perspective of decision-making, this provides a two level model of belief. There is a* credal level *at which beliefs are maintained and revised (using the D-S model or perhaps any other model which is thought to be appropriate in a given situation). Then when action needs to be performed, or a decision made, beliefs are transformed to the* pignistic level *(from the Latin word* Pignus *for bet) using the above transformation. At first sight the need to transform beliefs into point value probabilities would seem to weaken the justification for an alternative model at the credal level (i.e. D-S). However, Smets and Kennes (1989) provides a number of examples whereby this two level model provides qualitatively different answers to those obtained through framing the problem entirely within the Bayesian probability model.*

Based on the midpoints of the intervals the preference order for the options in Table 6.8 is $d_3 > d_2 > d_1 > d_4$.

Yager (1986) demonstrates the use of rules for choice using the Dempster-Shafer theory of evidence. The rules are analogous to the maximin, maximax, Hurwicz and least regret strategies for choice under strict uncertainty. Loui (1986) proposes an approach whereby the bounds on intervals are progressively narrowed until an option which dominates all others is identified, but the procedure for narrowing bounds without evidential justification seems to be somewhat arbitrary.

Experience with practising decision-makers suggests that it is the weight of evidence either for or against a proposition, as well as the level of uncertainty, which they take into account when making decisions based on intervals. The reaction of decision-makers to information expressed as intervals depends on their attitude to uncertainty. The attitude to uncertainty is analogous to the attitude to risk in normative decision theory. The conventional approach is that an uncertainty-averse strategy uses the lower bound whilst an uncertainty-prone strategy uses the upper bound. In that case the options in Table 6.8 would be ordered as follows.

Uncertainty-averse: $d_3 > d_2 > d_4 > d_1$

Uncertainty-prone: $d_1 > d_3 > d_2 > d_4$

The uncertainty-averse strategy corresponds to the "miniupper" rule (Dempster and Kong, 1987) when the interval expresses expected losses or evidence for failure, rather than the case here which is evidence for success. This miniupper rule is a generalisation of the minimax rule for choice under strict uncertainty.

Using the lower or upper bound alone as a basis of choice are extremes of a range of possible linear combinations of the form

$\lambda S_n(A_k|d_i) + (1-\lambda)S_p(A_k|d_i)$,

an approach that is analogous to utility functions, discussed in Appendices 2 and 4.

The value of λ represents the decision-maker's attitude to uncertainty.

$\lambda = 0$ represents extreme uncertainty-prone attitudes with decisions based only on the upper bound;

$\lambda = 0.5$ represents uncertainty-neutral attitudes;

$\lambda = 1$ represents extreme uncertainty-averse attitudes with decisions based only on the lower bound.

Interestingly, the human subjects engaged in the research case studies adopted what would be classified above as being a uncertainty-prone strategy but reasoned that because they were seeking to minimise the evidence against the success of an option is was in fact a uncertainty-averse strategy.

In order to include information carried by both the lower and upper bounds decision-makers may choose to adopt satisficing strategies. For example the decision-maker may set a limit to acceptable uncertainty ($S_n(A_k|d_i)$-$S_p(A_k|d_i)$) or to the evidence against the success of a decision option (1-

$S_p(A_k|d_i)$). Provided that satisficing level is surpassed another criterion can be used to select the preferred option.

There is, therefore, no unique of normative approach to choice based on intervals, though choice bases on the midpoint of the interval perhaps has the most normative appeal. Some flexibility in the choice mechanism is to be expected in an approach that recognises the open world more explicitly than the other choice mechanisms discussed in this chapter. It is by precisely defining the problem domain that normative models obtain the power of unique choice. If the model of the problem becomes open and is incompletely specified, as it is in open world decision theory, then the power of unique choice is lost.

6.9 Conclusions

1. Choice is the pivotal moment in the decision-making process. It is at the moment of choice that a preferred option is identified on the basis of the decision-maker's objectives. A choice mechanism defines the way in which a preferred option will be identified. A choice mechanism has been treated in this chapter as being a pre-defined rule or mathematical operation. Therein lies the dilemma of choice, which is one of matching a precisely defined mathematical operation to messy abstractions in an open world.

2. Ultimately choice is based on satisficing, optimising or a combination of the two. Satisficing involves identifying an option that satisfies a criterion or set of criteria, so it can be thought of as identifying some point in an acceptable region of decision space. Optimising, meanwhile, involves searching the decision space to identify the most advantageous point. To optimise requires a completely defined model of the decision space whereas satisficing is usually possible even in situations where information is vague or incomplete. In satisficing models subjective judgements are obvious as satisficing thresholds whilst in normative models they are less apparent by nonetheless present in the model construction.

3. Because normative models of choice are optimising models they have the power to identify a unique preferred solution. They have been the subject of much learned study and the axioms of rationality which underlie normative models should be abandoned with care. However, normative models of complex choice problems can be difficult to set up and at the end of a time-consuming process can still give the impression of having left out or misrepresented important aspects of the problem. The bounded rationality of normative models mean that the optimum actions they identify may be far from suitable in a complex, messy world. It is in complex open world situations, where decision-makers are confronted with the most difficult problems, that existing models are at their weakest.

4. Multi-attribute choice mechanisms involve either explicit or implicit trade-offs between attributes. Normative models of multi-attribute choice require a completely specified value function. Satisficing models of multi-attribute choice can operate with only partial information but because of that their consistency is not completely tested so they can prove to be incoherent.

5. Open world decision theory uses evidence expressed as intervals in situations of multi-attribute choice. It can therefore express the incompleteness inherent in evidence. It can represent uncertain value functions as well as nestedness and subtle interdependencies between attributes. The need to weigh up values cannot be avoided but legitimate uncertainty can be expressed. Open world decision theory can also be used to include evidence relating to the dependability of the process of obtaining and manipulating options and attributes. In order to do so it requires some judgement of the value of process dependability. To make a choice based on intervals requires some judgement of an appropriate choice mechanism, as there is no normative approach.

6. A contingency approach to choice involves characterising pertinent aspects of the choice situation and matching them to an appropriate choice mechanism. The decision-making context is a guide to an appropriate choice mechanism. Transparency and repeatability are usually important criteria for decisions relating to publicly funded coastal infrastructure. A contingency approach helps to ensure that relevant information, for example relating to process dependability, is take into account thus avoiding bounded rationality. By specifically addressing these issues inappropriate idealisations which distort the nature of problems of choice can be avoided. Ultimately, however, when dealing with open world situations, success of a choice mechanism itself must be partial or incomplete.

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CHAPTER 7

Uncertainty management for coastal defences on the East Coast of the UK

7.1 Objectives of Chapter 7

- to describe the development and application of hierarchical process modelling using Interval Probability Theory to support decision-making for two sea defence projects on the East Coast of the UK;
- to identify sources of uncertainty and sensitivities in the decision-making processes for these two projects;
- to demonstrate how hierarchical process modelling using Interval Probability Theory can be combined with probabilistic methods and observational strategies to manage uncertainty in a beach nourishment project;
- to draw general conclusions regarding the applicability and benefits of the uncertainty management concepts and associated tools and techniques develop in this thesis.

7.2 Introduction

The concepts, techniques and tools of uncertainty management developed in the preceding chapters have been applied to contrasting sea defences projects on the East Coast of the UK. The first application (Section 7.3) addresses the decision to implement a small managed retreat project at Orplands on the Blackwater estuary in Essex. The methods were then developed and applied to the strategic decision to implement a major beach nourishment scheme between Mablethorpe and Skegness on the Lincolnshire coast, know as the Lincshore project (Section 7.4).

These first two case studies were retrospective. The choice of preferred option had already taken place before the uncertainty analysis described was conducted. The uncertainty analysis involved assembling the evidence used in the decision, from documents and interviews with the managers involved. The uncertain dependability of the decision-making process was analysed using a hierarchical process model. Interval probabilities were attached to each of the sub-processes and used to express the logical relationship between the processes in terms of dependencies and conditional probabilities. This approach enabled the sources and significance of uncertainty in the decision-making process to be identified. The benefit of this retrospective analysis is that it was then possible to review the outcome of the decision and, in discussion with the decision-makers, reach a view on the merits of the approaches proposed in this thesis. In that sense these studies were conducted to test the hypothesis that process modelling with interval probability theory represent a useful and dependable approach. Reflections on the application of uncertainty modelling these two case studies are discussed in Section 7.5.

It was then possible to look forward with the Lincshore project to address the development of a beach management strategy for the five years 1999-2003 (Section 7.6). This third and final study was also an opportunity to demonstrate in principle how a range of uncertainty management techniques could be used in the future decision-making for the Lincshore project.

7.3 Orplands seawall

7.3.1 Background to the Orplands Seawall project

Estuary flood defences on the East Coast of the UK

The Environment Agency (EA) (formerly the National Rivers Authority (NRA)) is responsible for 440 km of seawalls in Essex (Leggett and Dixon, 1994). Of these a large proportion are within the extensive estuary systems on the Essex coast. Sixty per cent of the seawalls in Essex are fronted by saltmarsh, which provides the primary defence against wave action. If saltmarsh is eroded the flood defence embankment has to be armoured with revetments and the crest level raised. Thus the erosion of saltmarsh has very significant flood defence implications (Leggett and Dixon, 1994).

Estuaries are for several reasons a particularly challenging domain for the coastal manager (HR Wallingford, 1997). The hydraulic regime is driven by the combined effects of waves and tides and, to a lesser extent, fluvial flows. A wider range of sediment sizes is encountered than on open coasts, with cohesive sediments being much more prevalent. Estuaries are of tremendous economic importance, being the site of important ports, nuclear power stations and other major industries, amongst other things, and yet are also highly valued for environmental reasons. Indeed estuaries contain some of Britain's most precious natural habitats.

In recognition of the environmental importance of the Essex coast, in 1993 it became one of six new Environmentally Sensitive Areas (ESAs) designated by the government. The ESA scheme offered payments to farmers who undertook prescribed forms of environmentally beneficial management aimed at protecting and enhancing the characteristic landscapes and wildlife habitats of the areas. The payments reflect the costs involved and vary according to local circumstances (MAFF, 1993*a*). One such type of management, which fell within the remit of the ESA scheme, was managed retreat, whereby land which had previously been reclaimed for agricultural purposes is reinstated as saltmarsh. Managed retreat can therefore serve both a flood defence and a habitat creation purpose. Moreover, managed retreat provides space for the estuary shoreline to evolve naturally rather than being constrained by man-made defences. Consequently, it is said to help to relieve some of the stress which man-made defences have imposed on an estuary (HR Wallingford, 1997).

The designation of the Essex ESA represented an important change for coastal managers, as it provided a means by which individual farmers could be compensated for the economic losses associated with managed retreat. Without such payments farmers would have been unwilling to allow managed retreat and would have argued for continued flood protection for their land even though the cost of that protection may not have been justifiable from an economic point of view. In this way a policy decision widened the scope of flood defence alternatives available to flood defence managers.

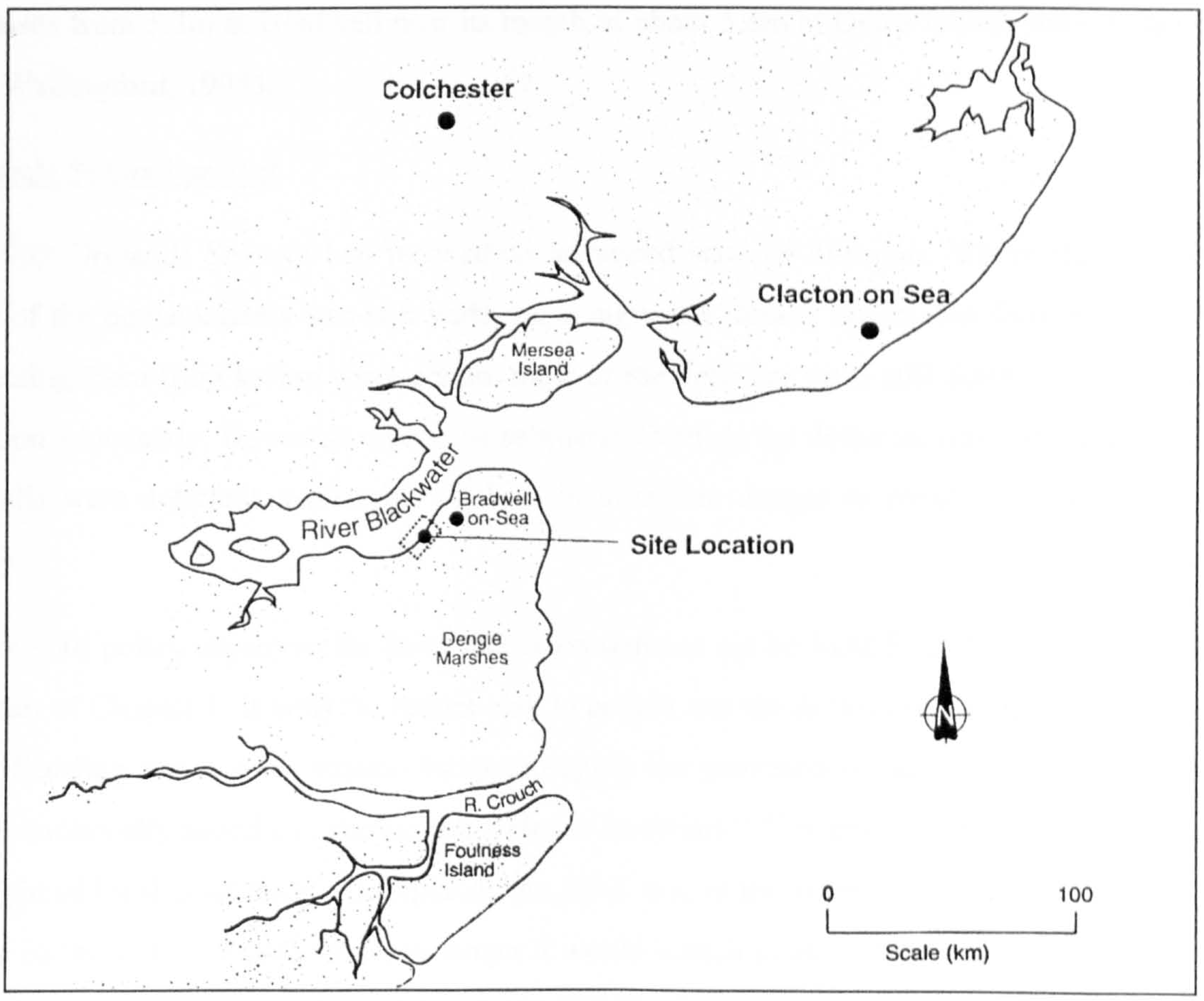

Figure 7.1 Location of Orplands seawall (after HR Wallingford, 1994)

Orplands Seawall

Orplands Seawall was a 5km long earth embankment, which protected 23.5 hectares of low grade agricultural land, situated on the southern side of the Blackwater estuary in Essex, 5km south-west of Bradwell nuclear power station (Figure 7.1). The area at risk from flooding was narrow, at most 350m wide. Beyond this strip the land naturally rises and is above the highest tide level. The land protected by the seawall was semi-improved grassland with conservation importance and was therefore part of a designated Site of Special Scientific Interest (Beecroft, 1994).

The defences at the site were earthen embankments, which in more exposed situations had been reveted with precast concrete "Essex blocks" or stone pitching. These were generally laid directly on the front slope of the clay embankment graded to a slope of 1:2. There was evidence of embankment raising in the past, probably as part of the major embankment-raising programme in the aftermath of the 1953 floods in East Anglia.

Because of the shape of the Blackwater estuary, waves generated in the North Sea become small by the time they reach the north-west facing Orplands seawall frontage. However, the frontage is exposed to waves generated within the estuary by winds from the west and north-west. Significant wave heights of over 0.5m can be expected to occur for some wave conditions (Force 7 and above from between 280 and 020 Degrees North). The Blackwater has a maximum tidal range that increases from 5.3m at Bradwell near its mouth to about 5.8m at Orsea Island, some 15km inland (HR Wallingford, 1994).

Orplands Seawall project

By 1993 Orplands Seawall had reached an advanced state of disrepair. Where the saltmarsh in front of the embankments was still wide, reducing water depths against the front face and hence protecting them from severe wave action, most of the defences were still deemed to perform their function adequately. However, where the saltmarsh fronting the defences was badly eroded the old seawalls were deficient, with some sections in imminent danger of breaching (HR Wallingford, 1994).

The overall policy objective for flood and coast defence set by MAFF (1993*b*) was introduced at the start of Chapter 1. It aims "to reduce risk to people and the developed and natural environment from flooding and coastal erosion by encouraging the provision of technically, environmentally and economically sound and sustainable defence measures." This objective has to be interpreted in the light of local conditions. At Orplands the NRA was in the situation where if the seawall were left in its present condition for much longer it would breach in an uncontrolled manner. This could have adverse impacts on adjacent frontages and the estuary regime as a whole. An unexpected breach would also attract criticism of the NRA, particularly from the owners of the land protected by Orplands seawall and users of the footpath along the seawall. Because of the potential consequences of a breach it was decided to review flood defence options for the Orplands frontage.

The flood defence at Orplands had to be a low cost solution. The low-grade agricultural land protected by the flood defence was of very limited economic value, the only source of income being Set Aside payments. Within the economic constraints, the flood defence at Orplands would also have to be environmentally sensitive and if possible enhance the environment, both locally and within the estuary system as a whole.

The generic options which MAFF (1993*c*) stipulates should be considered during an application for Grant Aid are

- the 'do nothing' or 'without project' case which involves no active flood or coastal defence expenditure: this represents the base case against which other options are appraised;
- a minimum feasible level of defence: examples are filling in low spots in an embankment or replacement of a revetment;
- a higher level of investment; and
- any other genuine alternatives such as managed retreat.

At Orplands three options were considered

- the 'do nothing' case in which no further resources would be invested in the defence which would be allowed to decay and breach in an uncontrolled manner;
- seawall maintenance, involving repair of weak spots, which corresponds to MAFF's minimum feasible level of defence even though this was the most costly of the options considered;
- managed retreat.

No higher level of investment was considered, as it was clear from the outset that even a minimum level of investment would be hard to justify in economic terms.

It was decided that the best solution from an economic, environmental and coastal management point of view was to retreat the defence line to a location approximately 350m inland where the land naturally rises above the high tide level. The managed retreat scheme involved

- constructing low earthen counter-walls to prevent adjacent sites from inundation;
- constructing new culverts in the drains at the landward limit of the site;
- excavating trenches in the retreat site with the aim of reproducing the natural saltmarsh drainage channels;
- constructing a low earth 'breakwater' adjacent to the breach location to prevent waves in the estuary from penetrating the site;
- breaching the sea defence embankment in two places.

These minor civil engineering works were carried out by a small local contractor in August 1995 at a capital cost of £108,000.

The Orplands Seawall project is considered to be a success by the managers involved, with the following benefits:

- reduction in maintenance costs for the flood defence;
- increased area of inter-tidal habitat;
- reduced stress on the estuary as a whole and an increased scope for natural estuary evolution.

Breaching of the embankments went smoothly and since then the cost of maintaining the project has been very small. The counter-wall has settled by 200-300mm. It had been built with an allowance of 400mm so this settlement is satisfactory from a flood defence point of view.

Table 7.1 Key dates in the Orplands Seawall project

Dates	Activity
10 August 1993	MAFF announce new ESAs
21 September 1993	NRA memo stressing urgency of action at Orplands
20 October 1993	NRA summary appraisal recommends Managed Retreat
3 February 1994	NRA memo stressing urgency of action at Orplands
Study conducted early 1994 (report issued March 1994)	Environmental assessment by Wildlife and Countryside Services
Study conducted March - May 1994 (report issued May 1994)	Hydrodynamic assessment by HR Wallingford
July - September 1994	Finalising design details between HR Wallingford and NRA
Study conducted August - October 1994 (report issued October 1994)	Economic appraisal by Scott Wilson Kirkpatrick
August - September 1994	Consultation
October 1994	NRA commissions pre and 5 year post inundation surveys of the land
August 1995	Managed retreat implemented.

The decision-making process as it actually materialised at Orplands has been traced through the documentation held by the EA and discussions with the engineers involved. It is summarised in Table 7.1. Before 1993 there was a fairly low level of monitoring and occasional maintenance activities. Between 1993 and 1995 there was a higher level of analysis, consultation and consensus building, culminating in implementation of the scheme in August 1995. Thereafter the activity associated with the Orplands seawall has been less, mostly associated with monitoring work. Management of the Orplands seawall is an ongoing process, but implementation of managed retreat stands out as the most important process in recent years, and the decisions associated with managed retreat are the subject of this analysis. The key decision which can be identified is the decision to implement managed retreat rather than another defence option (in this case, seawall maintenance or 'do nothing').

From study of the documentation relating to the Orplands Seawall the criteria for success of the project can be interpreted as follows:

- The design is demonstrably the most cost effective of all commonly available flood defence options.
- There are no adverse or knock-on effects on neighbouring frontages, in particular Bradwell marina or the estuary as a whole.
- There is no major objection to the scheme from statutory consultees (including environmental organisations), landowners and local organisations.
- A saltmarsh habitat is created between the abandoned sea defence embankment and the new (mostly natural) defence line.

The choice mechanism for the strategic decision was primarily one of satisficing these four criteria. Satisficing thresholds for the latter these criteria were defined in verbal terms and evolved during the course of the project with increasing understanding of the estuary system and the objectives of the stakeholders. From the economic point of view the scheme had to be shown to be the most efficient of all of the options considered. Although this is apparently a precise decision criterion it will be clear from the uncertainty analysis described presently that the economic attributes of the options were far from certain. Although the economic choice mechanism is nominally based on optimisation, in practice it was based on the satisficing criterion of the preferred option having a benefit-cost ratio in excess of unity. Optimisation was achieved by reducing costs, and there was no specific optimisation of costs and benefits.

7.3.2 The hierarchy of decision-making processes

The strategic decision to implement managed retreat at Orplands, which is the subject of this study, fits into a hierarchy of decision-making. The hierarchic nature of decision-making for coastal defences was introduced in Chapter 1. Some of the hierarchic influences that relate specifically to the Orplands Seawall project are summarised in Figure 7.2.

The analysis carried out in this study endeavoured to distinguish between the project decision to adopt managed retreat and more detailed design and implementation decisions. These different levels of decision-making are to some extent integrated in the documentation relating to the project. In particular the HR Wallingford report and subsequent correspondence between HR Wallingford and the NRA dwells upon the configuration of the managed retreat, which can be interpreted as a detailed design decision. However, if a feasible design cannot be identified then the feasibility of the project as a whole is jeopardised, so these detailed design activities may be

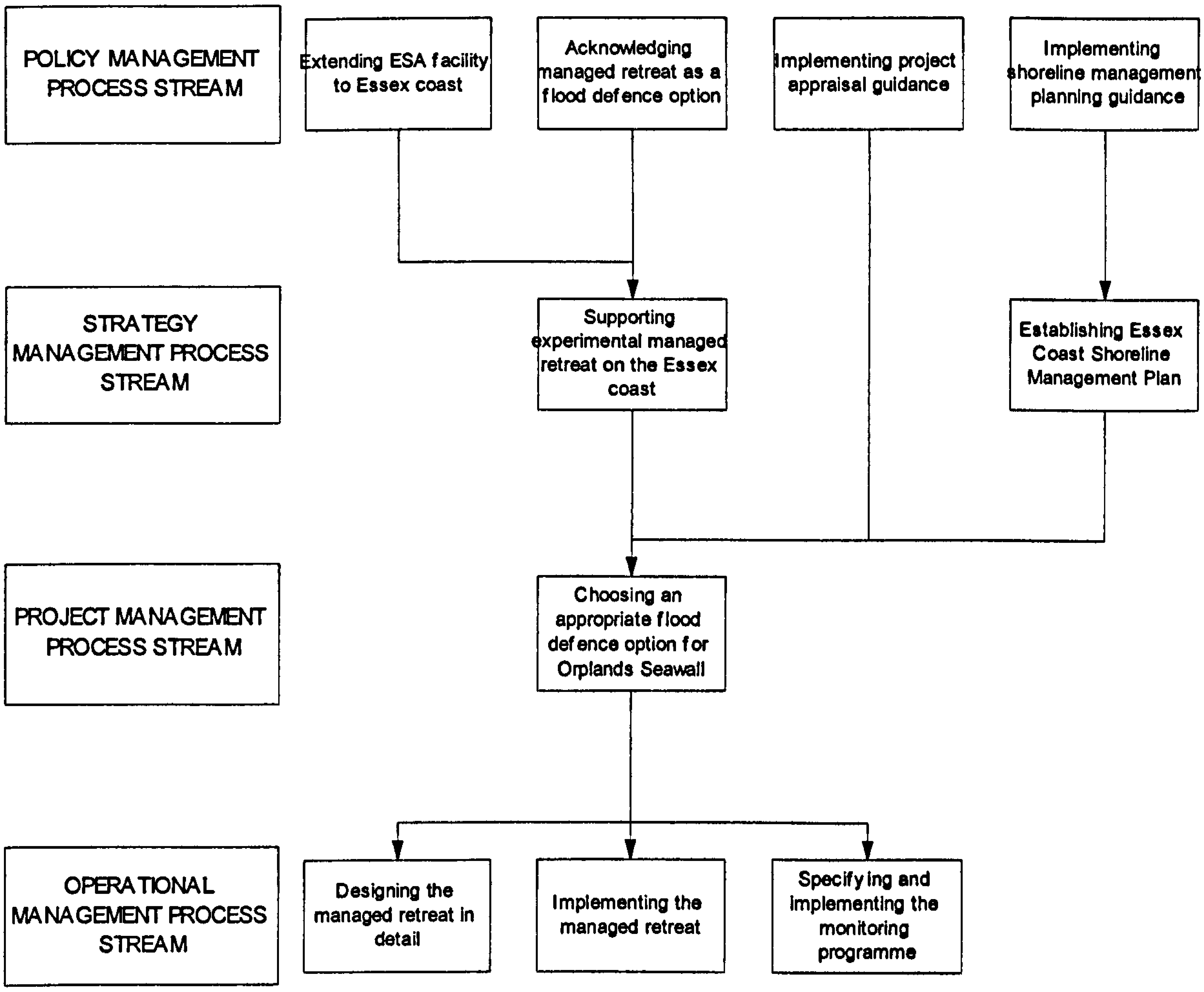

Figure 7.2 Hierarchy of decision-making processes

interpreted as necessary aspects of the project appraisal decision. Equally, it could be interpreted that in the real process, the decision to implement managed retreat had been effectively made before the project appraisal, during the NRA's summary appraisal of 20 October 1993 (see Table 7.1). The project appraisal studies merely served to confirm that decision and also fulfilled a design purpose. This difficulty in disentangling practical processes so that they fit into the structure of the hierarchical process model is a fundamental issue relating to the applicability of the approach, which is revisited in Section 7.5.

7.3.3 Case study 1: Uncertainty modelling for the Orplands Seawall project

The objective of the uncertainty model for the Orplands Seawall project was to provide a measure of the dependability of the decision-making process at Orplands. It identified sources of uncertainty in the process, and enabled sensitivities to process dependability to be explored.

Analysis of the evidence relating to the project indicated that there were four key activities that contributed to the choice to implement managed retreat (Figure 7.3):

- an assessment of the hydraulic impacts of managed retreat, which indicated that managed retreat was feasible in engineering terms and that the impact on neighbouring frontages and the Blackwater estuary as a whole would be negligible;
- an economic assessment of the options, which demonstrated that managed retreat was the most efficient option in economic terms;
- an assessment of the environmental impacts of the managed retreat, which indicated that retreat would have both positive and negative environmental impacts but on balance was a environmentally sound and sustainable engineering solution;
- a consultation exercise, which indicated that provided some mitigation measures were implemented there were no objections to implementing managed retreat.

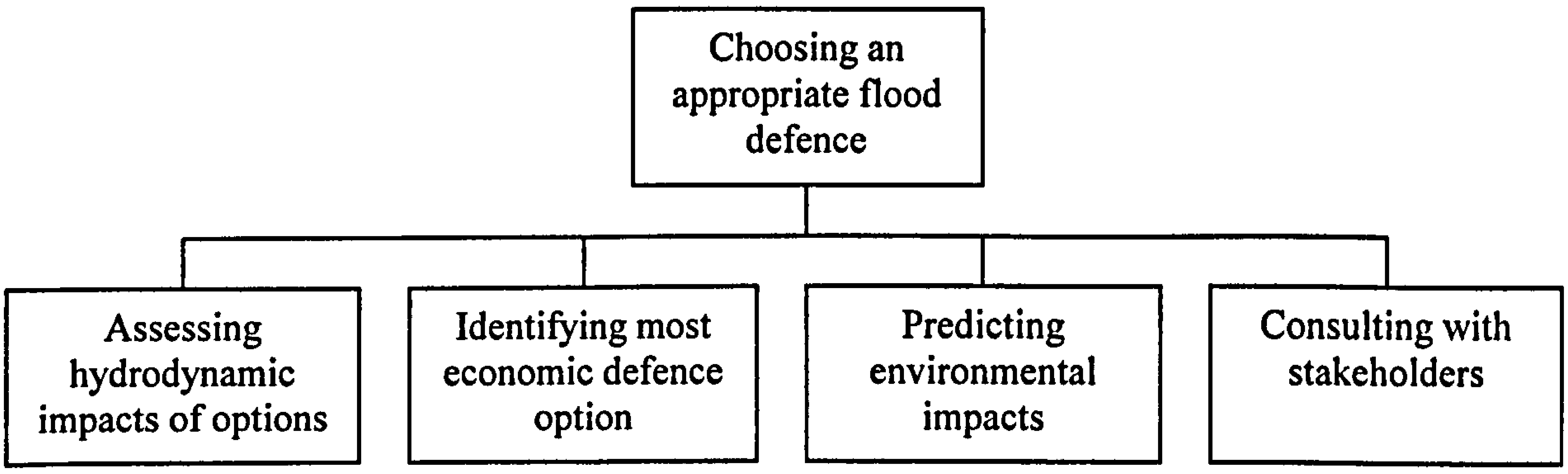

Figure 7.3 High level processes in the Orplands model

These four processes were broken down into further sub-processes. In all, the model of the processes leading up to the decision to implement managed retreat at Orplands comprised 117 processes (Figures 7.4 and 7.5). The manager involved confirmed that the model structure was a good description of the process as he perceived it. He commented that "it does tell the story".

The processes "estimating damage for do nothing" and "estimating flood levels" were used more than once in the hierarchy so were separated into modules (indicated by *M* in the process name). These modules are shown in Figures 7.6 and 7.7 respectively.

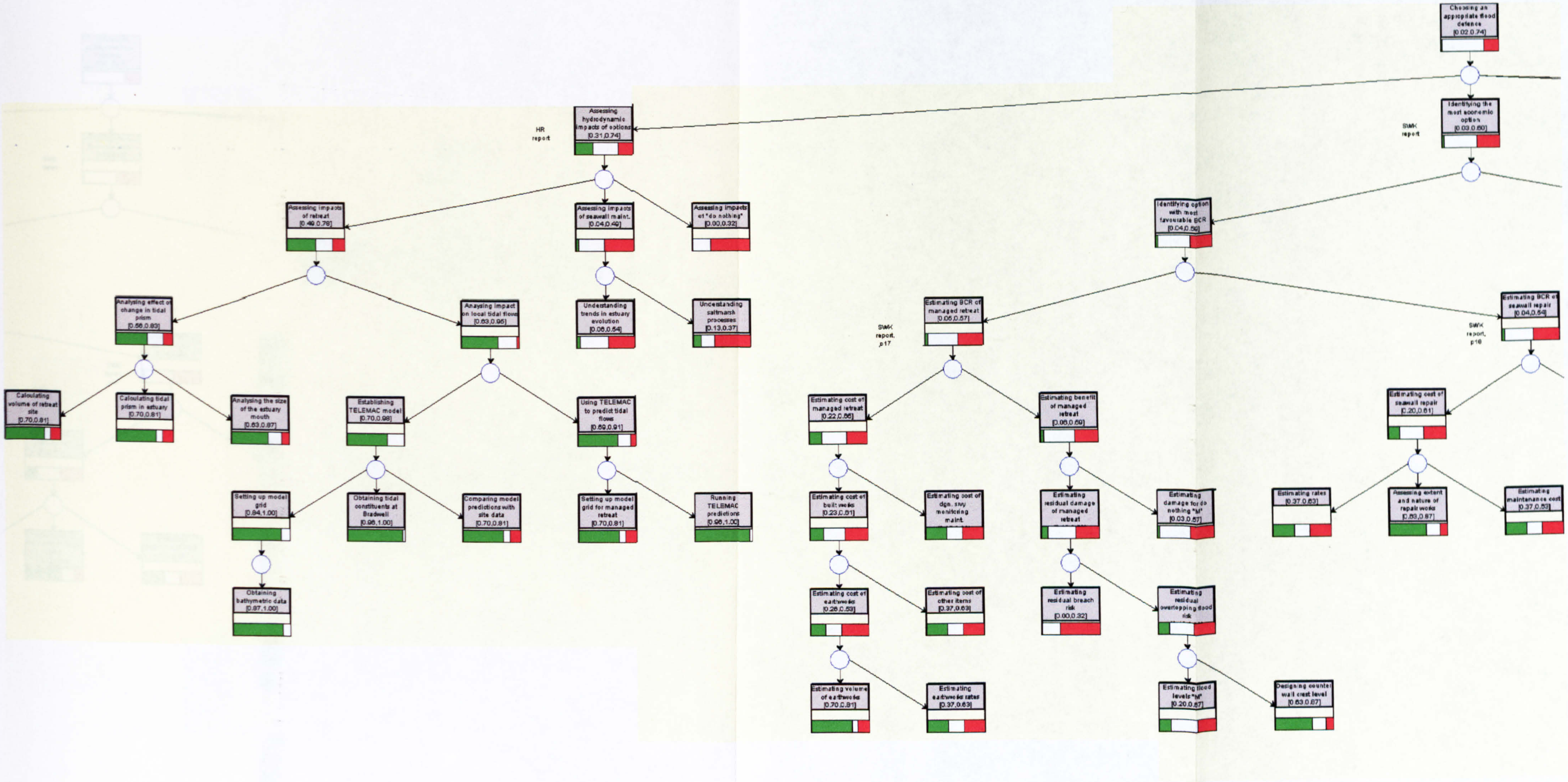

Figure 7.4 The Orplands process model (left hand side) after second pass

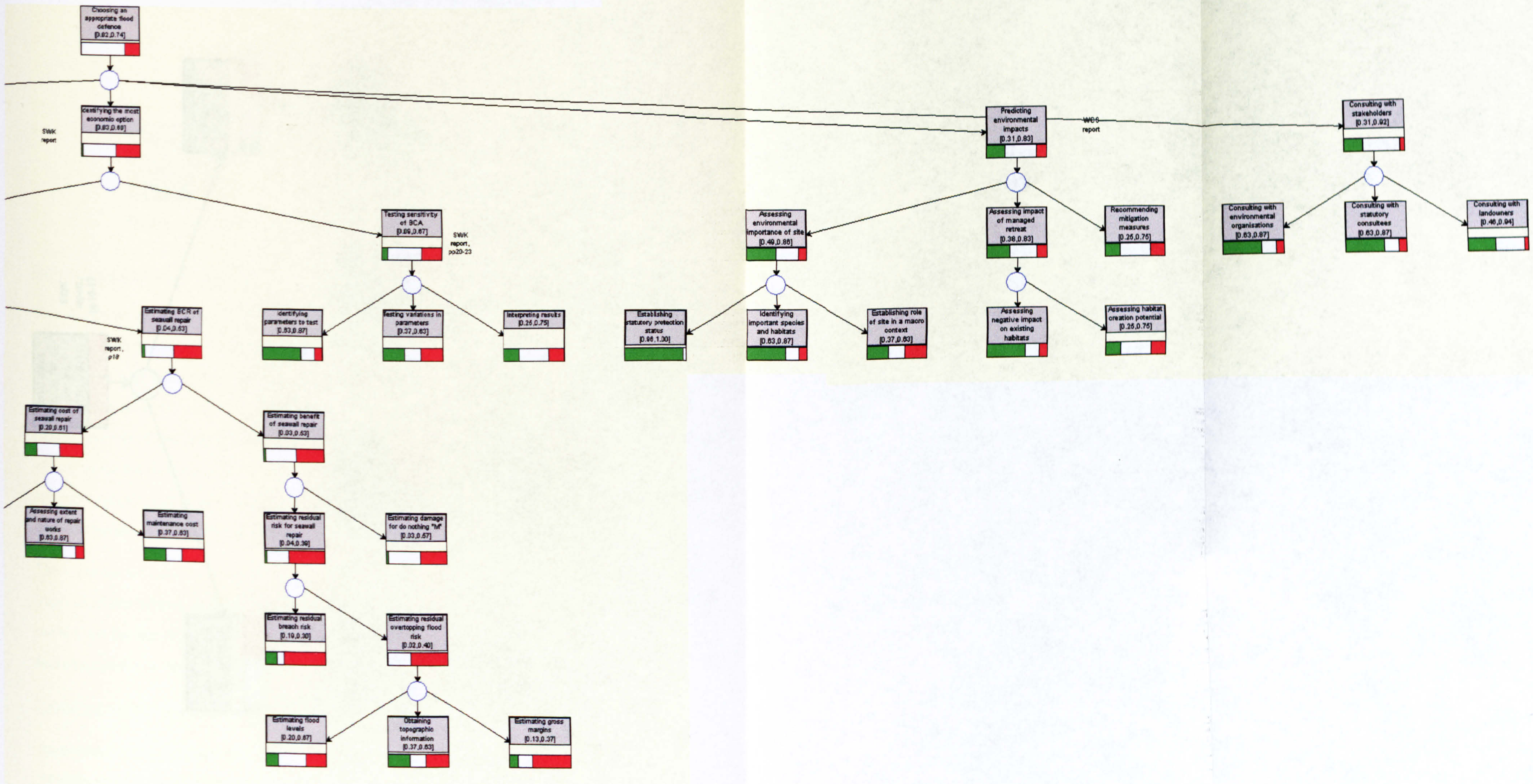

Figure 7.5 The Orplands process model (right hand side) after the second pass

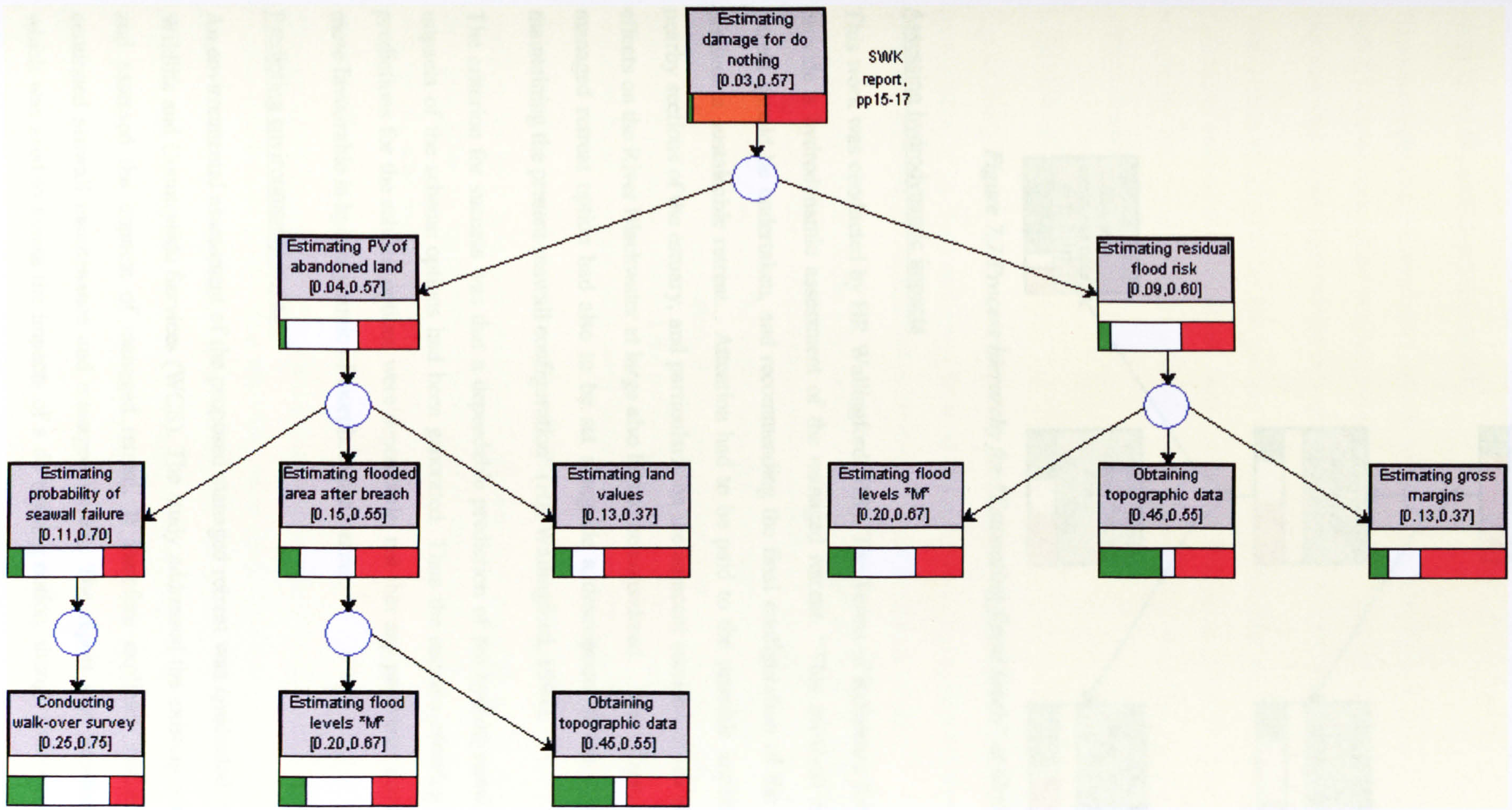

Figure 7.6 Process hierarchy for "Estimating damage for do nothing" at Orplands

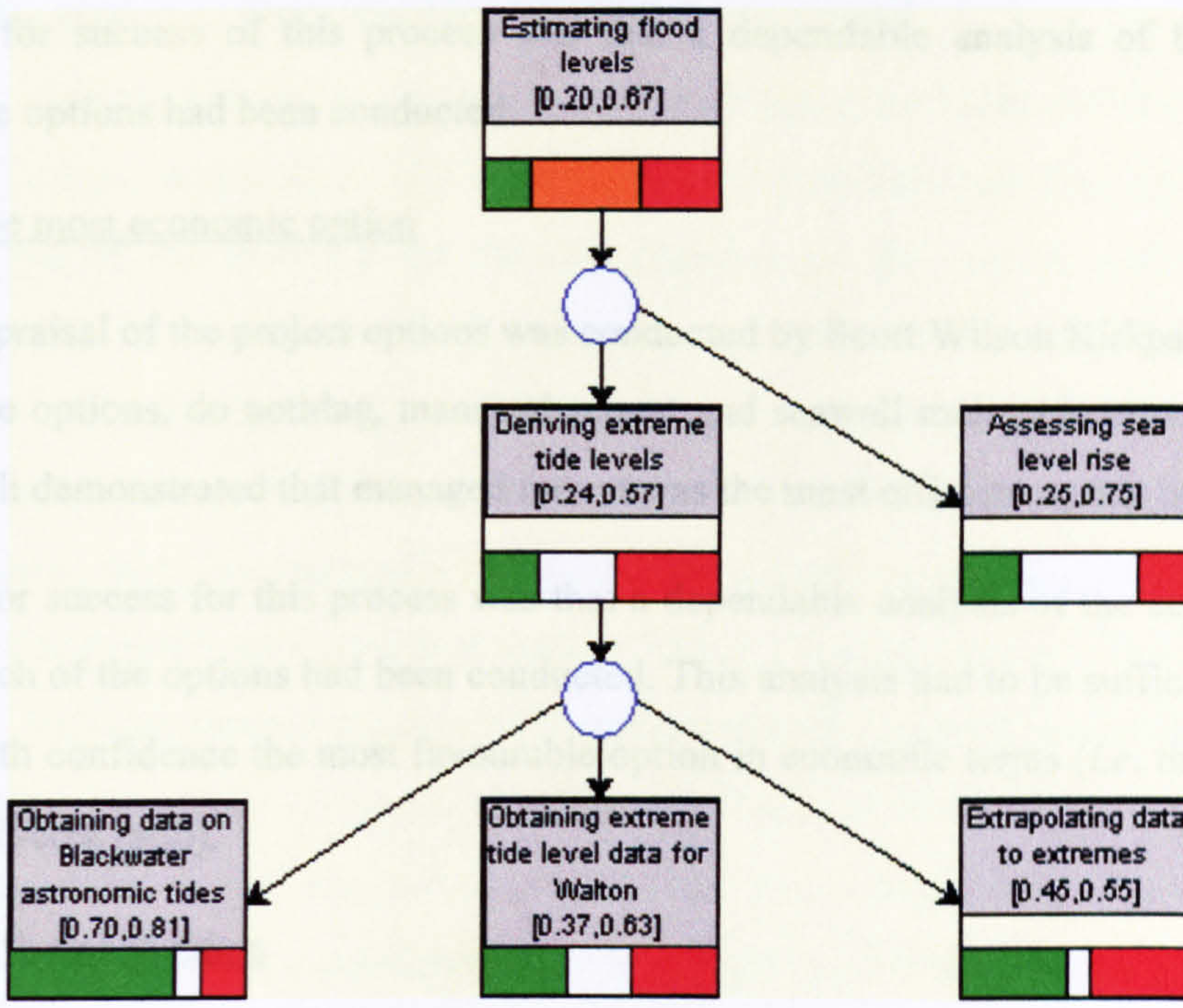

Figure 7.7 Process hierarchy for "Estimating flood levels" at Orplands

Assessing hydrodynamic impacts

This work was conducted by HR Wallingford (HR). The Terms of Reference for HR's study was to provide a hydrodynamic assessment of the managed retreat. "This involved considering how the retreat should be undertaken, and recommending the final configuration of the seawall in order to produce a sustainable retreat... Attention had to be paid to the possible impacts of the retreat on nearby sections of the estuary, and particularly on the adjacent sections of sea defences. The overall effects on the River Blackwater at large also had to be considered… The effects of the recommended managed retreat option had also to be set alongside a description of the effects and impacts of maintaining the present seawall configuration" (HR Wallingford, 1994).

The criterion for success was that a dependable prediction of the hydrodynamic and morphological impacts of the scheme options had been generated. Thus the success criterion was that the model predictions for the scheme options were dependable, not that any particular option was shown to be more favourable in hydrodynamic of morphological terms.

Predicting environmental impacts

An environmental assessment of the proposed managed retreat was conducted by Roger Beecroft of Wildlife and Countryside Services (WCS). The study addressed the existing environment at the site and assessed the impacts of managed retreat. It therefore explicitly addressed the impacts of continued seawall maintenance and managed retreat. The report also provided some information which was used to assess the impacts of a do nothing option, though this option was not explicitly addressed.

The criteria for success of this process was that a dependable analysis of the environmental impacts of the options had been conducted.

Identifying the most economic option

Economic appraisal of the project options was conducted by Scott Wilson Kirkpatrick and Partners (SWK). Three options, do nothing, managed retreat and seawall maintenance were studied in the analysis which demonstrated that managed retreat was the most efficient option in economic terms.

The criteria for success for this process was that a dependable analysis of the economic costs and benefits of each of the options had been conducted. This analysis had to be sufficiently dependable to identify with confidence the most favourable option in economic terms (*i.e.* the option with the highest benefit-cost ratio).

Consulting with stakeholders

Unlike the other key process, which were conducted by consultants, consultation was carried out by the NRA. It involved contacting 28 local and national organisations to obtain their views on the proposed managed retreat and obtain the necessary statutory approvals.

To be completely successful a favourable consensus about the project, consistent with the NRA's proposals, should be established amongst the consultees. Consultation is an interactive process and should be conducted with the expectation that the engineering proposals may be changed as a consequence of the consultation. In the case of Orplands the statutory consultees were largely in favour of the scheme, provided the public footpath along the abandoned seawall was satisfactorily re-routed. Minor changes had to made to the scheme to reach agreement with the landowners.

At some sites it is impossible to reach this type of consensus, because of underlying value conflicts. To be a successful consultation process, at a minimum all of the stakeholders should have been given an opportunity to participate in the process and should recognise its legitimacy, with the scope for appeal to higher authority.

At Orplands the consultation process and the environmental impact assessment were distinct. The manager involved commented that it is more customary for these two processes to be merged. This enables stakeholders to articulate what they value about the environment, at the same time as the rather more scientific process of classifying the environment at the site and the assessing the impact of the proposed project. From the point of view of uncertainty analysis, this type of merged political/scientific process would have been rather more challenging to model than the separated process.

7.3.4 Results obtained from the Orplands process model

When constructing the process models a mapping analogous to a fuzzy membership function was used to help the experts map their beliefs onto numerical values (Figure 7.8). The expert made two judgements, one of the evidence for the dependability of the process and secondly of their confidence in their first judgement. Both of these judgements were made by choosing one verbal measure from "very low", "low", "moderate", "high" and "very high". The first judgement, evidence dependability, enabled one of the five curves shown on Figure 7.8 to be selected. The second judgement, which represented a measure of the expert's confidence in their first judgement, enabled one of five levels on the vertical scale in Figure 7.8 to be selected. The intersection of the selected level, with the selected dependability curve enabled an interval probability to be obtained. So, for example a judgement of "low" evidence, with "high" confidence in that judgement, would give an interval [0.19, 0.30]. The curves are such that any judgement of "very low" confidence gives an interval of [0,1] and any judgement of "very high" confidence gives a point probability. In this way the number of alternative input intervals is restricted to 21. Obviously the user is free to input other values if appropriate, but greater consistency of input interval probabilities is achieved by generally restricting judgements to intervals corresponding to linguistic labels. It is possible to map probabilities back onto verbal descriptors by selecting the descriptors that most closely correspond to a calculated interval.

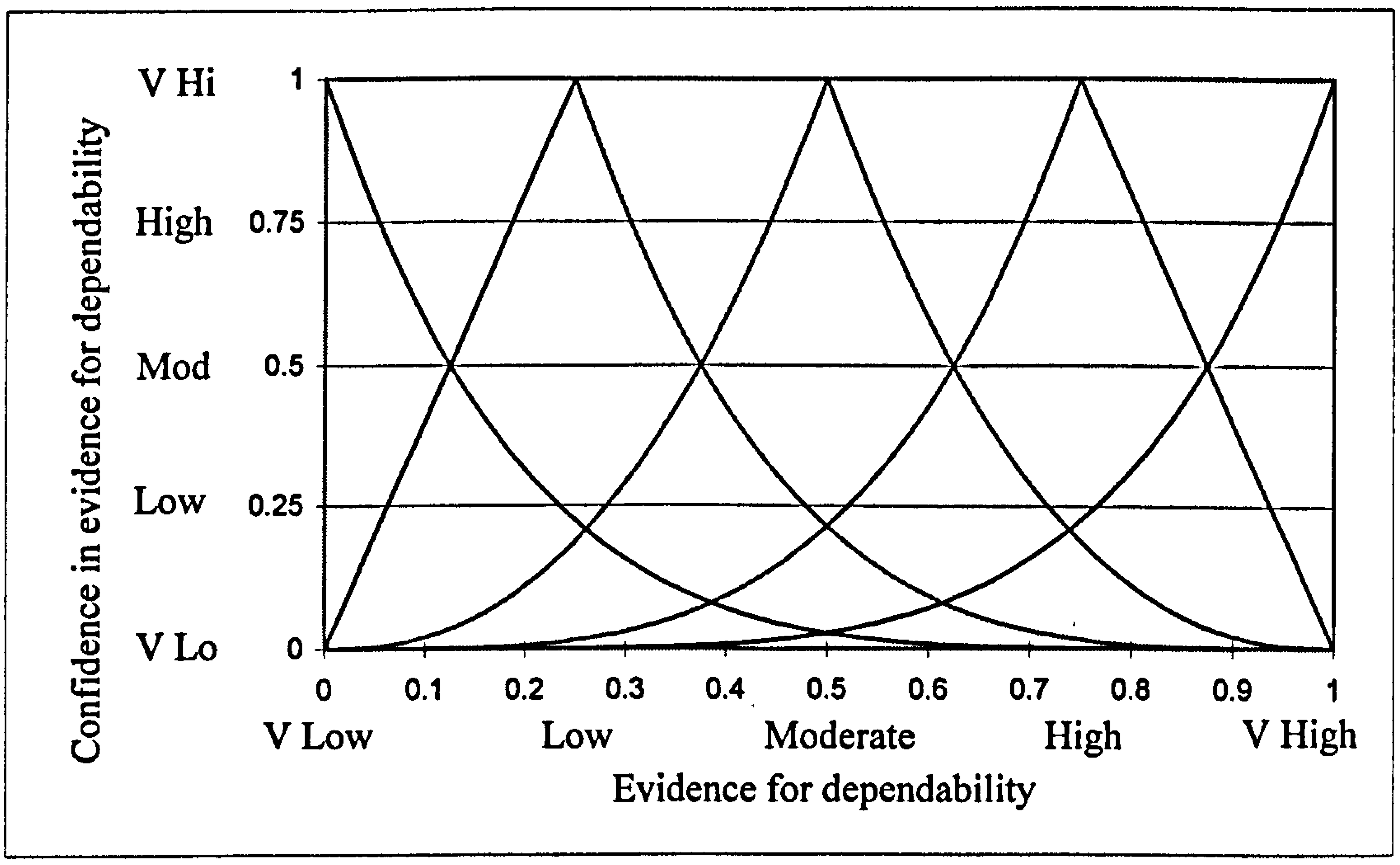

Figure 7.8 Membership function for mapping expert judgements onto interval numbers

Results from the preliminary model construction

When interval measures of dependability and uncertainty were included in the model on the basis of the documentary evidence, the top process of "choosing an appropriate flood defence" had a calculated support interval of [0.00, 0.64]. This interval number corresponds in verbal terms to "very low" evidence for dependability of the process with "low" to "very low" confidence in the assessment. Orplands was a novel and rather experimental project so a high level of uncertainty is to be expected. In particular the failure mechanism and potential evolution of the site under the various options is difficult to predict, being at the limits of scientific understanding of estuary processes. Nonetheless, the professionals working on the project judged that in general the analysis which had been carried out was sound and, whilst the scheme was rather novel, there was a reasonable expectation of success, and, in the short term at least, this expectation seems to have been realised. To try and account for this inconsistency between the model and professional

Table 7.2 Support for sub-processes (after the first pass through the model)

Sub-process	Support interval	Verbal mapping	Comments
Assessing hydrodynamic impacts of options	[0.31, 0.74]	Moderate dependability with moderate confidence	The calculated measures concur with intuition. The analysis was carried out by a reputable consultant using the best available models. However, the nature of the analysis was quite unusual and the modelling techniques were being stretched to their limits. Some of the data necessary to conduct the analysis (tidal data, topography of site) was only of moderate dependability.
Identifying the most economic option	[0.01, 0.58]	Low dependability with low confidence	The calculated measure was considered to be rather pessimistic. The analysis was based on some significant assumptions (e.g. breach probabilities) which had limited evidence to support or refute them. However, other aspects of the analysis, in particular capital costing, can usually be estimated with moderate dependability. The overall measure of dependability was dominated by the logical necessity of both costs and benefits in the cost-benefit analysis process.
Predicting environmental impacts	[0.31, 0.83]	Moderate to high dependability with low confidence	The measure concurs with the general nature of the environmental assessment which English Nature's officer described as a "succinct but adequate review of all the relevant aspects of the scheme" (English Nature, 1994). Environmental assessments are typically characterised uncertainty because of the complexity and poorly understood nature of ecological systems.
Consulting with stakeholders	[0.13, 0.82]	Moderate dependability with low to very low confidence	Evidence for or against the success of this process can be expected was scarce, not least because of the arguable nature of the criteria for success.

judgement, the model was examined in more detail. The support intervals for the four high level sub-processes are summarised in Table 7.2.

Results from the second pass through the model

In view of the comments summarised in Table 7.2 the process model of the economic analysis ("Identifying the most economic option") was revisited. The following revisions were implemented:

1. For the seawall repair option the support for "Estimating maintenance cost" was increased from [0.06, 0.54] to [0.37, 0.63], which represents an increase from "low evidence with low confidence" to "moderate evidence with moderate confidence". The conditional probabilities were increased to reflect low impact of maintenance cost on overall cost for this option. This change had the effect of increasing the support for "Estimating cost of seawall repair" from [0.11, 0.54] to [0.20, 0.61]. However, no reason could be found to question the low dependability of the benefit assessment. This was both due to the uncertainty associated with the breach probability and the evaluation of the economic value of subsidised land (retreated or protected). Benefit assessment is a necessary sub-process in a benefit-cost assessment (BCA) process, so the change in the costing processes only increased the dependability of the BCA from [0.02, 0.51] to [0.04, 0.54].
2. The conditional probabilities relating to the process "Estimating the benefit of managed retreat" were changed to bring them into line with the conditionals for estimating the benefits of seawall repair. Minor adjustments were made to other conditional probabilities to make them more consistent.

The overall effect of revisiting the benefit-cost assessment was a minor change in the evidential support for the BCA from [0.01, 0.58] to [0.03, 0.60].

Scrutiny of the processes involved in "Assessing the hydrodynamic impacts of options" and the "Predicting environmental impacts" revealed no aspects in obvious need of revision. The intervals assigned to the sub-processes of "Consulting with stakeholders" were judged to be rather low in some cases. Consulting with landowners was increased from [0.25, 0.75] to [0.46, 0.94]. Minor adjustments were made to the conditional probabilities and dependencies. This had the overall effect of increasing the support for the consultation process from [0.13, 0.82] to [0.31, 0.92].

Following these revisions the support for the top-level sub-processes was as shown in Table 7.3, giving an overall dependability for the top process of [0.00, 0.67].

Table 7.3 Support for top level sub-processes after revisions

	Sub-process	Support interval	Verbal mapping
A	Assessing hydrodynamic impacts of options	[0.31, 0.74]	Moderate dependability with moderate confidence
B	Identifying the most economic option	[0.03, 0.60]	Low dependability with low confidence
C	Predicting environmental impacts	[0.31, 0.83]	Moderate to high dependability with low confidence
D	Consulting with stakeholders	[0.31, 0.92]	Moderate dependability with low to very low confidence

The conditional probabilities and dependencies associated with the top level processes were as follows:

$p(H \mid A \cap B \cap C \cap D) \in [1.00,1.00]$ $\quad$ $p(H \mid \overline{A} \cap B \cap C \cap D) \in [0.06,0.54]$

$p(H \mid A \cap B \cap C \cap \overline{D}) \in [0.25,0.75]$ $\quad$ $p(H \mid \overline{A} \cap B \cap C \cap \overline{D}) \in [0.00,0.32]$

$p(H \mid A \cap B \cap \overline{C} \cap D) \in [0.63,0.87]$ $\quad$ $p(H \mid \overline{A} \cap B \cap \overline{C} \cap D) \in [0.00,0.32]$

$p(H \mid A \cap B \cap \overline{C} \cap \overline{D}) \in [0.25,0.75]$ $\quad$ $p(H \mid \overline{A} \cap B \cap \overline{C} \cap \overline{D}) \in [0.00,0.13]$

$p(H \mid A \cap \overline{B} \cap C \cap D) \in [0.13,0.37]$ $\quad$ $p(H \mid \overline{A} \cap \overline{B} \cap C \cap D) \in [0.00,0.32]$

$p(H \mid A \cap \overline{B} \cap C \cap \overline{D}) \in [0.00,0.32]$ $\quad$ $p(H \mid \overline{A} \cap \overline{B} \cap C \cap \overline{D}) \in [0.00,0.13]$

$p(H \mid A \cap \overline{B} \cap \overline{C} \cap D) \in [0.00,0.32]$ $\quad$ $p(H \mid \overline{A} \cap \overline{B} \cap \overline{C} \cap D) \in [0.00,0.04]$

$p(H \mid A \cap \overline{B} \cap \overline{C} \cap \overline{D}) \in [0.00,0.13]$ $\quad$ $p(H \mid \overline{A} \cap \overline{B} \cap \overline{C} \cap \overline{D}) \in [0.00,0.00]$

The dependencies were also assigned based on the linguistic mapping shown in Figure 7.8 extended to a [-1, 1] scale:

$Dep(AB) \in [0.37, 0.63]$ $\quad$ $Dep(AD) \in [-0.13, 0.13]$ $\quad$ $Dep(BD) \in [0.00, 0.32]$

$Dep(AC) \in [0.25, 0.75]$ $\quad$ $Dep(BC) \in [-0.13, 0.13]$ $\quad$ $Dep(CD) \in [0.00, 0.32]$

In discussion with the managers involved in the project it had not been particularly difficult to explain the role of conditional probabilities, even to someone admitting to only basic knowledge of probability theory. However, it was much harder to elicit numerical values of these conditional probabilities, particularly for multiple sub-processes. It was possible to order the importance of sub-processes. However, even this ordering was occasionally contradicted by the subsequent discourse of the manager. The dependency between processes was rather easier to explain in principle but still in practice proved to be a challenge to obtain numerical values. Following analysis of the discourse of the managers involved the dependencies were reviewed and changed to the following values:

$Dep(AB) \in [0.63, 0.87]$ $\quad Dep(AD) \in [-0.13, 0.13]$ $\quad Dep(BD) \in [0.00, 0.32]$

$Dep(AC) \in [0.25, 0.75]$ $\quad Dep(BC) \in [0.13, 0.37]$ $\quad Dep(CD) \in [0.25, 0.75]$

which increased the dependability of the top process from [0.00, 0.67] to [0.01, 0.68]. The conditional probabilities were then reviewed and eight of the interval probabilities were changed as follows:

$$p(H \mid A \cap \overline{B} \cap C \cap D) \in [0.25, 0.75] \qquad p(H \mid \overline{A} \cap B \cap C \cap D) \in [0.25, 0.75]$$

$$p(H \mid A \cap \overline{B} \cap C \cap \overline{D}) \in [0.06, 0.54] \qquad p(H \mid \overline{A} \cap B \cap C \cap \overline{D}) \in [0.06, 0.54]$$

$$p(H \mid A \cap \overline{B} \cap \overline{C} \cap D) \in [0.06, 0.54] \qquad p(H \mid \overline{A} \cap B \cap \overline{C} \cap D) \in [0.06, 0.54]$$

$$p(H \mid A \cap \overline{B} \cap \overline{C} \cap \overline{D}) \in [0.00, 0.32] \qquad p(H \mid \overline{A} \cap B \cap \overline{C} \cap \overline{D}) \in [0.00, 0.32]$$

These changes had the effect of increasing the support interval of the top process to [0.02, 0.74]. Thus, all of the changes in the second pass through the model had the combined effect of increasing the support for the top process from [0.00,0.64] to [0.02, 0.74]. The possible support increased but the uncertainty also increased. The necessary support only changed marginally. The second pass through the model confirmed that it was quite robust to changes in individual parameters. The support for the top process is a function of the combined effect of all of the sub-processes and the relationships between them.

Sensitivity testing with the model indicated that to increase the dependability of the economic appraisal process to a "moderate" level would require the assessments of flood risk for the "do nothing" scenario and the residual risk associated with the options to be of "high" dependability, all other processes remaining the same. To do so would require analysis costing perhaps half of the scheme cost (at the time estimated to be £86,590) of the managed retreat project. The economic data generated in the economic appraisal indicated that managed retreat was clearly the most

Table 7.4 Summary of economic analysis (after Scott Wilson Kirkpatrick, 1994)

	Do nothing	Managed retreat	Maintain seawalls
Cost	0.00	86.59	504.17
Damages	102.58	0.00	2.60
Benefits (Damages avoided)		102.58	99.98
NPV		15.98	-404.19
Benefit-cost ratio		1.18	0.20

favourable option (Table 7.4). Combining the evidence from the uncertainty model with data from the economic appraisal could be used to inform the decision on whether to invest in improving the flood risk assessment process. In this case it would seem not to have been justified.

7.3.5 Summary of findings from the uncertainty model of the Orplands Seawall project

1. The process of choosing an appropriate flood defence for the Orplands Seawall project was found to have a dependability of [0.02, 0.74] which corresponds in verbal terms to "low" dependability with "low" to "very low" confidence in that measure of dependability. The uncertainty modelling demonstrated that there was substantial uncertainty in the overall process of choosing an appropriate defence option. Considerable uncertainty is to be expected in a rather experimental project of this type.

2. The process of choosing an appropriate flood defence was a function of four principle criteria. The chosen option had to be economically efficient, environmentally sound and sustainable, sound in engineering terms and politically acceptable. Uncertainty in each of these processes contributed to the overall uncertainty in the decision on which scheme to implement.

3. The main reason why the dependability was judged to be low was because of the low support for the economic appraisal, which has a great influence on the overall dependability of the process. The economic appraisal was found to have dependability [0.03, 0.60]. The lower bound on this interval is a dominant influence on the lower bound of the top process.

4. The economic appraisal was found to be of low dependability because of the complexity of the failure mechanisms at the site, upon which the economic assessment of flood risk depended. Failure probabilities (which are necessary aspects of the benefit assessment) were assigned using expert judgement and were not particularly dependable. To reduce uncertainty in the decision-making process would require investment in an improved assessment of flood risk.

5. Of the other three high level processes, "Consulting with stakeholders" made the greatest contribution to uncertainty, followed by "Predicting environmental impacts" and finally by "Assessing hydrodynamic impacts of options".

6. The findings of the uncertainty model should not be used in isolation but should complement the information relating to the options which was generated during the economic appraisal, hydraulic and environmental assessment and consultation exercise. These indicated that managed retreat is a favourable option which is robust to changes in key parameters in the decision-making process. Testing of the uncertainty model suggested that for a project of this size (the estimated cost of the project was £87k) it would on balance be appropriate to proceed cautiously without further analysis, especially in view of the high cost associated with reducing

the uncertainty of key processes. A more costly project would have merited more detail analysis in order to secure the economic case for investing public funds.

7. Construction and use of the uncertainty model forced reflection on how the different activities and studies that had been undertaken contributed to the strategic decision to implement managed retreat. The final model structure was confirmed by the EA's project manager to be a good representation of the processes leading up to the strategic decision.

8. The process model revealed considerable disequilibrium in the levels depth of analysis contributing to the four high level processes. The more analytical processes were represented in to a greater level of detail in the hierarchical model, whilst less structured activities like consultation, could not be decomposed to such a detailed degree of granularity. This disequilibrium is a reflection of the ease with which different processes can be represented in a process model. It may also be a reflection of the tendency to invest more effort in engineering design and analysis activities and less effort in consultation and environmental impact assessment processes.

9. Construction and use of the uncertainty model also raised some more general issues relating to hierarchical process modelling with interval probability theory which are discussed in Section 7.5.

7.4 Lincshore sea defences project

7.4.1 Background to the Lincshore project

The sea defences between Mablethorpe and Skegness

The 24 km of sea defences between Mablethorpe and Skegness (Figure 7.9) protect an area of some 20,000 hectares of low lying land including in excess of 15,500 residential properties and 18,000 residential caravans as well as extensive agricultural, commercial and service related activities (Posford Duvivier, 1991), one of the largest coastal flood risk areas in the UK. Drainage of the low-lying land protected by the sea defence is via a system of six major outfalls, which pass through the defences.

Prior to 1953 the sea defences between Mablethorpe and Skegness consisted of revetments of various types, including concrete slab and stepped structures with wave return walls. In 1953 the defences were breached in several places and, in addition to causing widespread flooding, a total of 41 people died. There followed in the aftermath of that disaster a major programme of reconstruction of the sea defences (Posford Duvivier, 1991).

In the 1970s when many of the post-1953 defences were starting to decay a plan for phased reconstruction and upgrading was drawn up. In the 1980s it became clear that in view of the ongoing beach erosion between Mablethorpe and Skegness it would be necessary to consider options of safeguarding the investment in seawall reconstruction.

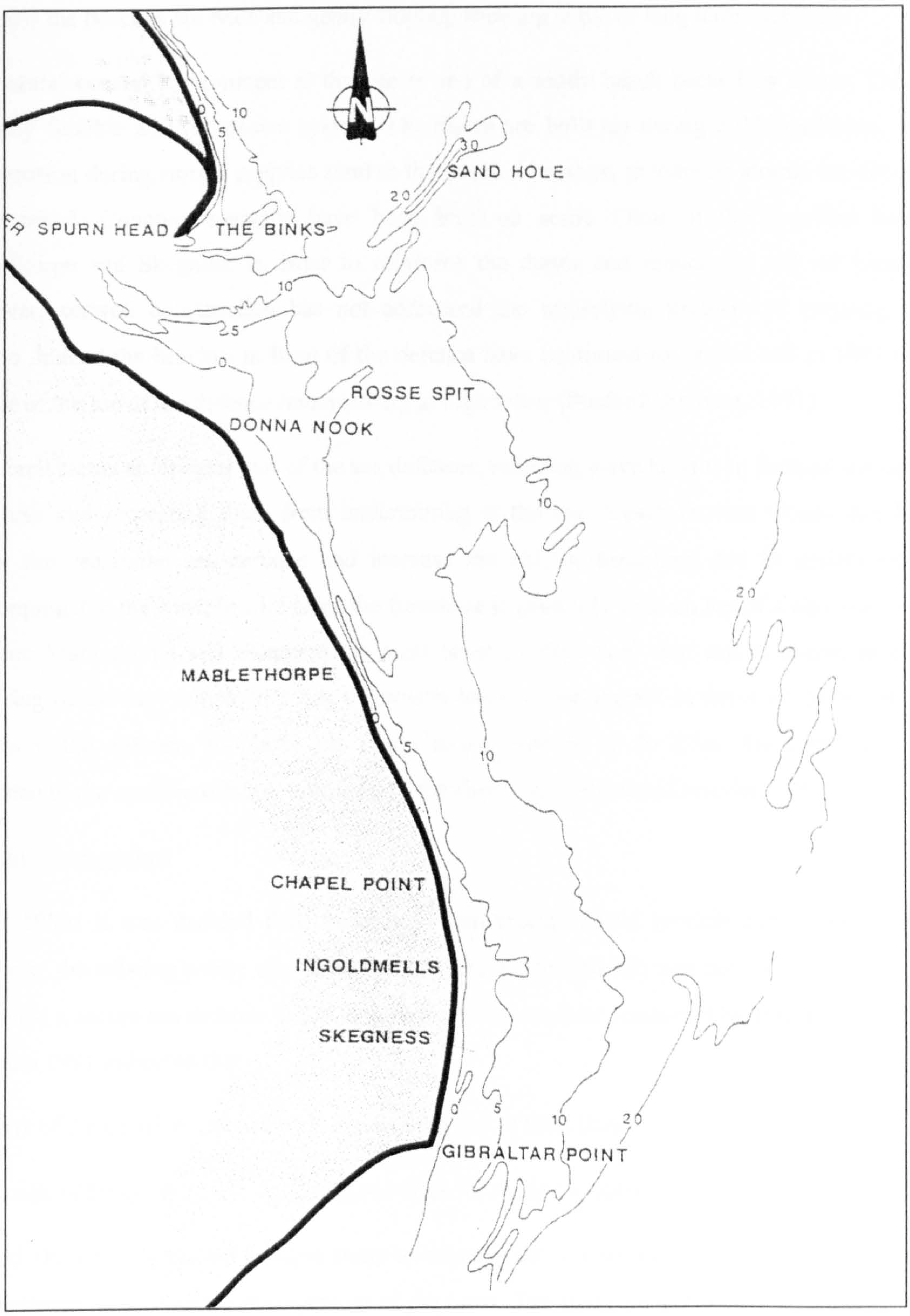

Figure 7.9 Location of the Lincshore project (after Posford Duvivier, 1991)

The coastal system

The coast between Mablethorpe and Skegness forms part of coastal cell 2 (Brampton and Motyka, 1993) which extends from Flamborough Head to the Wash. The coast is gently convex and between Mablethorpe and Skegness is characterised by narrow and relatively steep beaches, which are symptomatic of the erosive conditions that prevail. The low water mark on the beaches has been retreating at a rate of 1-2m/year (Halcrow, 1988). North of Mablethorpe and south of Skegness the beaches are wide and gently sloping, showing signs of long term accretion.

The natural coastal environment at the site is one of a sandy beach backed by dunes. This is a naturally flexible and responsive system. The dunes are built up during mild conditions, whilst dune erosion during storms supplies sand to the beach. However, in extreme storms the dunes can be breached. Concrete seawalls have been built on some 19km of the coastline between Mablethorpe and Skegness in order to reinforce the dunes and reduce the risk of breaching. However, seawall construction has not addressed the underlying problem of ongoing beach erosion. Indeed the beaches in front of the defence have continued to narrow and in 1991 only a quarter of the toe of the defence remained dry at high water (Posford Duvivier, 1991).

The beach forms an integral part of the sea defences, reducing wave heights in front of sea defence structures and protecting them from undermining at the toe. Beach erosion means that higher waves can reach the sea defence and increase the risk of breaching due to undermining or overtopping. On the Lincolnshire coast the foreshore is generally sand on top of a clay substratum. Between Mablethorpe and Skegness the sand layer is often very thin and is absent in places. Lowering of the clay substratum due to erosion has a major impact in terms of increased wave heights at the defence. In places the beach levels vary by up to 2.5m, the variability being attributed to the onshore/offshore transport of the finer sands (Posford Duvivier, 1991).

The Lincshore project

In the 1980s it was decided that, because of continuing coastal erosion and lowering of the foreshore, the existing policy of phased replacement of the seawalls was not on its own sufficient to provide a secure sea defence in the long term. An assessment conducted by Posford Duvivier in 1990 and 1991 indicated that

- many of the existing defence were nearing the end of their lives
- the risk of failure due to overtopping was high for many structures.

Posford Duvivier conducted a major study to advance the understanding of the coastal processes and recommend on strategic management of the coast. The study included field investigations and model studies by HR Wallingford (1987, 1990), Delft Hydraulics (1991*a*, 1991*b*), EGS (1990*a*, 1990*b*) and Nottingham University (1991).

In the development of the strategy the following alternatives were considered either singly or in various combinations.

- Do nothing

- Retreat
- Linear rock protection (seawalls)
- Beach nourishment
- Rock groynes
- Offshore breakwaters / Reefs
- Artificial headlands

Six options were subsequently selected for detailed consideration as follows:

Seawall approach:

1. Sustain present standard of service
2. Improve to a 1:100 year standard
3. Improve to a 1:200 year

Nourishment approach:

4. Nourishment alone
5. Nourishment with rock groynes
6. Nourishment with breakwaters

The Posford Duvivier study concluded that Option 4 was preferred for environmental, technical and economic reasons. The first phases of beach nourishment have now been complete together with some seawall and outfall works. The total costs amount to £71 million over the first five years and £146 million over the 50 year lifetime of the project (Zwiers *et al.*, 1996).

The progress of the Lincshore scheme from the 1980s to the present day is summarised in Figure 7.10. The principal participants in the scheme at this level are

- the NRA (subsequently the EA) who are responsible for the scheme and the client body for the various consultants and contractors involved;
- MAFF who set national policy for flood and coast defence and who Grant Aid capital works;
- Posford Duvivier who carried out the strategic study of options for Lincshore, and several subsequent studies; and
- Halcrow who were involved with the strategic review of the scheme and tender assessment for Phase 2.

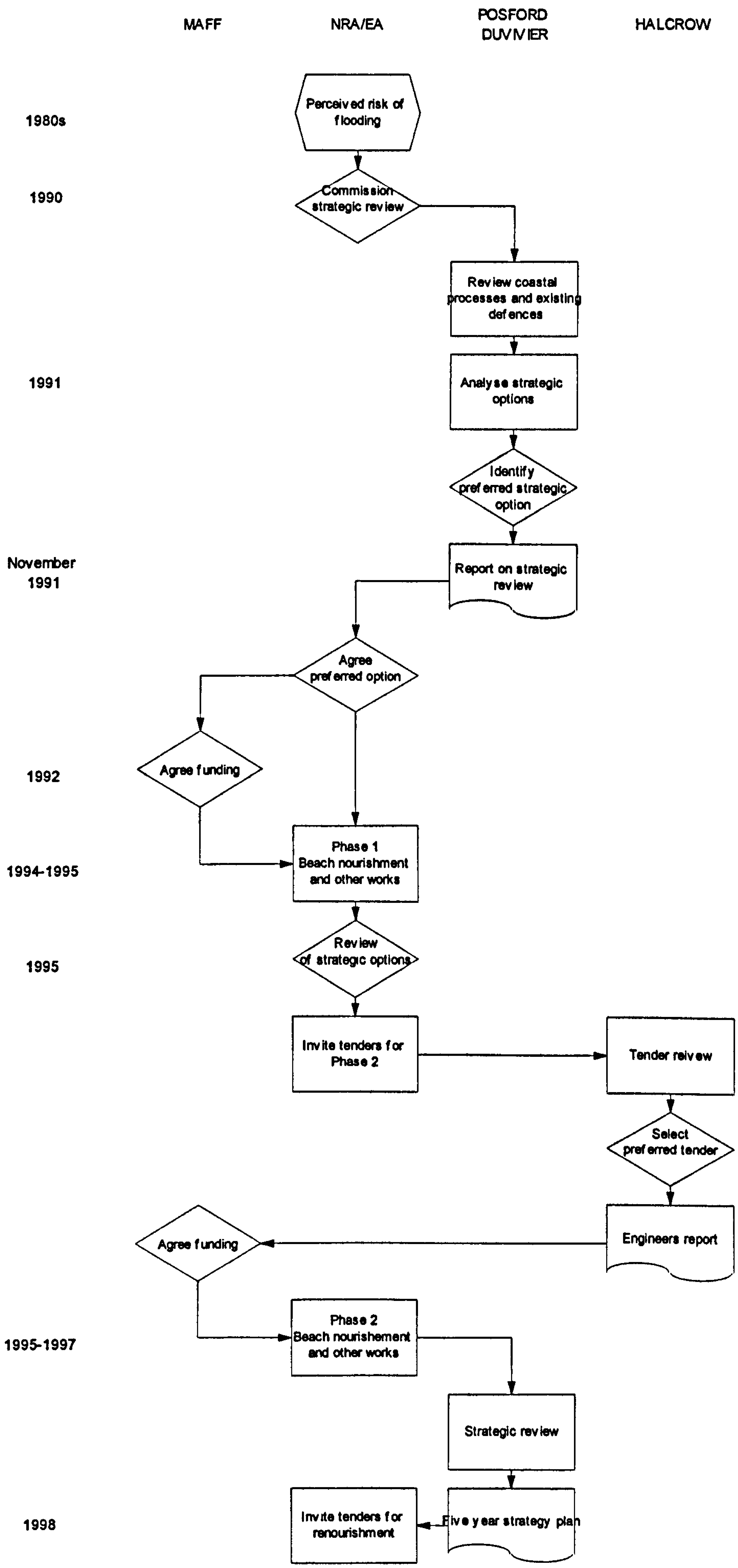

Figure 7.10 Overview of the Lincshore scheme

Some of the key decisions are shown in Figure 7.10 but as should become clear in this report the decision-making processes were complex and widely distributed.

The Lincshore scheme is well documented. Over the years much modelling and analysis has been carried out. Individual projects as well as the overall strategy have been the subject of Post Project Evaluation studies (Babtie, 1995*a*, 1995*b*, 1995*c*, 1995*d*, 1995*e*) which provide some quite independent evidence about the effectiveness of the project and previous decisions. The project is widely regarded as having been successful by most of the usual criteria for success.

The data on the Lincshore scheme used in this study was obtained from the following sources:

- a two-day visit to the site and to the Manby offices of the EA from where the Lincshore project is managed on a day-to-day basis, where the many reports relating to the scheme were studied;
- a visit to the headquarters of MAFF's Flood and Coast Defence Division in London where Post-Project Evaluation reports on the project are held;
- discussions with engineers within the EA and consultants who have been involved with the scheme.

The Lincshore project represents a particularly good example with which to demonstrate the concepts introduced in this thesis. It is an example of coastal defence as an ongoing process. For many years there has been ongoing investment in the defence between Mablethorpe and Skegness. It is well recognised that the nourishment scheme is another element in that ongoing coastal management process and will require a commitment to investment for many years to come.

Neighbouring sections of the Lincolnshire coast are clearly interacting. A strategic approach, which takes these interactions into account, is therefore essential. However, the nature of interactions and the quantity of sediment which is transported between adjacent sections of coast cannot be predicted with certainty because of the year on year variability in wave and tide conditions and because of the complexity of the sediment transport process. The interactions vary from the very local to the impacts of fine sediments eroded from the clay platform, which may influence coastal processes tens or hundreds of miles away in the southern North Sea.

7.4.2 The hierarchy of decision-making processes

Decision-making for the Lincshore scheme is now well established in a hierarchical framework. The Shoreline Management Planning process has involved widespread consultation and identifies in general terms the type of defence option which is appropriate for specific lengths of coastline. Between Mablethorpe and Skegness the Shoreline Management Plan favoured a policy of holding the line. Strategy plans involve more detailed technical and economic review of options. A strategic approach to the sea defences was initiated in 1990, long before MAFF published its guidance on strategy planning (MAFF, 1997). The strategy identified beach nourishment, together with works to repair and upgrade the seawalls as the favoured defence option. Individual seawall and drainage outfall projects and phases of beach nourishment have been implemented as part of that strategy. At the lowest level in the coastal management hierarchy are the day to day decisions on operation of the defences and implementation of the various defence projects which are taking place at any given time. These different levels of decision-making for the Lincshore project were illustrated in Figure 4.6, as an example of the flow of information in a hierarchical decision-making process.

The following section of this report analyses the strategic decision to implement managed retreat. This is a retrospective analysis similar to the one conducted at Orplands. In Section 7.6 current evidence was used to analyse the decision-making process for the next five years of beach management.

7.4.3 Case study 2: Uncertainty modelling of the strategic decision to implement beach nourishment

The strategic decision to implement beach nourishment was based on economic, environmental and technical analysis of the scheme options. The six options (listed on p216) developed and analysed in detail were all considered to be technically achievable, so the choice of option was based on economic and environmental criteria. On the economic side, a degree of optimisation did take place inasmuch as different defence standards for seawalls and beach nourishment were analysed in order to find the most cost-effective defence standard. The environmental assessment was based on satisficing criteria for environmental impact.

The consultants had undertaken sensitivity testing during the economic analysis to test the robustness of the preferred options to likely changes in important variables (Posford Duvivier, 1991). According to the tests performed, the choice was robust to these changes so the implied criterion of robustness was satisfied.

The uncertainty analysis described below focuses on the processes of predicting the economic efficiency of each of the options. The objective of this analysis was to provide measures of the dependability of these processes. These measures of dependability could then have been used, along with the economic, technical and environmental information to compare options and identify needs for further analysis. The uncertainty analysis in this study therefore differs from the Orplands Seawall study discussed previously where an overall measure of the dependability of the decision-making process was generated which did not relate to any specific option.

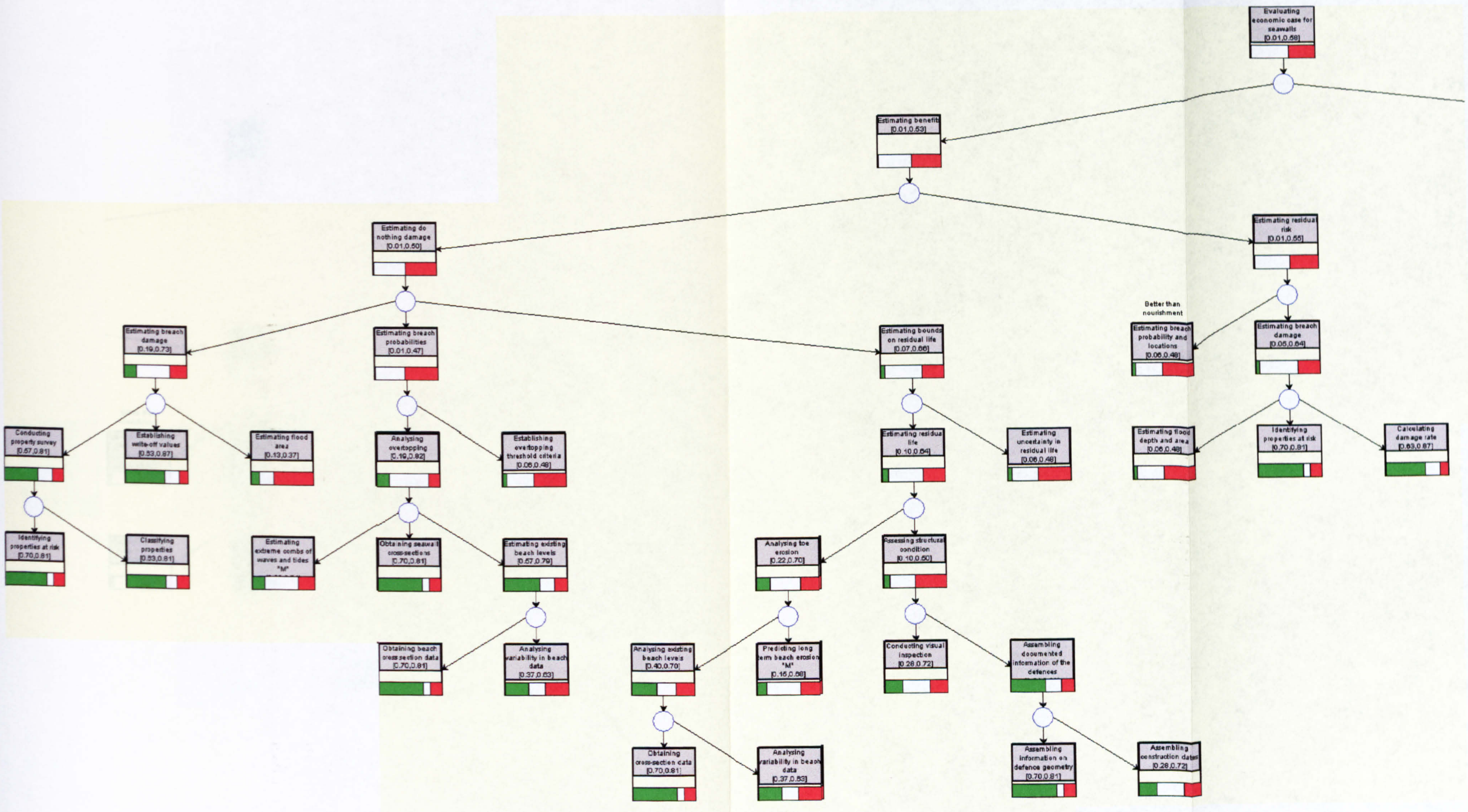

Figure 7.11 Process model for economic appraisal of seawall options (left hand side)

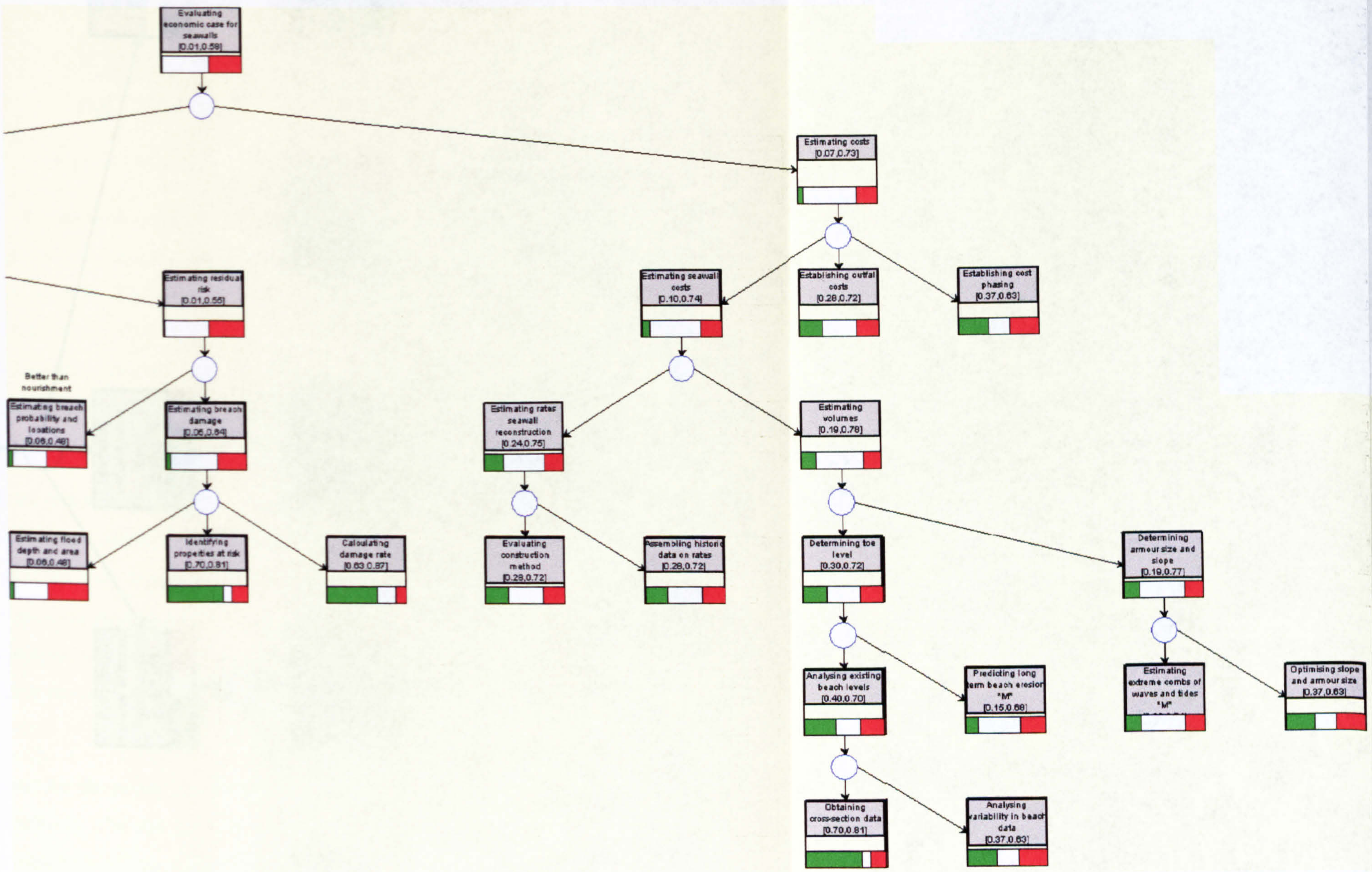

Figure 7.12 Process model for economic appraisal of seawall options (right hand side)

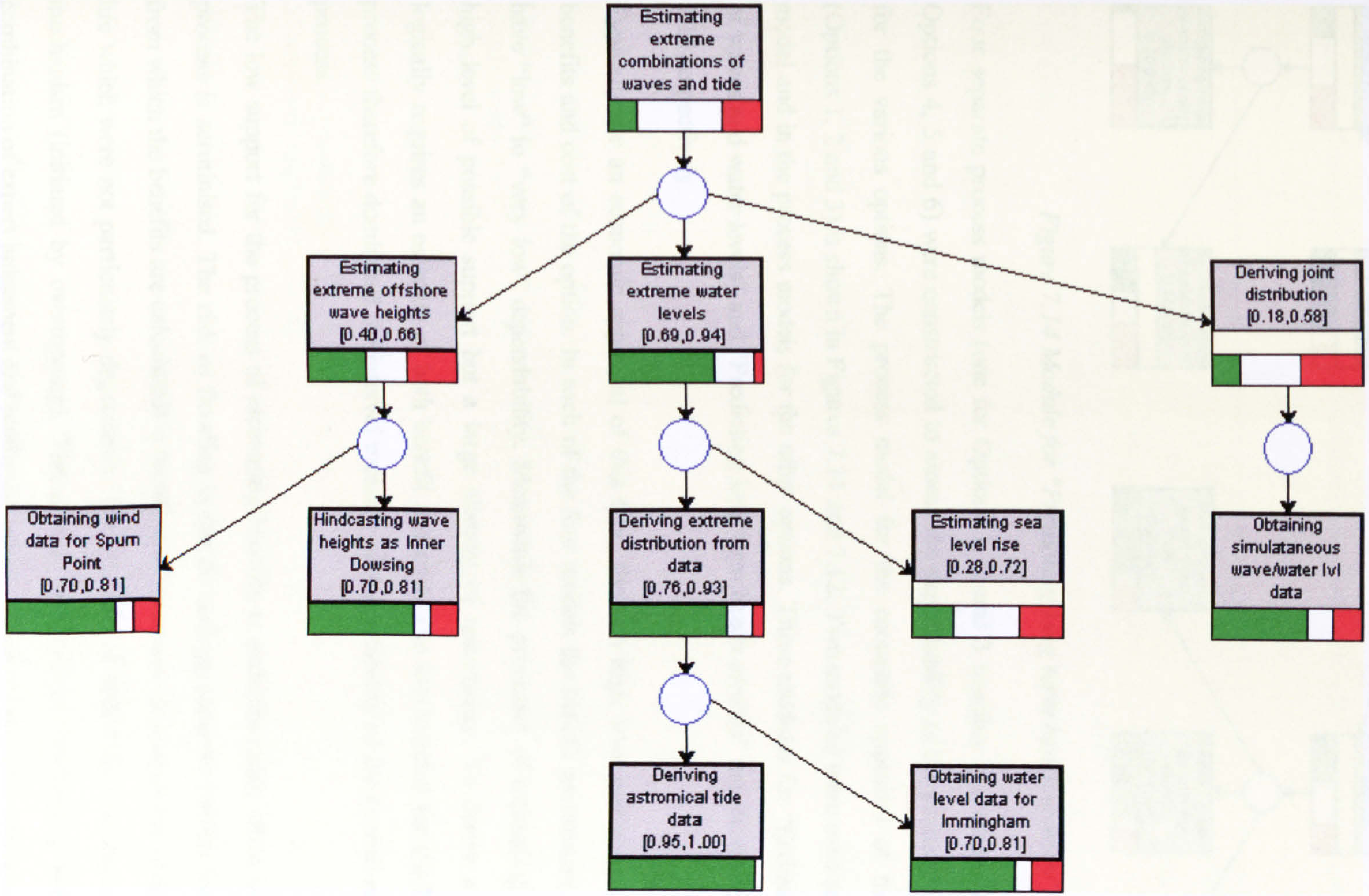

Figure 7.13 Module for "Estimating extreme combinations of waves and tides"

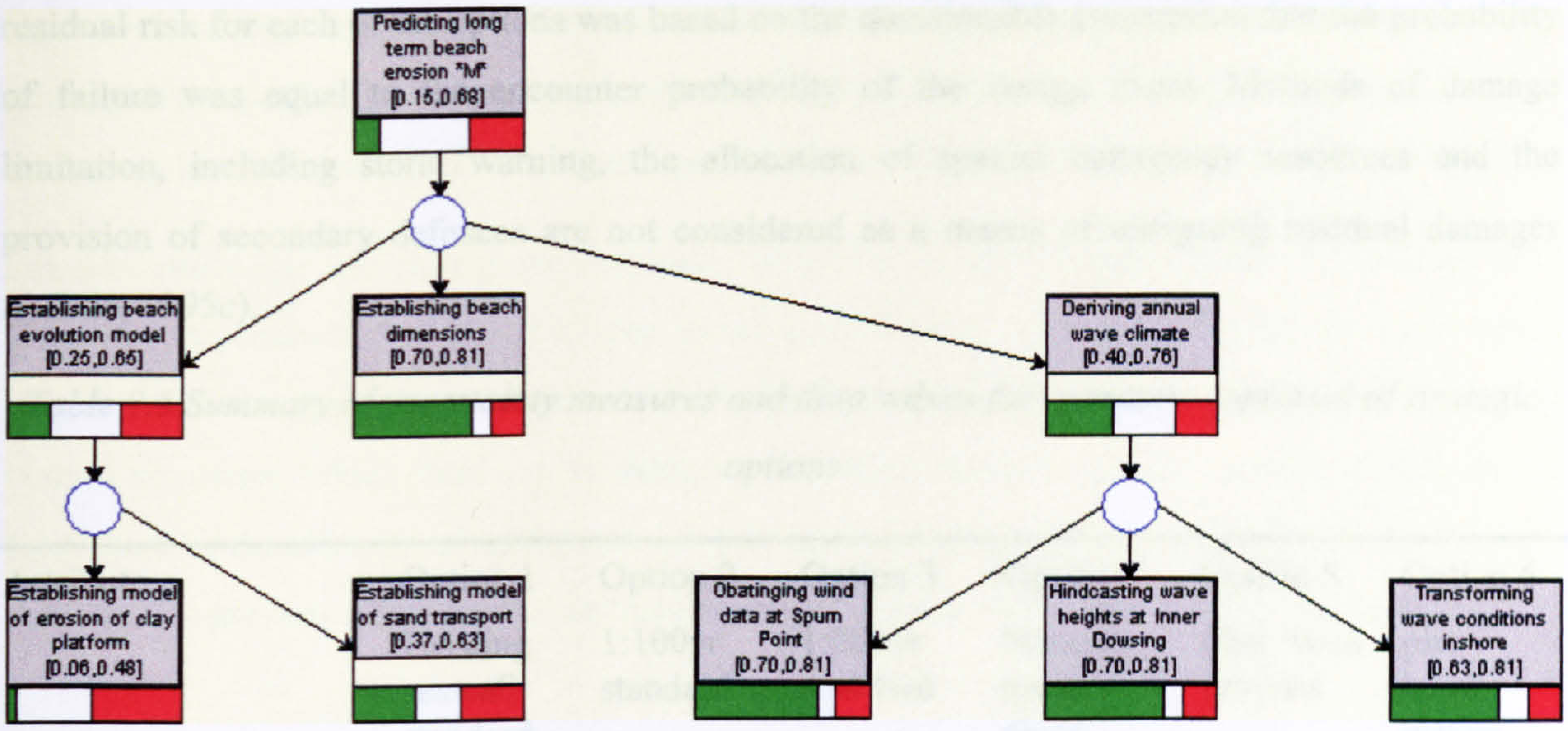

Figure 7.14 Module for "Predicting long term beach erosion"

Four separate process models (one for Options 1, 2 and 3 together and three separate models for Options 4, 5 and 6) were constructed to assess the dependability of the economic appraisal process for the various options. The process model for the economic appraisal of the seawall options (Options 1, 2 and 3) is shown in Figures 7.11 and 7.12. Two modules were used several times in this model and in the process models for the other options. These modules for "Estimating combinations of waves and water levels" and "Predicting long term beach erosion" are shown in Figures 7.13 and 7.14 respectively.

Typically for an economic appraisal of this type, the two high level processes are evaluating the benefits and cost of the option. In each of the four models the benefit assessment process proved to have "low" to "very low" dependability. Meanwhile the processes of estimating costs had quite a high level of possible support but a large element of uncertainty. To derive a benefit-cost ratio logically requires an estimate of both benefit and cost. The low support for the benefit assessment process therefore dominated the interval measure of dependability of the overall economic appraisal process.

The low support for the process of estimating benefits is understandable when the risk assessment process is scrutinised. The risk of flooding in the do nothing scenario (which forms the base case from which the benefits are calculated) is based on assessments of probability of failure and residual life which were not particularly dependable. The analysis of breaching considered only one failure mechanism (initiated by overtopping). The critical threshold of overtopping was set based on a combination of expert judgement and published values, which are not well substantiated by evidence. The analysis of residual life was based largely on expert judgement (Posford Duvivier, 1991). Breaching processes are extremely difficult to analyse, and the analysis that was undertaken represented typical practice at the time. Only recently has residual life assessment attracted more research attention (Sayers and Simm, 1997). Similarly the analysis of residual risk for each of the options was based on the questionable assumption that the probability of failure was equal to the encounter probability of the design event. Methods of damage limitation, including storm warning, the allocation of special emergency resources and the provision of secondary defences are not considered as a means of mitigating residual damages (Babtie, 1995*c*).

Table 7.5 Summary of uncertainty measures and data values for economic appraisal of strategic options

Attribute	Option 1 Existing seawall standard	Option 2 1:100yr standard	Option 3 1:200yr standard	Option 4 Nourish-ment alone	Option 5 plus rock groynes	Option 6 plus break-waters
Cost (£m)	77.7	95.4	104.4	80.3	91.6	107.8
Dependability of cost estimating process	[0.07, 0.73]	[0.07, 0.73]	[0.07, 0.73]	[0.04, 0.69]	[0.03, 0.67]	[0.02, 0.64]
Residual damages (£m)	253.3	64.2	55.0	57.6	106.3	133.6
Dependability of process of estimating residual damage	[0.01, 0.55]	[0.01, 0.55]	[0.01, 0.55]	[0.01, 0.49]	[0.00, 0.43]	[0.00, 0.43]
Amenity benefit (£m)	-	-	-	20.9	20.9	20.9
Dependability of process of estimating amenity benefit	-	-	-	[0.10, 0.73]	[0.85, 0.65]	[0.85, 0.65]
Total benefit (£m)	690.2	879.4	888.6	906.8	858.1	830.8
Dependability of benefit assessment process	[0.01, 0.53]	[0.01, 0.53]	[0.01, 0.53]	[0.00, 0.57]	[0.00, 0.55]	[0.00, 0.55]
Benefit-cost ratio	8.9	9.2	8.5	11.3	9.4	7.7
NPV (£m)	612.5	784.0	784.2	826.5	766.5	723.0
Overall dependability of economic appraisal process	[0.01, 0.58]	[0.01, 0.58]	[0.01, 0.58]	[0.01, 0.58]	[0.00, 0.56]	[0.00, 0.55]
Sensitivity (B/C) to:						
• Overtopping limits	7.3	9.0	8.4	11.0	8.9	7.3
• Residual lives	8.3	8.8	8.3	11.1	9.1	7.4
• Sea level rise	8.3	8.9	8.2	11.2	9.1	7.3
• Foreshore erosion	8.3	8.4	7.8	N/A	N/A	N/A
• Increased losses	N/A	N/A	N/A	9.1	7.9	7.1

The dependability of the cost estimates for beach nourishment options is lower than for the seawall options (Table 7.5). Beach nourishment on the scale planned for Lincshore is rare in the UK so there was, at the time, limited experience of costs. Commercial considerations (for example relating to the availability of dredgers and dredging licenses) also meant that estimating the cost of the beach nourishment scheme was less dependable than other more routine construction work.

For the breakwater and rock groyne options, the processes of estimating costs and residual risks were less dependable than for the open beach nourishment. This was because the open beach nourishment was the subject of more data collection and analysis than the options with beach control structures, which were merely designed in outline. Because of their relative novelty on the East Coast of the UK estimating the cost of rock breakwaters is more uncertain than estimating the cost of rock groynes which are rather more common and are on the whole more straightforward to construct.

The findings from the analysis of the economic appraisal process can be used in conjunction with the economic data that was output from the analysis, which is also summarised in Table 7.5. For each of the options the benefit-cost ratio is comfortably in excess of unity. The sensitivity analysis (no probabilistic analysis of variability was carried out at strategic decision stage) had indicated that the benefit-cost ratio was robust to plausible changes in key variables. However, the uncertainty analysis suggested that the appraisal process by which these data were generated was of "low" to "very low" dependability. At a strategic appraisal stage the effort involved in highly dependable assessments is not necessarily justifiable in circumstances where the economic case for investment is strong, as was the case for the Orplands project. In the case of Lincshore, the uncertainty analysis did provide evidence, which was not previously obvious, to suggest that whilst the economic case for constructing some kind of flood defence was strong, it may not have been sufficiently dependable to distinguish between the options.

There are, however, other environmental and flood defence reasons, which were not evaluated in economic terms, in favour of beach nourishment. Soft engineering techniques like beach nourishment have the important benefit of flexibility. They adapt and can be modified in response to changes in the coastal environment, for example climate change and long term geomorphological evolution. In an uncertain and dynamic environment this benefit of flexibility is a valuable characteristic (Collingridge, 1980). Harder solutions like seawalls are much less flexible and adaptable, fixing the coast on a particular alignment. If long term changes in morphology or climate are tending to change the coast then the presence of a fixed line will build up stress in the system (Klein, 1987). In the Post-Project Appraisal report on the Lincshore project (Babtie, 1996) it was commented that

> *The solution for nourishment alone* [Option 4] *is the one that offers the maximum opportunity for future flexibility, modification and enhancement. There is a lot still to learn about coastal processes in general and about this section of coastline in*

> *particular. A nourishment alone option does not preclude opportunities for: environmental enhancement in the future (perhaps in association with limited managed retreat); the introduction of groynes or headlands in response to the observed development of the new shoreline; the improvement of existing rigid defences should these be shown to be necessary and cost-effective in particular areas; the introduction of recycling as a beach management procedure with the aim of reducing ongoing maintenance costs.*

On the other hand beach nourishment schemes can be regarded as being rather more uncertain in their behaviour than more established hard solutions like seawalls, resulting in uncertainty in their expected cost and the standard of service they will provide. In the short to medium term this is probably true. Thus beach nourishment options can be considered to be "more risky" (Floyd, 1996) than hard engineering solutions, even if in the medium to long term they are more resilient and sustainable.

The Lincshore scheme has aspects of both hard and soft engineering. Beach nourishment is a soft solution, but was implemented to safeguard investment in a fixed seawall. The overall policy for the shoreline between Mablethorpe and Skegness is to hold the shoreline in its current position. The strip of development immediately behind the seawall means that, in the absence of a radical initiative and very strong political will, the shoreline has become fixed. Nonetheless, beach nourishment clearly has advantages of flexibility over the competing seawall options and to a lesser extent the nourishment options with groynes or breakwaters. At the time of the strategic decision the advantage of flexibility was mentioned but not analysed. The beach nourishment scheme has been accompanied by intensive monitoring. The renourishment is being planned in the light of that monitoring information. The following comments date from 1996 (Babtie, 1996):

> *The nourishment that has taken place so far appears to be performing better than anticipated...It is a point of general note that there are advantages in undertaking major replenishment schemes over an extended period such that maximum benefit may be obtained from site observations.*

Also mentioned but not evaluated for the strategic decision was the beneficial impact of beach nourishment on down-drift beaches when compared with seawalls or schemes with beach control structures which will tend to starve the down-drift beaches of sediment.

7.4.4 Summary of findings from the uncertainty model of the strategic decision to implement managed retreat

1. The process of economic appraisal of the various options for the Lincshore project was found to have dependability of [0.01, 0.58] for Options 1 to 4 (three seawall options and the open

beach nourishment option), whilst for Option 5 (beach nourishment with rock groynes) it was [0.00, 0.56] and for Option 6 (beach nourishment with offshore breakwaters) it was [0.00, 0.55]. These interval probabilities correspond in verbal terms to "low" to "very low" dependability. They demonstrate that, in terms of the dependability of the analysis and design process, there is very little to distinguish between the options. The options are more clearly distinguishable in terms of flood risk reduction and cost (see Table 7.5).

2. Whilst the economic case for constructing some kind of flood defence was strong, the process model indicated that the analysis of options may not have been sufficiently dependable to distinguish between the options.

3. The economic appraisal process was a function of two sub-processes of evaluating benefits and estimating costs. Of these the benefit assessment process contributed the most uncertainty to the economic appraisal process.

4. The principal source of uncertainty in the benefit assessment process was the evaluation of flood risk in the "do nothing" situation and the residual flood risk for each of the project options. This is the first area where additional analysis effort would have to have been directed in order to reduce uncertainty.

5. The uncertainty model of the economic appraisal process for options involving groynes or offshore breakwaters were of slightly lower dependability than the other options, on account of uncertainty both in the assessment of their performance and in estimating their costs. The model also indicated that the assessment of residual risks and costs for the beach nourishment options were less dependable than for seawall options. However, beyond these subtle differences the process model indicated that the analysis and design processes for the different options were of similar dependability.

7.5 Discussion of case studies 1 and 2

7.5.1 Compatibility of uncertainty modelling with the decision-making process

The first two case studies were conducted as retrospective analyses of projects, which had already been implemented. They were opportunities to reflect on how uncertainty modelling could be integrated with the decision-making process in a live project.

The process models were constructed from the documentary evidence relating to the project and discussions with the experts involved. Several passes were made through the documentation until it was felt that the model was a reasonable reflection of the processes which had lead up to the decision to implement managed retreat. In a live project the dynamics of model construction would have been rather different. Processes are enacted through time and so the evidence used in a decision gradually accumulates. At the outset of a live project there would be an overview of the processes necessary to undertake the project but few of the sub-processes would actually have been carried out so there would be very limited evidence relating to their dependability. In practice therefore the model would begin as a skeleton, with uncompleted processes being very uncertain. As the processes were completed, evidence relating to their dependability would come available and reduce the uncertainty in the model. A process model is a snap shot of the evidence at a given moment. To map the model onto the dynamics of the decision-making process requires the model owner to continue updating the model as evidence materialises.

Experience of applying similar uncertainty models in the oil industry (Davis and Hall, 1998) has demonstrated the benefits of a collective approach to model building. For example in situations where an asset team is engaged in uncertainty modelling, the model has become a catalyst for reflection on process dependability and the structure of decision-making problems. The model itself has 'faded into the background'. This collective aspect was not possible in the first two case studies, but it is worth highlighting as one of the most beneficial ways in which uncertainty modelling can support the decision-making process.

7.5.2 Proliferation of uncertainty

A frequent problem with uncertainty models, and especially large models, is that uncertainty is compounded throughout the model and the resulting measure is very uncertain and consequently rather difficult to interpret. The proliferation of uncertainty is a logical consequence of the numerical measures entered in the model by the user. IPT (which has been described in detail in Chapter 5) finds the least conservative bounds on the calculated support for the compound proposition. The calculus itself therefore introduces no spurious uncertainty into the model. It is argued therefore that the uncertainty calculated in the high level process is real, and a logical consequence of the judgements input into the model, whilst the intuition which says that the uncertainty in the top process should be less is biased. Human subjects, when asked to make judgements of uncertainty, have been shown under some circumstances to significantly underestimate the uncertainty (Tversky and Kahneman, 1974). Indeed the justification for using a numerical uncertainty model is that the model reduces the impact of the heuristics and biases which human subjects tend to adopt when making judgements under uncertainty and helps to force more logical, rational thinking. This is particularly important in complex situations where human cognitive facilities are stretched. Nonetheless, in Section 5.11 it was explained how in Support Logic Programming, evidence from different proof paths can be used reduce the uncertainty in inference situations. This is a facility that is not included in the current implementation of IPT, which could limit the proliferation of uncertainty.

A practical consideration is that when interval judgements are being made uncertainty should not be exaggerated or introduced indiscriminately. Some of the changes which were made during the second pass through the Orplands model were to reduce the uncertainty in interval measures where, upon reflection, they were considered to be unnecessarily conservative.

7.5.3 The meaning of support intervals

A fundamental issue in uncertainty modelling with numerical structures (*e.g.* interval probabilities) is how an individual interprets the numerical measures. In uncertainty modelling with numerical structures one is attempting to map a messy real world situation onto a precise numerical structure. The real world situation has to be structured in a suitable way to map onto that calculus. In other words the mathematical syntax is defined but the semantics of the real world situation has to be interpreted in terms of that syntax. The practical implications of this issue are often expressed in the question "but what does it all *mean*?"

One way of addressing this issue is to make sure that the real world situations one is attempting to attach numerical measures to are as clearly defined as possible. In this way ambiguity and confusion is avoided. At the same time it needs to be recognised that there can legitimately be different interpretations of the same situation, an issue that is addressed in more detail below.

The use of linguistic labels provides a natural expression of model inputs and results. Linguistic labels assisted in the elicitation of interval probabilities and in the communication of results. It would have been convenient to automate the mapping from linguistic labels to interval probabilities, so that the expert's judgement was articulated exclusively in linguistic terms. The evidence provided by the process model could also be mapped automatically onto linguistic labels in order to generate a script describing and explaining the evidence.

7.5.4 Judgement of structure

Process model construction essentially involves:

- identifying the processes, naming them and summarising their attributes;
- structuring the model by grouping related processes together and creating logical links between them (the strength of the relationship between processes is expressed in conditional probabilities and dependencies);
- making a judgement of the evidence relating to the dependability of the 'leaf' processes.

Experience with the first two case studies has demonstrated the following:

Identifying processes, naming them and summarising attributes

Identifying low-level processes, for example detailed investigation or analysis activities, is not particularly difficult. It is more difficult to group these low-level processes together and name the super-process to which they contribute. However, the set of top processes that was eventually identified did seem to capture the key items of evidence leading up to the decision.

The two studies have demonstrated the importance of identifying the objectives of the model and naming the top-level process accordingly. For the Orplands model the objective was to provide an overall measure of the dependability of the decision-making process whilst for the Lincshore model it was to address specific options and to scrutinise the dependability of the economic and environmental aspects of the appraisal process.

Judgements of evidence (expressed as interval numbers)

Making judgements of evidence for the 'leaf' processes did not present major difficulties. It was assisted by the use of mappings from a verbal scale. Because the set verbal measures was fewer than the total number of possibilities on a continuous scale of interval numbers it helped to avoid inconsistencies between judgements that were not substantiated by evidence.

To make a judgement of evidence does require a clear understanding of the criteria for success. In most circumstances it was straightforward to identify what were the criteria for a process to be dependable. For example, for the process "Analysing extreme water levels", the process would be considered to be dependable if the estimate of extreme water levels generated by the process formed a dependable basis for conducting a risk assessment and designing the flood defences. However, in other cases the criteria for success were less clear and perhaps multiple or conflicting. This was the case for the 'softer' processes such as the environmental assessment and especially the consultation exercise.

Judgements of structure

Judgements of structure, which are expressed both in the layout of the model and in the conditional probabilities and dependency measures, are difficult to make and can be hard to substantiate on the basis of evidence. They therefore often have a high level of uncertainty associated with them. For logically ordered activities, process model structure is straightforward to establish. For other messier activities like consultation processes, model construction is rather more delicate.

Assigning conditional probabilities to high level processes is particularly difficult. At lower levels the dimensionality of the problem and the nature of the transformation process provides revealing information about the relationship between processes. Some low-level activities are much easier to structure than others. Benefit-cost assessment has a logical framework so did not present difficulties in developing an appropriate structure, whilst environmental assessment is a less strictly structured process.

At high levels, when very different types of information are being weighed up, the relationship between the processes can be problematic to define. If it is considered that each of the top-level processes is logically necessary then, in the evidential reasoning framework adopted in this study, the uncertainty in the top-level process is high and the evidence against is compounded. Logical necessity is a very strong condition.

The problem of judging structure means that different individuals may construct a model in different ways. This may be due to a legitimate difference in perception of the process structures. On the other hand, differing judgements may merely be a consequence of confusion on the part of the model builders about the meaning of conditional measures.

The model was constructed by the researcher, using evidence from domain decision-makers and documents. However, if these methods are to be more widely used in practice the researcher will have to withdraw and leave model construction to practitioners. They will construct the model based on their own interpretations of IPT and problem structure, and attach their own meaning to the input values and the calculated answers.

The author constructed two process models of the strategic decision to implement beach nourishment for Lincshore. The construction of these models was based on the same evidence but the second model was constructed approximately a year after the first (in the meantime the software for uncertainty propagation was being developed). No reference was made to the first model while the structure of the second was being developed. Comparing the two models gave an opportunity to assess the consistency of judgements of uncertainty that can be achieved by one individual. The structures were generally very similar, though the earlier model tended to go to deeper levels of detail than the later model. This was driven by a quest for completeness during the earlier stages of research, whereas in the light of the experience of construction several process models a more appropriate level of detail emerged.

Hierarchical representation of fuzziness

Hierarchy was introduced as a fundamental systems concept in Chapter 3, and in Chapter 4 it was explained how hierarchy could be used to represent concepts at varying levels of precision. In Chapter 5 the idea of hierarchical representation of fuzziness was compared with the more formal mathematization of fuzziness. In the development of IPT, fuzziness has been addressed through adopting hierarchical structures, rather than by formally embracing the mathematics of fuzziness, though fuzzy sets and fuzzy logic have been an important influence in the development of IPT.

The implementation of IPT, which has been demonstrated in this chapter, is based on a hierarchical structure. High-level decision processes have been progressively decomposed into more and more precise analysis and design processes. Meanwhile, another, and potentially conflicting, influence on model structure has been logical analysis of the flow of information in the lead-up to a decision. Hierarchical representation of fuzziness and faithful representation of the flow of information need not be conflicting influences. In Section 4.6.1 a model of the flow of information in the lead-up to a decision was developed, which was consistent with hierarchical ordering of the granularity of processes. However, under some circumstances, analysis and design processes structured according to flow of information are all at a similar level of granularity, in which case the model does not represent features of fuzziness. This tension between hierarchical representation of fuzziness and faithful representation of the flow of information in the lead-up to a decision is reflected in the process models constructed for the case studies.

7.6 Case study 3: Analysis of the Lincshore beach renourishment strategy for the next five years

7.6.1 Introduction

In the years following the strategic decision to implement beach nourishment two major phases of nourishment were undertaken involving placement of approximately 7.5 million m^3 of dredged sand (Zwiers *et al.*, 1996). In 1998 the EA were in the process of developing a beach renourishment strategy for the following five years to make up for natural reduction in the volume of sand on the beach and preserve the defence provided by the beach. Developing the renourishment strategy involved optimising the options for volume, phasing, location and grain size of renourishment, and contractual arrangements for implementation.

The decisions associated with developing the future renourishment strategy are the subject of the final case study described in this thesis. The objective of this final case study was to demonstrate how uncertainty methods might have been employed during the development of the renourishment strategy for Lincshore. Although new risk-based modelling approaches are proposed, the main emphasis of this case study in on more general issues of uncertainty management and choice. Unlike the preceding case studies it is not based on descriptive analysis of the processes which led up to the decision. The proposed approach has been developed independently of the work of the EA and their consultants, who were nonetheless contacted in order to obtain up-to-date data on the project.

7.6.2 The decision problem

Constraints

Unlike the previous two case studies, the decision problem for the Lincshore renourishment was fairly constrained. The strategic option to adopt open beach nourishment, which was the subject of the previous case study, had already been taken in the early 1990s and had been endorsed in the Shoreline Management Plan (Environment Agency, 1996). Thus other strategic options, such as managed retreat, and indeed other technical options, such as beach nourishment with groynes or detached breakwaters, had been eliminated from consideration. The decisions relating to the renourishment operations were therefore fairly low in the hierarchy of decision-making for the Mablethorpe to Skegness coast, and were constrained by higher level and previous decisions.

Variables

The decision was one of identifying quantitative values of decision variables, rather than choosing between qualitatively different options on the basis of their attributes. The variables in the decision that had not been previously specified were the volume, location and phasing of beach renourishment operations. The renourishment sediment size was to some extent a variable, but at the same time constrained by availability. Although only one type of solution was to be considered, there was potentially a vast range of options to be considered within that one type of solution.

Objectives

The objectives of the decisions surrounding the renourishment strategy can be summarised as:

- economic efficiency (in terms of reduced economic consequences of flood risk, relative to scheme cost);
- reduced risk of loss of life, health and social impacts of flooding;
- environmental sustainability;
- preservation and enhancement of the amenity and recreational value of the coast.

The range of options for volume, phasing and location of renourishment was mostly distinguishable in terms of their cost and their impact on flood risk. The options were less distinguishable in terms of the impact on the environment and on amenity. Dredging operations will tend to have some impact on the environment, but the marginal impact on the environment of increasing or decreasing the volume of dredging operations will on the whole tend to be small. The different options for volumes and phasings of beach renourishment will therefore tend to be indistinguishable in environmental terms. Beach renourishment operations will disrupt beach amenity during construction, but afterwards will result in a wider beach for amenity purposes.

Perspectives

The decision relating to the Lincshore renourishment can be addressed from several of the perspectives introduced previously in this thesis. From a national point of view (which is the perspective adopted by MAFF) the objective is efficient risk reduction. MAFF's national perspective means that an application for funds for Lincshore needs to be weighed up against competing applications nationally.

The EA have responsibility for efficient implementation of the project on time and to budget. Efficient project implementation contributes to the national objective of risk reduction, but the emphasis of the operating authority is slightly different, and dwells upon the effective management of flood risk on a day to day basis. Timely implementation of the project is a key consideration, with full regard for the flood risk profile throughout the project period.

Local residents have the objective of reducing risk to their lives and property, but also have an interest in the amenity value of the coast. The strategic decision to implement beach nourishment had been demonstrated to be consistent in principle with both safety and amenity objectives of local stakeholders.

7.6.3 The process

The proposed process for developing and implementing the renourishment strategy is shown in Figure 7.15. The process includes the conventional steps of

- design of the renourishment programme,
- Grant Aid application,
- tender advertisement and assessment

and then a cyclic observational approach for the renourishment implementation. According to this observational approach the renourishment works will be specified in the light of the most recent information on beach conditions, and commercial and operational influences as they materialise during the five-year programme.

The key choices in the process are:

- approval of the Grant Aid application by MAFF,
- tender selection,
- continuous review of the implementation during the five year renourishment programme,

- strategic review after five years (not considered here).

The main uncertainty management strategies in the process are:

- *Risk-based optimisation of a wide range of strategies at the start of the project.* This differs from the practice adopted in the first phase of the Lincshore project when only a few different beach nourishment configurations under a handful of loading conditions were analysed. The optimisation model can be used in preparing the Grant Aid application, in assessing tenders and in subsequent detailed specification of the beach renourishment operations.

- *Analysis of the dependability of the renourishment design and optimisation process using a process model with uncertainty representation using IPT.* The closed world probabilistic optimisation is thereby extended into an open world. The process modelling is used to inform decision-makers about the sources and sensitivities to uncertainty.

- *Use of evidence from the process model and the risk-based optimisation to design the beach monitoring strategy.* Areas identified in the process model as being sources of uncertainty or in the optimisation model as being determinants of the failure mechanism are identified as areas for monitoring attention.

- *Providing a high degree of flexibility in the tender document, to enable prospective contractors to optimise sources of material and construction programming.* Contractors are given the opportunity to offer materials and construction sequences that they can most economically provide. It is recognised that the works actually implemented may be somewhat different from those envisaged at tender.

- *Tender assessment that uses a range of tendered strategies in the optimisation model.* Each contractor's bid is assessed by testing it in the optimisation model. An impression is thereby gained of the likely cost of the renourishment in a very wide range of possible conditions.

- *Tender assessment based on the theory of multi-attribute choice based on intervals developed in Section 6.9.* The tender is selected according to a range of desirable attributes in a transparent way, which nonetheless does justice to the uncertain nature of evidence about option attributes and client preferences.

- *An observational implementation strategy that responds to natural changes on the beach as they materialise over the five-year project.* An observational approach (see Section 6.5) is adopted so that the actual renourishment programme can be adjusted in response to feedback from monitoring. In this way the strategy is less vulnerable to the inevitable deficiencies in the predictive models used in the probabilistic optimisation. The strategy is flexible and responsive.

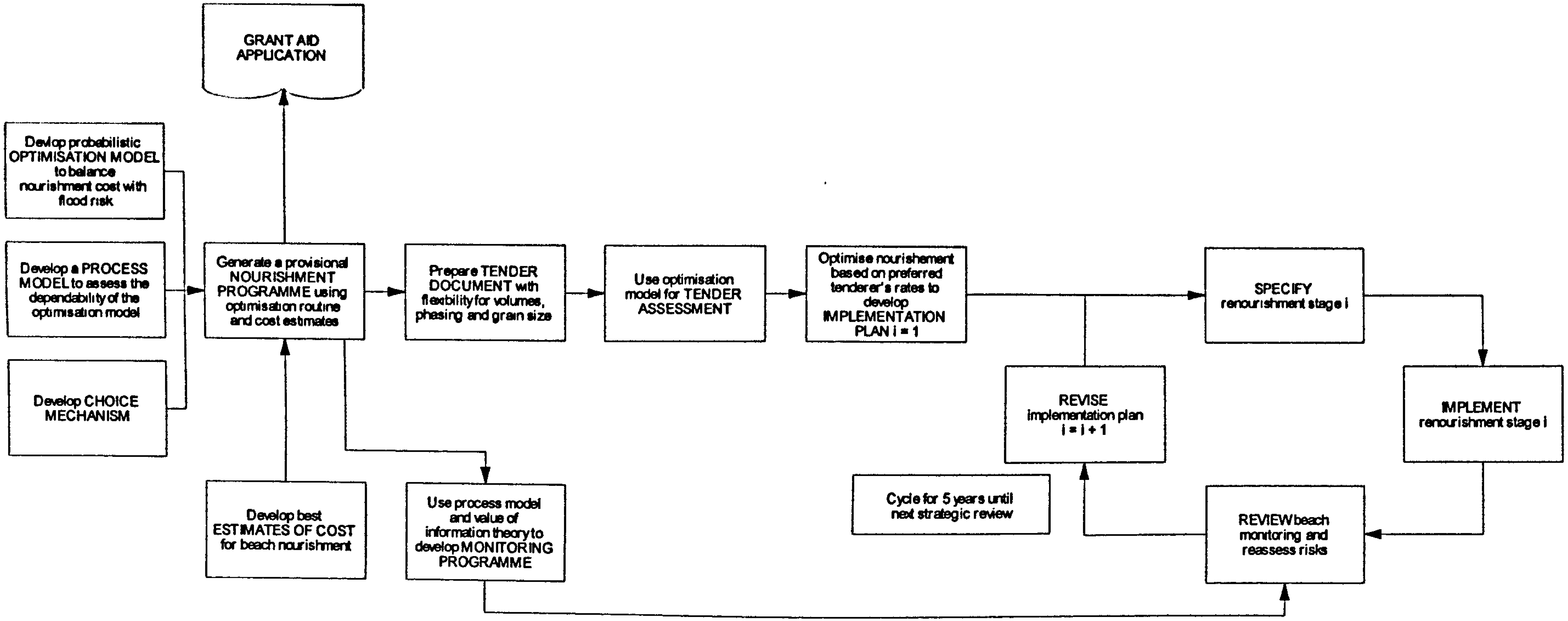

Figure 7.15 Outline model of the stages involved in developing and implementing the renourishment strategy

7.6.4 Risk-based optimisation

A risk based optimisation model to balance flood risk with the cost of beach renourishment is a key element of the renourishment strategy. It is used to

- develop a renourishment programme to be included in the Grant Aid application;
- compare the tenders for renourishment work;
- periodically re-evaluate the renourishment operations during the five year programme.

Note that this optimisation model is used jointly with a process model to assess its dependability, and in an observational mode so that project implementation is updated on the basis of observations of actual beach behaviour. Predictive models are therefore being used in the context of a wider treatment of uncertainty in order to address some of the potential pitfalls of risk-based methods identified in Appendix 1 and elsewhere by Hall (1999).

Risk-based optimisation is used to weigh up the cost of beach renourishment with the marginal reduction in flood risk. The optimum beach renourishment programme is identified by comparing the cost of a wide range of possible renourishment volumes and phasings with the benefit in terms of risk reduction relative to the 'do nothing' scenario.

In theoretical terms this is a more efficient approach than the approach adopted in Case Study 2 based on a target standard of protection. However, it depends on models of seawall failure and estimates of flood damage, which are rather uncertain. Care should be taken when adopting this type of approach, so that

- the proposed renourishment strategy is reasonably consistent with the sort of solution that would be obtained by previous deterministic methods, acknowledging the tacit knowledge embedded in these methods;
- the dependability of the models used is monitored using a process model (see Section 7.6.5);
- criteria for acceptable risk are used in addition to economic optimisation.

A number of risk-based optimisation strategies were tested by students under the author's supervision (Dawson and Flory, 1999). The optimisation was based on joint distributions of wave heights and water levels in the North Sea and involved:

- establishing a nearshore wave transformation model;
- developing a cross-shore beach profile and seawall failure model;
- developing a long-shore beach evolution model;

- establishing a simple inundation model of the hinterland;
- using data on potential flood damage.

The cost of each tested renourishment strategy was estimated by multiplying the renourishment volume by a unit rate for renourishment operations. The costing element of the optimisation can evolve as the project proceeds. When it is used in the Grant Aid application, estimated rates for renourishment (expressed as a point value or a distribution) based on the cost of previous operations on the Lincolnshire coast are adopted. When used in the tender assessment, each of the tender's rates are used to compare the tender and optimise the programme on the basis of the preferred tenderer's rates. Actual rates can be input into the model when it is used to optimise the observational strategy during implementation.

The number of possible configurations of the optimisation variables volume, location and phasing is enormous. Because the optimisation linked a number of different models together without being entirely automated it was not practical to address the problem with established mathematical optimisation tools. In stead a wide range of possible renourishment configurations was tested to identify the types of strategy that offered the greatest benefit, and then these were examined in more detail to locate an optimum.

It was demonstrated that the optimum renourishment configuration in a series system like the Mablethorpe to Skegness coastline occurs when renourishment is concentrated at beach cross-sections with the highest probability of failure. This does not necessarily correspond to the cross-section where beach erosion is at its most rapid, as probability of failure is governed by the seawall as well as the beach. It was demonstrated that the least renourishment volume for a given risk reduction can be achieved by phasing the renourishment programme fairly uniformly over the five years. This did not however account for any commercial considerations of plant availability, which might result in lower unit costs if the renourishment operation could be concentrated in one or two phases.

The results from the optimisation modelling are typically of the form shown in Figure 7.16. The horizontal axis is renourishment volume. The benefit-cost ratio (BCR) is calculated from the maximum benefit achievable in terms of risk reduction for any given volume. The BCR comfortably exceeds unity. Incremental increases in benefit with increasing volume decrease, because losses from very large renourishment operations will tend to be larger than losses from small. This evidence can then be used in the Grant Aid application (see Section 7.6.7).

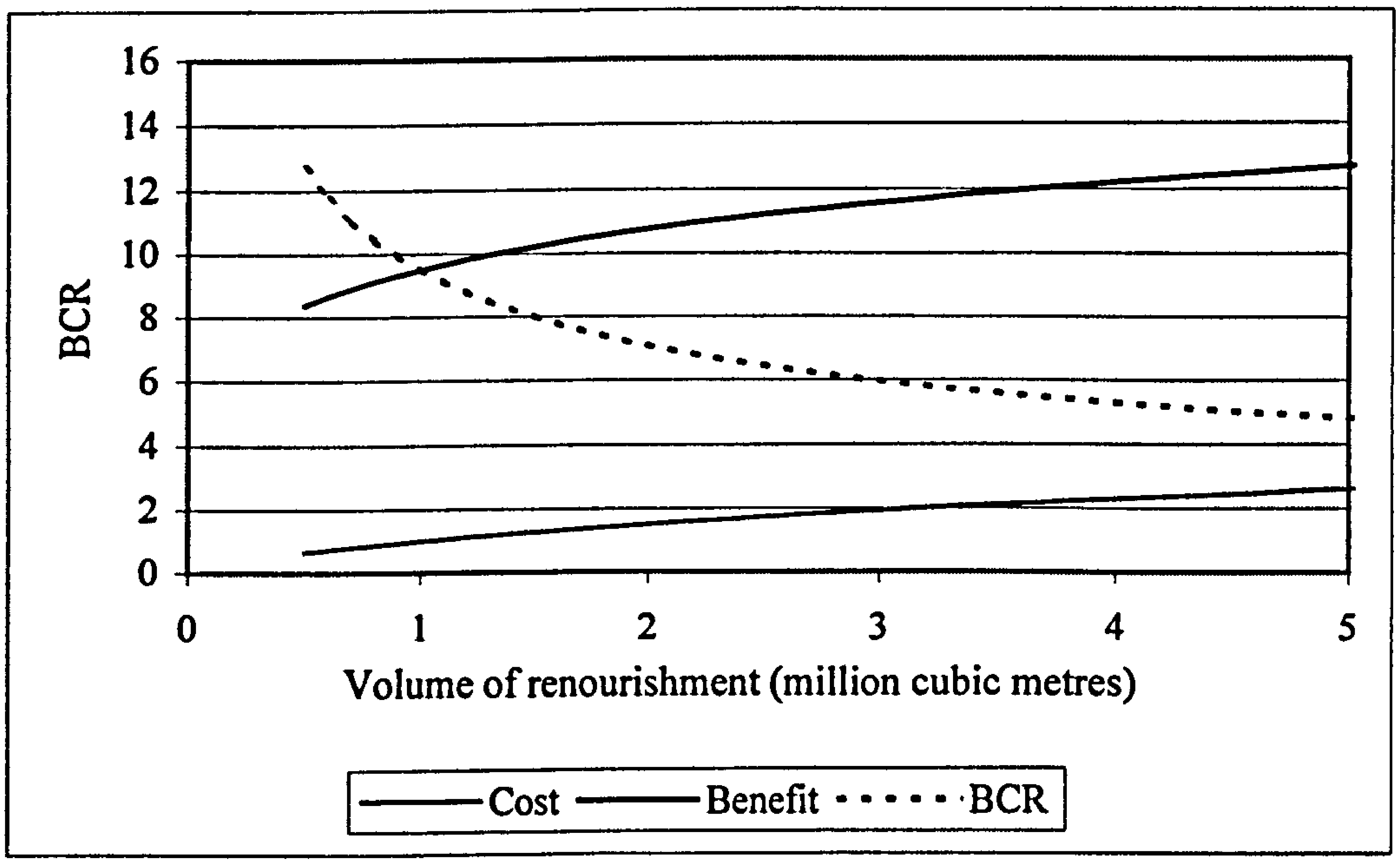

Figure 7.16 Typical results from economic optimisation model

The optimisation model was an early demonstration of how beach renourishment operations could be designed from a risk-based point of view. Further refinement could be achieved by including

- simulating different storm sequences, in order to analyse the effects of storm persistence;
- improving models of the breaching mechanism;
- simulating of observational response to severe storms (see Section 7.6.9);
- improving flood propagation modelling, and simulation of the effects of flood warning;
- in the benefit assessment including estimates of the economic impact of environmental damage, amenity benefits and disbenefits, health and loss of life, and the impact of renourishment on neighbouring beaches;
- in the costing including the impact of phasing on unit rates *e.g.* winter working.

7.6.5 Modelling process dependability

The process model was used to provide a commentary on the optimisation process in order to highlight sources of uncertainty to decision-makers and guide the design of monitoring activities. The process of estimating breach probabilities and flood risk is illustrated in Figure 7.17 and Figure 7.18. The risk modelling process is broadly similar to the one used for the strategic decision described in Case Study 2, but differs as follows:

1. The proposed modelling approach is fully probabilistic rather than being based on analysis of a handful of design conditions. Therefore, full joint distributions of wave heights and water levels are used to calculate flood risk.

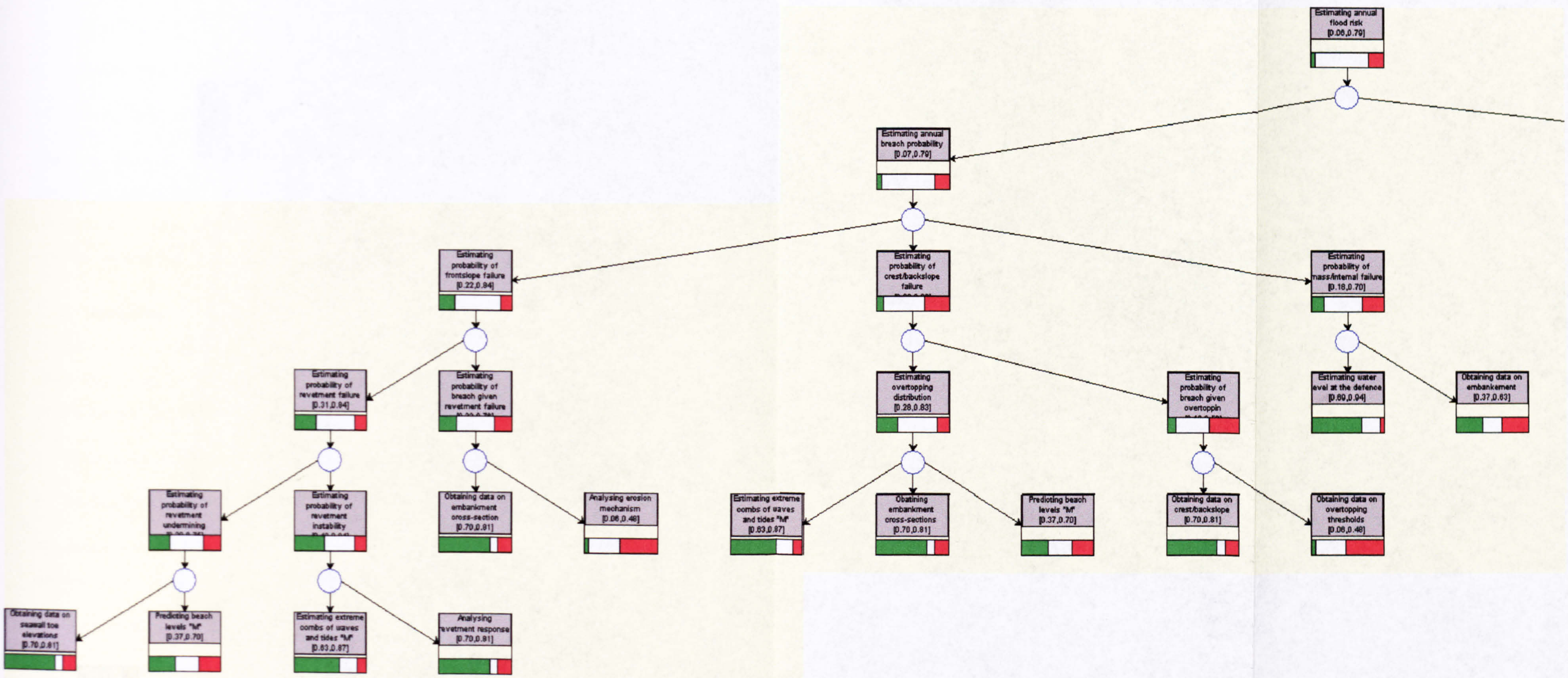

Figure 7.17 Process model for probabilistic modelling of flood risk (left hand side)

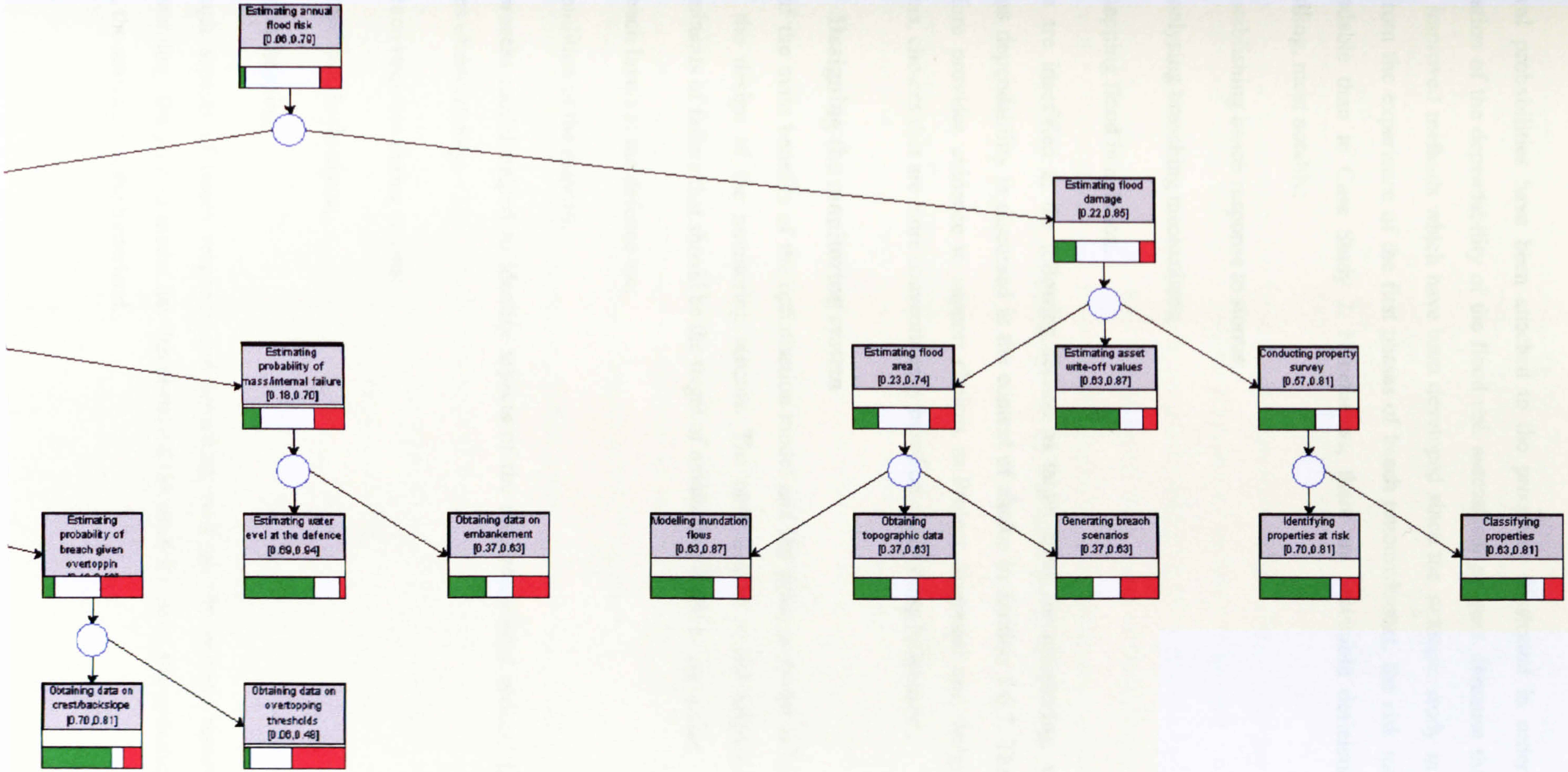

Figure 7.18 Process model for probabilistic modelling of flood risk (right hand side)

2. A number of seawall breaching mechanisms are addressed.

Interval probabilities have been attached to the processes illustrated in order to generate an indication of the dependability of the flood risk assessment process. Because the model benefits from improved methods which have been developed since the strategic study in the early 1990s and from the experience of the first phases of beach renourishment, the risk modelling is more dependable than in Case Study 2. Nonetheless, there are inevitable deficiencies in the risk modelling, most notably:

- establishing beach response to storms;
- analysing breaching mechanisms;
- mapping flood inundation.

These are identified in the following section as target areas for monitoring, and the issue of process dependability is discussed in the context of choice in Section 7.6.7. The process model therefore provides evidence to support choices in Project Appraisal and design of monitoring systems, choices that are more conventionally based on engineering judgement.

7.6.6 Designing the monitoring system

One of the main benefits of the optimisation model and the process model is to inform choices about the design of the monitoring scheme. The optimisation model helped to identify key determinants of failure that should be the target of monitoring work, in particular:

- beach levels at the defence toe;
- condition of the seawall.

The process model helped to identify aspects of the risk assessment process that are the key sources of uncertainty:

- beach response during storms;
- breaching mechanisms;
- flood mapping.

Although aspects of beach response and breaching well require generic research to improve understanding, the process model has demonstrated the need for more site-specific analysis of the beach, the seawall and the hinterland.

7.6.7 Grant Aid application

The choice mechanism to be used during the Project Appraisal in the Grant Aid application is constrained by MAFF's (1993) Project Appraisal Guidance Notes. The existence of this high level guidance on choice helps to achieve some of the meta-level decision-making objectives identified in the discussion of the contingency approach to choice in Chapter 6. In particular, transparency, repeatability and optimality, and, to a lesser extent, criteria of understandability, legitimation and ease of application are achieved through use of MAFF's guidance.

The decision rule set out in MAFF's Project Appraisal Guidance Notes is based on maximisation of benefit-cost ratio, subject to satisficing criteria of Indicative Standard of protection. The Indicative Standard is intended to ensure some uniformity of defence standard (defined in terms of design load) for sites with similar land use. However, the meaning of the Indicative Standard in the context of a risk-based approach is not clear because it is expressed in terms of the return period of the design event, which the optimisation model has demonstrated does not correspond to the annual probability of failure.

The optimisation model demonstrated that the BCR of the renourishment operations comfortably exceed unity even at very large renourishment volumes. The decision rule provides provision for moving to a higher standard of protection than is achieved when maximising BCR, provided the incremental BCR is greater than unity. Figure 7.16 demonstrates how progressive addition of beach material results in a declining incremental BCR. However, the actual value of the incremental BCR depends on the size of the increment. The meaning of incremental BCR is therefore rather difficult to interpret in situations where decision variables are continuous or near continuous. This is an issue that is not addressed in MAFF's guidance.

The approach proposed here also uses a satisficing criterion for acceptable risk to life. Given the density of housing immediately behind the seawalls at sites on the project frontage it seems likely that in the event of a sudden breach in the seawall there will be fatalities. Some evidence for this hypothesis is provided by experiences in 1953, when 40 people lost their lives on the Lincolnshire coast. These days flood warning and evacuation procedures can be expected to reduce that death toll, but for residents immediately behind the seawall there may be little warning in the event of a sudden failure. Detailed analysis of risk to life on the Lincolnshire coast has not been conducted, so at this stage it is not possible to set an appropriate satisficing threshold. Moreover, there is variability in the criteria for acceptable risk used across government (Health and Safety Executive, 1989, Royal Society, 1992, Parliamentary Office of Science and Technology, 1996, Vrijling, 1996, Interdepartmental Liaison Group on Risk Assessment, 1996). The intention here is to demonstrate how such a criterion could be used in the choice mechanism for Lincshore.

In summary therefore there are three elements to the choice at Project Appraisal stage:

1. economic evidence from the risk-based optimisation;

2. evidence indicating the extent to which the range of options satisfy a criterion for acceptable risk to life;

3. evidence from the process model about the dependability of the various models employed.

Since the nominal BCR is comfortably in excess of unity. The preferred renourishment programme should be the one which maximises BCR whilst satisfying the threshold for acceptable risk. However, the estimate of BCR and the bounds for acceptable risk are of imperfect dependability. To explore the impact of uncertain dependability involves interactive use of the optimisation model and the process model. By making plausible changes to the optimisation model, which the domain expert judges is consistent with the evidence from the process model, it is possible to explore the robustness of the choice. Although the process model indicates rather uncertain process dependability, which could possibly be rather high, it is not sufficient to suggest that the BCR will not exceed unity. However, the evidence from the process model indicates that the optimisation model is not sufficiently dependable to identify a unique optimum. Rather, it is appropriate to carry forward in the Grant Aid application and tender document a cluster of possible renourishment options with performance close to the acceptable risk criterion. These options are however, broadly similar, and all satisfy the criteria set down by MAFF. As more information becomes available and as the scheme materialises the options are narrowed, converging towards the renourishment programme that is actually implemented.

7.6.8 Tender process

The meta-level objectives (see Section 6.3.2) for the tender process in the public sector are dominated by issues of transparency, repeatability and optimality. It is necessary to identify a unique preferred contractor at the end of the choice. Issues of understandability and legitimation are less important.

Commercial issues associated with tendering and letting the contract for dredging and placement of beach material are a major influence on the development of a beach renourishment programme. The philosophy in the proposed approach is to give the contractor the flexibility to propose their preferred renourishment material phasing and volume. In this way individual contractors can make the most of materials that are readily available at a competitive price and adopt an efficient method of working so as to offer a competitive price to the client. Nonetheless, different configurations of renourishment volume, location, phasing and sediment size have different implications in terms of risk reduction and other impacts. The configurations offered by tendering contractors therefore have to be evaluated in order to identify the strategy that is expected to deliver best value. The tender assessment will involve taking the prices submitted by each of the tenders and rerunning the optimisation on the basis of those prices. The tender whose prices generate the lowest expected cost strategy should be identified.

At the same time as offering flexibility to the contractor the EA should retain flexibility in the year-on-year programme in order to respond to events as they materialise on the beach. In this way the EA reduces vulnerability to inaccuracies in the predicted beach behaviour.

Increasing emphasis is being placed on selecting contractors by an overall assessment of the value they bring to the project, rather than merely one criterion of least cost (Jackson-Robbins, 1998). Tender selection therefore becomes a problem of choice against a range of criteria. For the Lincshore project, key criteria in the selection of a contractor will be

- estimated cost;
- variability in cost for a range of weather conditions;
- suitability of the contractor for observational mode of working (change management);
- safety systems and record;
- environmental systems and record.

The performance of the tenderers against each of these criteria has uncertainty associated with it. For the cost-related aspects the uncertainty will be expressed in probabilistic terms, whereas the evidence of performance against the other criteria will tend to be vague and incomplete. The approach to choice based on intervals developed in Chapter 6 has therefore been adopted.

Suppose that four contractors bid for the renourishment work, and that their bid rates have been entered into the optimisation model to obtain the probability distributions of cost shown in Figure 7.19. The cost range of each options (using the two standard deviation bounds as an estimator) has been normalised onto a [0,1] scale to give the cost attributes entered in Figure 7.20. Evidence of the contractor's performance against the other criteria have also been mapped onto a interval scale in Figure 7.20 using the mapping from verbal measures illustrated in Figure 7.8.

The dependencies between each of the attributes are listed below. Safety management and environmental management are often part of the same function in contracting companies so tend to be strongly dependent. Efficient contractors are more adept at coping with change, which is an essential aspect of observational methods. The relationship of cost with the other criteria is rather uncertain. There is evidence to suggest that good change management, safety and environmental practices save cost. However, at tender stage there is a possibility that inexperienced contractors may underestimate the cost of efficiently implementing an observational strategy and working safely on the coast without harming the environment, in which case the dependency between cost and these other attributes will be low.

$Dep(A_1A_2) \in [-0.3, 0.7]$ $Dep(A_1A_4) \in [-0.3, 0.7]$ $Dep(A_2A_4) \in [0.0, 0.5]$

$Dep(A_1A_3) \in [-0.3, 0.7]$ $Dep(A_2A_3) \in [0.0, 0.5]$ $Dep(A_3A_4) \in [0.3, 1.0]$

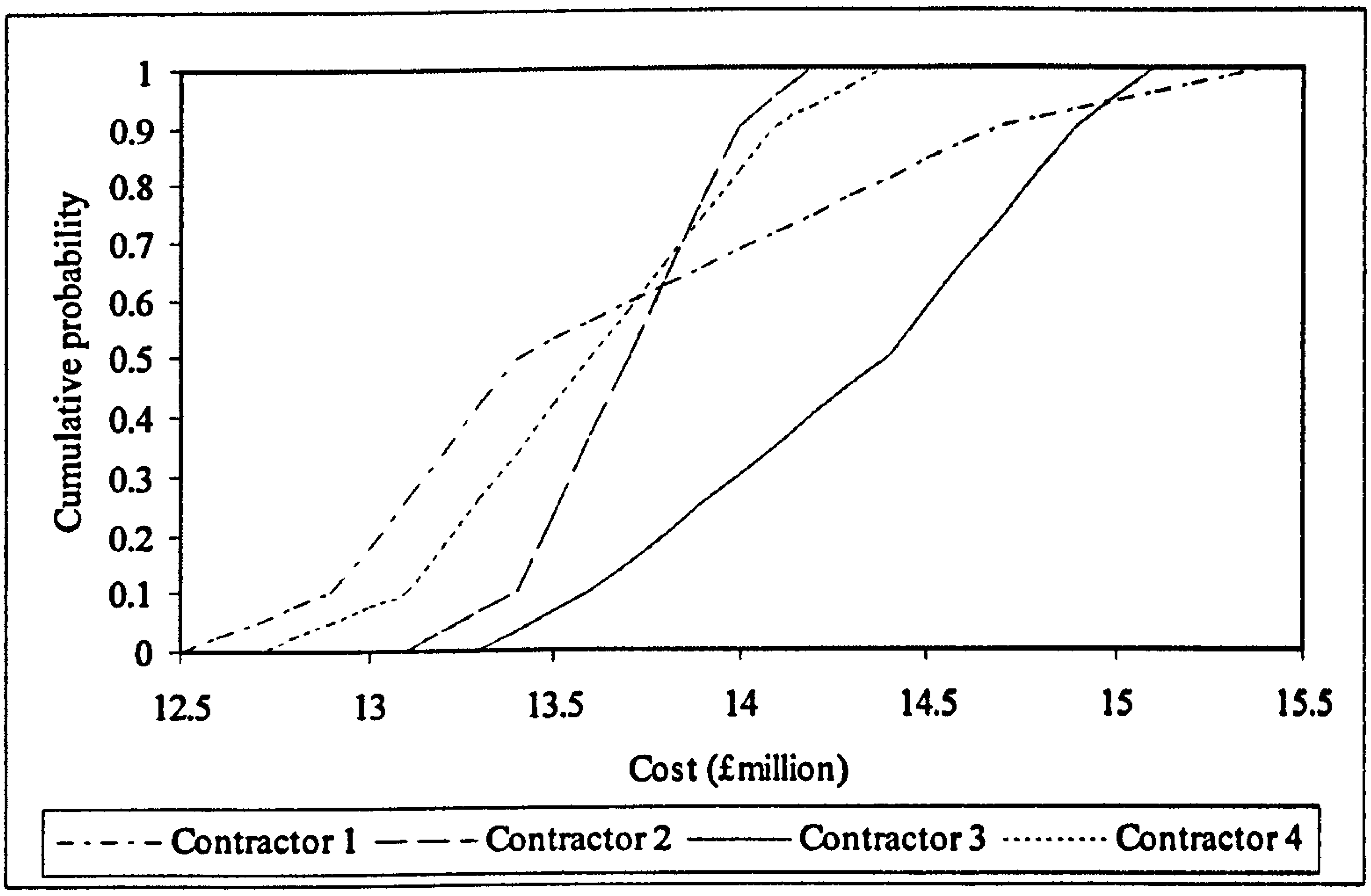

Figure 7.19 Cost distribution for 4 tendering contractors

The conditional probabilities reflect the relative importance of each of the attributes, and are based on mapping from verbal measures, as shown in Figure 7.8. Options that have low cost carry high weight. However, options that do not have low cost are not necessarily unsuitable, provided they are valued in other respects. Without capability at observational working the contractor will not be particularly suitable, and this is reflected in the conditional probabilities attached to sub-sets including $\overline{A_2}$. The same is true of the safety and environmental criteria, and corresponding sub-sets containing $\overline{A_3}$ and $\overline{A_4}$.

$p(H \mid A_1 \cap A_2 \cap A_3 \cap A_4) \in [1.0, 1.0]$

$p(H \mid A_1 \cap A_2 \cap A_3 \cap \overline{A_4}) \in [0.70, 0.81]$

$p(H \mid A_1 \cap A_2 \cap \overline{A_3} \cap A_4) \in [0.70, 0.81]$

$p(H \mid A_1 \cap A_2 \cap \overline{A_3} \cap \overline{A_4}) \in [0.37, 0.63]$

$p(H \mid A_1 \cap \overline{A_2} \cap A_3 \cap A_4) \in [0.44, 0.56]$

$p(H \mid A_1 \cap \overline{A_2} \cap A_3 \cap \overline{A_4}) \in [0.37, 0.63]$

$p(H \mid A_1 \cap \overline{A_2} \cap \overline{A_3} \cap A_4) \in [0.37, 0.63]$

$p(H \mid A_1 \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_4}) \in [0.0, 0.23]$

$p(H \mid \overline{A_1} \cap A_2 \cap A_3 \cap A_4) \in [0.76, 1.0]$

$p(H \mid \overline{A_1} \cap A_2 \cap A_3 \cap \overline{A_4}) \in [0.37, 0.63]$

$p(H \mid \overline{A_1} \cap A_2 \cap \overline{A_3} \cap A_4) \in [0.37, 0.63]$

$p(H \mid \overline{A_1} \cap A_2 \cap \overline{A_3} \cap \overline{A_4}) \in [0.06, 0.48]$

$p(H \mid \overline{A_1} \cap \overline{A_2} \cap A_3 \cap A_4) \in [0.13, 0.37]$

$p(H \mid \overline{A_1} \cap \overline{A_2} \cap A_3 \cap \overline{A_4}) \in [0.06, 0.48]$

$p(H \mid \overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap A_4) \in [0.06, 0.48]$

$p(H \mid \overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_4}) \in [0.0, 0.0]$

The results of the analysis are shown in the super-process box in Figure 7.20, which on this occasion indicates that the interval support for Contractor 2 dominates the other contractors so is the preferred tenderer. Of course it will not always be the case that both the lower and upper

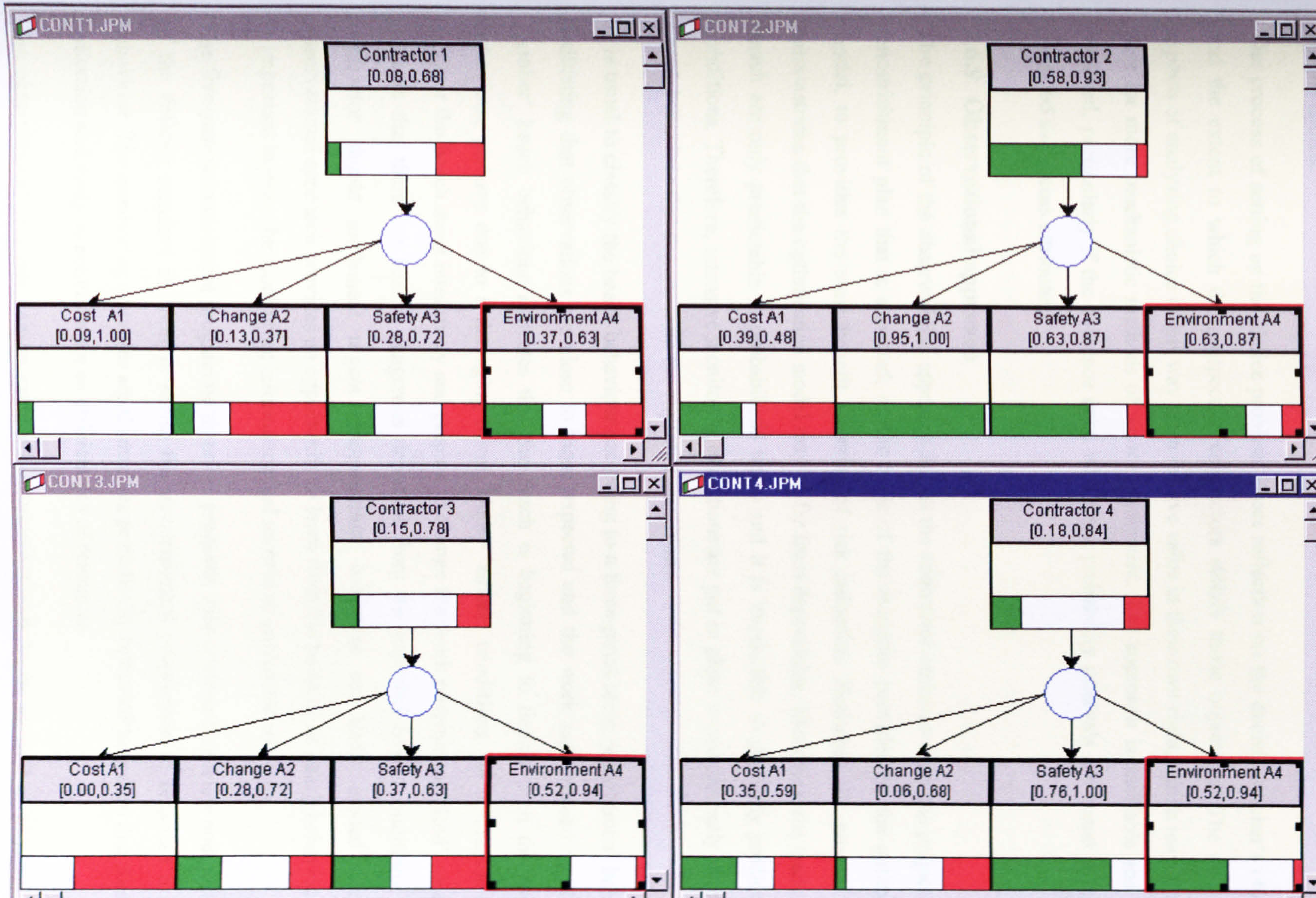

Figure 7.20 Multi-attribute tender assessment

bounds will dominate, in which case the choice is less straightforward. Strategies for choice based on intervals were discussed in Section 6.8. Under some circumstances it will be necessary to gather more information or enter into further negotiations with a contractor in order to reach a unique choice.

The process of setting up the choice problem forces reflection on the decision-maker's objectives and the extent to which the prospective contractors satisfy those objectives. The deliberative aspects of analysing choice in this way seem to have value in their own right, which may not be the case for more mechanistic methods of tender assessment. The approach is auditable and can be revisited, particularly if the evidence upon which the probability intervals are based is carefully recorded as process attributes.

7.6.9 Observational approach

The principle of the observational approach is that the contractor embarks upon the project with a renourishment plan that is expected, on the basis of the evidence provided by the optimisation model, to provides the most benefit in terms of risk reduction. However, the process model demonstrates that the optimisation model may be far from dependable. Moreover, the loads on the beach are only predictable in probabilistic terms and it is impossible to precisely predict storm conditions. Therefore, intensive monitoring operations are put in place to provide early warning of beach behaviour that departs from the expected conditions.

It is usual to classify the beach behaviour according to a three point scale, with 'green' behaviour indicating that observations are close to those expected and the work can proceed as planned. 'Amber' beach behaviour indicates that the beach is beginning to depart from the expected behaviour, perhaps due to a damaging storm. Under 'amber' conditions the contractor should monitor the beach more intensively and prepare to change the work programme. 'Red' conditions indicate that there has been a dangerous departure from the expected beach conditions so the contractor should implement urgent renourishment works at the badly eroded site. The observational approach provides an opportunity to learn from the patterns of beach behaviour, so it is important to store the monitoring information and records of contractor response.

The five-year renourishment programme therefore proceeds with continuous review and evaluation of the defence standard in order to revise the renourishment programme in response to beach behaviour. The monitoring activities are themselves periodically reviewed to ensure that these data collection and analysis activities are an efficient use of resources.

The observational principle can be extended to organisational issues as well as physical beach response. Evidence from the Heathrow Express tunnel collapse, which was mentioned in Chapter 1, demonstrates the importance of having effective organisational systems in order to cope with observational models of operation. It is important to monitor organisational performance to ensure that the systems are functioning effectively. Key performance indicators could, for example, relate to the delay between a 'red' observation and implementation of remedial works.

7.6.10 Discussion

This case study has addressed the assembly of evidence and the actual choices at Project Appraisal and tender assessment, as well as issues of coastal monitoring and observational implementation. The decisions surrounding the Lincshore renourishment operations were fairly constrained, and this has influenced the analysis of uncertainty and the choice mechanisms used.

The main criteria distinguishing between options at Project Appraisal stage were risk and cost. The problem was essentially one of single attribute choice, based on the neutral attitude to risk that is prescribed by government. At Project Appraisal stage the choice mechanism is to a great extent prescribed by MAFF. It has been demonstrated how interactive use of the process model and optimisation model indicate that the beach renourishment could be expected to have BCR in excess of unity for all plausible levels of model dependability. However, there was not sufficient evidence to justify a single optimum solution, so a range of solutions should be carried forward to Project Appraisal and tender stage.

The use of Indicative Standards of protection has been shown to be problematic in a risk-based analysis. The meaning of an incremental benefit-cost ratio, which also features in the MAFF (1993) decision rule, has also been shown not be applicable in to choice problems based on continuous variables.

The tender assessment was a problem of multi-attribute choice, recognising that best value is not achieved by merely selecting the lowest tender. It has been demonstrated how multi-attribute choice based on intervals can be applied in practice. In this case study the interval support for one option dominated the other intervals so it was possible to identify a unique preferred option. This will not generally be the case so, as discussed in Section 6.8, it will be necessary to adopt a further rule for choice based on intervals in order to identify a unique preference. Nonetheless the approach has been demonstrated to be a useful way of expressing some of the uncertain evidence used in tender assessment.

The process modelling and the optimisation modelling has informed the design of the monitoring programme. The implementation should be conducted in a learning mode, with the optimisation model and process model being up-dated in the light of research developments and site-specific monitoring.

The meta-level objectives of the decision-maker have guided the selection of the choice mechanism for the Project Appraisal and tender assessment. Moreover, the contingency approach to choice (Chapter 6) proposes that in a given decision situation the choice mechanism should

- make the most of available information;
- use information in a form which is natural and comprehensible to the decision-maker;
- avoid distorting the information by changes of format;
- recognise that information exists in an open world.

At Project Appraisal stage most of the information was in a probabilistic format, which was reduced ultimately to a single value of BCR. This represents a loss of information, but the use of the process model helped to proved a commentary on the quality of that information and so redress some of the problems of information loss. During the tender assessment the distribution of cost was mapped onto an interval scale, which represents a change of information format. Interval probabilities were an appropriate format for most of the attributes, based as they were on open world expert judgements.

The proposed renourishment strategy combines the following benefits:

1. The observational beach renourishment strategy is flexible and responsive so embodies the key characteristics of a resilient coastline. Moreover, the observational approach reduces vulnerability to inaccuracies in the predictive models employed in the optimisation.
2. A comprehensive simulation approach to optimisation means that the economic appraisal is a much better reflection of the likely costs and benefits of the strategy than deterministic approaches that are customary in current practice.
3. Uncertainty is handled with appropriate tools. Parameter uncertainty is handled probabilistically whilst IPT is used to represent the dependability of the modelling process.
4. The process modelling provides an overview of the process. It indicates where data collection activities should be directed and where contingencies should be developed.
5. Different choice mechanisms have been adopted at Project Appraisal and tender assessment stages to reflect the nature of the available information and the meta-level objectives of the decision-makers.

7.7 Conclusions

1. The decision to implement managed retreat at Orplands in Essex and the decision to implement beach nourishment on the Lincolnshire coast between Mablethorpe and Skegness have been

retrospectively analysed to test the uncertainty modelling techniques developed in this thesis. Using documented evidence and the testimony of the experts involved, process models have been constructed to identify the sources of uncertainty and the sensitivity of the decision process to the dependability of the various activities leading up to the decision.

2. The experts involved in the decisions have confirmed that the process models were a good representation of the processes that actually took place in the lead-up to the decisions. The process models provided new information about the decision-making process. The uncertainty modelling approach provided a method of assembling evidence from documentary sources and externalising expert knowledge, forming a basis for consensus building and informed decision-making.

3. For logically ordered activities process model structure is straightforward to establish. For other messier activities like consultation processes, model construction is rather more delicate. Large complex process models tend to generate uncertain high-level interval probabilities. This logical consequence of the intervals input into the model can making high level interval measures difficult to useful interpret. When structuring a process model there is a tension between hierarchical ordering of concepts according to granularity of information and logical analysis of the flow of information in the lead-up to a decision.

4. Uncertainty management techniques have been demonstrated in the five-year strategy for renourishment of the beaches between Mablethorpe and Skegness. It has been demonstrated how uncertainty modelling with IPT can be combined with probabilistic analysis and an observational approach to express and take account of the different types of uncertainty in a complex infrastructure management process. This approach is economically efficient and responsive to natural coastal process. Incompleteness in the models upon which optimisation decisions are based is represented by using IPT. Choice mechanisms at Project Appraisal and tender assessment stage have been scrutinised.

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CHAPTER 8

Conclusions

8.1 Objectives of Chapter 8

- to demonstrate how the objectives set for this research in Chapter 1 have been achieved;
- to summarise the principal research contributions;
- to review the potential of the proposed uncertainty management tools and techniques;
- to indicate directions for further research.

8.2 Research objectives revisited

The research described in this thesis has involved

- empirical analysis of current decision making practice and sources of uncertainty;
- development of concepts of uncertainty and dependability;
- theoretical work on uncertainty representation;
- implementation of the theoretical developments in process modelling software;
- demonstration of the techniques on coastal defence projects.

Each of the objectives set for the research in Chapter 1 will be reviewed, before more general reflection on the value and implications of the research findings.

8.2.1 Objective 1: *to provide a critical analysis of the sources of uncertainty for coastal engineers and managers in the UK and review current methods for coping with uncertainty in decision-making*

Data on uncertainty and decision-making in coastal engineering were obtained from:

- Grounded Theory analysis of semi-structured interviews with coastal engineering practitioners;
- case studies of coastal engineering projects;
- published literature.

The Grounded Theory analysis in Chapter 2 demonstrated that sources of uncertainty in decision-making are diverse but can be categorised in essence as being modelling issues, values issues, communication, and environmental constraints.

The interview data, case studies and literature review demonstrated that intuitive and implicit methods of taking uncertainty into account in decision-making are currently much more prevalent than explicit methods. Issues of model dependability are not always addressed and there is no consistent format for communicating information about dependability.

8.2.2 Objective 2: *to define uncertainty and to characterise the sources of uncertainty in the evidence assembled in the lead-up to a coastal engineering decision*

Uncertainty has been defined in Chapter 4 as being a function of both information content and dependability of evidence. In order to make good decisions engineers ideally require evidence with high information content that is also highly dependable. There are therefore two sides to uncertainty in evidence, both of which need to be addressed to gain an impression of uncertainty.

1. <u>The information content of the available evidence</u>. A single item of information may be uncertain in an ambiguous (possibilistic and/or probabilistic) or vague/fuzzy sense. Consideration of a body of evidence may reveal conflict.

2. <u>The dependability of the available evidence</u>. The dependability of the evidence is a reflection of the process by which it was obtained. The evidence will originate from some model, where the term 'model' is used in the most general sense. A dependable model must be well tested and the test results must indicate that, in the context of the decision, the theory is not refuted. Truth is a sufficient but not necessary condition for dependability.

Interview data and analysis of case studies were combined with review of the literature of decision-making to develop understanding of how information is used in the decision-making process. Decision-making can be idealised as a cyclic process, which comprises problem definition; generating and analysing options; choice of a preferred option; implementation and monitoring. In practice it often involves looping between sub-processes which proceed in parallel until the moment of choice. The dependability of a decision is a function of the dependability of each of the sub-processes. Choice is based on evidence about option attributes, which are compared with decision objectives (Chapter 6).

8.2.3 Objective 3: *to identify the requirements for uncertainty management in coastal defence systems*

In Chapter 1 it was argued that coastal engineers are now confronted with increasing pressure for economic efficiency in public investment in coastal defences at a time of reduced in-house engineering capacity. The move towards soft engineering is intensifying the information processing demands placed on decision-makers at a time of increased public interest in coastal issues and the impacts of flooding. It cannot be assumed that traditional methods of dealing with uncertainty are as efficient as they could be or that they will continue to be effective in increasingly complex coastal management systems in which more interactions and sensitivities are taken into account. There is therefore a need for improved decision-making in general and improved management of uncertainty in particular.

Descriptive analysis demonstrated that the coastal defence system is far from straightforward. It is a messy problem, which does not readily succumb to the conventional tools of reductionist analysis. Therefore it was clear that any treatment of uncertainty should be able to handle the diversity of issues which occupy decision-makers. Key themes for uncertainty management emerged from the Grounded Theory analysis described in Chapter 2.

- It is important to take a comprehensive view of uncertainty and address the diverse sources of uncertainty.
- Techniques aimed at improving the management of uncertainty in coastal engineering need to be able to handle a wide range of types of uncertain information. This includes precise modelling data but also much more vague information about values and organisational constraints.
- The process of assembling evidence in the lead-up to a decision is a complex one, often combining a multi-disciplinary range of activities at very different levels of definition. It desirable that all of these activities can be represented in one model to provide a clear picture of the interaction of processes. Processes are rather project-specific.
- There is an increasing need for transparency in decision-making. To achieve this requires tools for externalising expert judgements and improving communication. Methods must be rigorous and repeatable.

These themes were developed through the discussion of systems and processes in Chapter 3, of uncertainty and decision-making in Chapter 4, of the mathematics of uncertainty in Chapter 5 and of choice in Chapter 6. The following requirements for uncertainty management emerged.

- In order to understand uncertainty in a decision it is necessary to understand the dependability of the processes leading up to that decision.
- Evidence about dependability should be taken into account in that decision, ideally by some explicit mechanism. That mechanism should reflect the nature of the available information and the meta-level objectives of the decision-maker.

- It is of great value to provide an overview of process and to clarify process attributes such as objectives, ownership, and criteria for success in the overall decision.

- Uncertainty should be represented with an appropriate mathematical syntax (see Objective 4, which is discussed in the following section).

8.2.4 Objective 4: *to review approaches to representing uncertainty with numerical structures and identify an appropriate syntax for representing the uncertainty in the evidence assembled in the lead-up to a coastal engineering decision*

Probability theory is the most established numerical representation of uncertainty and was the starting point for the examination of uncertainty calculi in Chapter 5. Probability theory is a special case of the various formulations of fuzzy sets and fuzzy measures that were critically reviewed in the context of coastal engineering. A set of criteria was established in order to identify an appropriate syntax for representing the uncertainty in the evidence assembled in the lead up to a coastal engineering decision. In summary:

- the syntax should represent an open world view in which the problem domain need not be completely specified in order to obtain meaningful inferences;

- the axioms should not be so weak as to provide inferences that are of limited practical use yet on the other hand they should be not artificially constrain the problem, implying less uncertainty than is in fact the case;

- there should be an explicit recognition of ignorance, to allow non-committal statements, and of inconsistency, to identify potential conflicts and factors contributing to the conflict;

- the syntax should be able to represent varied dependency relationships between evidence;

- the syntax should be reasonably straightforward to use in practical decision-making situations.

Probability theory does not reflect the aspects of vagueness, ambiguity, conflict and incompleteness that characterise the decision-making situations identified in this research. The axioms of probability theory can constrain an inferential problem in a way that is an unwelcome distortion of the decision-maker's actual state of knowledge or ignorance.

Interval Probability Theory (IPT) was found to satisfy the criteria for an uncertainty calculus. In IPT interval numbers are use to represent the probability measure, in order to capture in a relatively simple manner, features of ambiguity and incompleteness. Features of fuzziness can be captured by hierarchical ordering of concepts according to granularity of information. The dependence parameter ρ generalises other inference rules that assume a specific dependence relationship between evidence. Conflict between items of evidence, which can be unavoidable but also informative, is measured and propagated by the calculus.

A new and general approach to logical inference in IPT based on the total probability theorem has been presented. Conditional probabilities are used to represent the structure of the inferential situation. Judgements of incompleteness and relevance are explicitly required. The approach has proved through use in practice to be a relatively straightforward way of representing uncertainty in hierarchical process models.

8.2.5 Objective 5: *to implement theoretical developments in uncertainty representation in a decision support tool*

Uncertainty handling with IPT was implemented in a Windows-based software package for hierarchical process modelling. The implementation was described in outline in Chapter 5 and demonstrated in more detail in Chapter 7. The process model is constructed by starting with the high level process of the decision in question. Each of the sub-processes that contribute evidence to that decision are then assembled in a logical hierarchy. Interval numbers are attached to each of the low level process in the hierarchy to represent the evidence for and against the dependability of those processes. The relationship between sub-processes and super-processes (expressed as a conditional probabilities) and the dependency between sub-processes are also input as interval numbers. At each level in the hierarchy the least conservative bounds on the evidential support for the super-process are calculated and represented graphically. The software was tested in a range of conditions and found to be robust.

8.2.6 Objective 6: *to demonstrate how different types of uncertain information can be used when making a choice*

Discussion of the decision-making process in Chapter 4 identified choice as the pivotal moment in the decision-making process. In Chapter 6 it was demonstrated how different types of uncertain information, including information expressed in probabilistic, fuzzy and interval terms can be used in a choice. A contingency approach to choice involves characterising pertinent aspects of the choice situation and matching them to an appropriate choice mechanism. The decision-making context is a guide to an appropriate choice mechanism. A contingency approach helps to ensure that relevant information, for example relating to process dependability, is taken into account.

8.2.7 Objectives 7: *to demonstrate the new developments by application to case studies of UK coastal engineering projects*

In Chapter 7 the developments in process modelling and uncertainty representation using IPT were applied to two Environment Agency projects on the East Coast of the UK. Using documented evidence and the testimony of the experts involved, process models were constructed to identify the sources of uncertainty and the sensitivity of the decision process to the dependability of the various activities leading up to the decision. It was also demonstrated how uncertainty modelling with IPT can be combined with probabilistic analysis and an observational approach, to express and take account of the different types of uncertainty in a complex infrastructure management process.

8.3 From current practice to uncertainty management

8.3.1 Current practice

This research has, for the first time, addressed uncertainty from a systems point of view in coastal engineering. In a domain so strongly characterised by uncertainty, most obviously in the physical loading of the system, uncertainty has naturally attracted attention in the past. So, for example, the statistics of waves and water levels, and reliability analysis of flood defences are well-established topics. However, at the start of the research, the impact of uncertainty on the coastal defence system as a whole was uncharted. Hence, of necessity, the starting point of this research was to analyse uncertainty in the coastal defence system in as comprehensive a way as possible through empirical study of the sources and implications of uncertainty. The approach was to work with decision-makers to explore the issues that they considered significant in their daily practice. Documented evidence, for example from the Bye report (Bye and Horner, 1988) and the House of Commons Agriculture Committee (House of Commons, 1988), also contributed to the empirical analysis.

The picture which was established from these various empirical sources was one of widespread concern about uncertainty, though that concern was not necessarily explicitly articulated. In keeping with this tacit acknowledgement of uncertainty, many of the ways of dealing with it were intuitive or heuristic, for example through use of locally established conventions on freeboard allowances.

There was also a conviction that risk-based techniques held some answers to the problems that bothered practitioners. Probability and statistics had proved to be effective ways of dealing with the randomness of hydraulic loads. There was also attractive work, primarily from the Netherlands (CUR/TAW, 1990), on the use of reliability methods for the analysis of flood defences. Reliability methods from the process and nuclear industries had been applied to the uniform and highly engineered flood defence that are to be found in some areas of the Netherlands. Their application to strongly three-dimensional systems, which are subject to long term change and do not have a clearly defined 'failure surface', has proved to be much more challenging. The case was made in Chapter 3 that reductionist approaches like fault tree analysis will inevitably prove to be unsatisfactory in systems where emergent properties are significant. Reductionist reliability theorist have tended to side-step human and societal interactions with the flood defence system, or naïvely idealise them (see Appendix 1).

Thus, despite some fairly isolated treatment of uncertainty from a technical perspective, there was little in the way of analysis of uncertainty in the coastal defence system as a whole. Moreover, there was no conceptual framework for a holistic treatment of uncertainty in the coastal defence system. In the absence of such a framework the potential of existing tools and techniques for uncertainty management was not being fully realised. Establishing a conceptual framework of uncertainty in coastal engineering has been one contribution of this thesis.

The research came at a time of significant changes in coastal defence in the UK, which made the need for systems approaches, and a systematic treatment of uncertainty in particular, more evident. Coastal engineering is to do with dynamic management of a socio-technical system. This integrated view attaches as much importance to the human and organisational issues surrounding coastal defence systems as it does to the physical processes on the coast. There is an increasing emphasis on strategic planning, softer engineering techniques and a recognition of the socio-political context of coastal engineering.

MAFF is beginning to address some uncertainty issues in its new Project Appraisal Guidance on risk (to be published in 1999), following earlier work on the scope of risk and uncertainty in flood and coastal engineering (Meadowcroft *et al.*, 1997). Risk and uncertainty are also at the centre of the proposed research and development strategy for MAFF and EA (MAFF, 1999). The research described in this thesis therefore came at an exciting time of growing awareness of uncertainty issues, and the author was invited to contribute to all three of the documents just mentioned. There is a realistic prospect of the vision of uncertainty management promoted in this thesis being implemented in practice.

8.3.2 Uncertainty management

The aim of uncertainty management is decision-making that accounts for uncertainty in all its guises. Therefore, at the moment of choice decision-makers are provided with evidence that communicates the different sources and implications of uncertainty and guided towards appropriate choice mechanisms and strategies for implementation, monitoring and control. It aims to address the dependability of evidence about option attributes, the option generation process itself and the process of agreeing decision objectives, which are based on vague, perhaps conflicting, underlying values.

The dependability of the evidence will be a reflection of the process by which it was obtained. A central message in this thesis, which has been established in conceptual terms in Chapter 4, is that the dependability of the processes that contribute to a decision is as important as the (uncertain) evidence about option attributes upon which the decision is based. Issues of dependability and information content are intimately linked in the concept of uncertainty proposed in this thesis.

Engineers intuitively take account of the dependability of the processes they are engaged in when they are making decision. However, it has been argued that in more complex multi-disciplinary projects there is no reason to believe that intuition will continue to be a dependable guide. There is a need for more explicit ways of communicating issues of process dependability, a problem that has previously attracted scant attention. It is this problem that has been a focus of attention in this thesis. The contributions of this thesis to addressing the issue of process dependability has been

- to develop the conceptual framework for handling issues of process dependability;
- to develop hierarchical models for representing process dependability;
- to identify an appropriate calculus for representing uncertain dependability in hierarchical process models;
- to develop a new approach to uncertain inference using IPT based on the total probability theorem;
- to implement the developments in Visual C++ and integrate them with a Window's based hierarchical process modelling package developed by co-researchers;
- to demonstrate the new techniques in practice on two of the Environment Agency's sea defences projects. This is the first application of non-probabilistic methods of uncertainty representation to coastal engineering.

The benefits of the proposed approach are summarised as follows:

- Hierarchical process modelling provides an integrated overview of complex multi-disciplinary projects. It helps to clarify the contribution that diverse processes make to high level objectives.
- The uncertainty modelling approach provides a method for assembling evidence from diverse sources including documentary evidence and expert knowledge, so supports more informed decision-making.
- By externalising expert judgement the approach makes decision-making more transparent and aids communication, forming a basis for consensus building.
- Hierarchical modelling has revealed sensitivities to uncertain evidence in a decision. It has identified considerable disequilibrium in the level of evidence gathering and analysis that takes place in some important coastal engineering decisions.

The research on modelling process dependability has endeavoured to redress the balance that has tended to be strongly weighted towards representing uncertainty in information content, primarily in the conventional probabilistic paradigm. However, the aim of this thesis has been to develop a comprehensive view of sources and responses to uncertainty in decision-making in coastal engineering, and a systems-based view of coastal management in general. The final case study in Chapter 7 was a pointer in this direction. It was shown in principle how probabilistic optimisation methods could be combined with analysis of dependability of the optimisation models. Also, and crucially, strategies were adopted which were flexible and adaptable in the face of unforeseen conditions. At a time of increasingly apparent, and perhaps more erratic, global change resilient strategies will be of increasing importance.

8.3.3 Outstanding challenges

Whilst promoting a vision of uncertainty being addressed and managed in all its guises, the research described in this thesis has demonstrated the challenge presented by that vision as well as its potential. More explicit treatment of uncertainty can highlight issues that decision-makers may in practice be reluctant to acknowledge. In Chapter 7 practitioners were surprised by the high level of uncertainty in the evidence about decision options, even though this uncertainty measure was merely a consequence of the interval probabilities input lower in the hierarchy. It seems that in complex situations decision-makers may be inclined to under-estimate the level of uncertainty. The model provided an important message about the absence of dependable evidence and helped to identify where evidence-gathering activities should be directed to remedy the situation.

A move from normative decision theory to the more flexible, context-dependent approach of the contingency approach to choice also potentially brings new difficulties for decision-makers. They will have to reflect more deeply on the situations they are in and acknowledge that any choice mechanism is a compromise. This may not be an immediately welcome recognition but does better justice to the complexities of the coastal defence system and should ultimately result in improved decision-making.

The mathematics of uncertainty can be off-putting for some practitioners. However, the use of a graphical user interface has made uncertainty representation more accessible to the non-specialist. The graphical representation of interval probabilities provides a convenient overview of the complex sources and implications of uncertainty in a decision. An interval representation has proved to be attractive to a range of practising decision-makers. It is straightforward to explain and seems to reflect the decision-makers varying level of uncertainty surrounding the use of probabilistic statements. The approach provides decision-makers with information in a simple format, which at the same time can reflect the complexity of the inference problem and the richness of available evidence. One of the potential benefits of process modelling, which was identified in Chapter 3, was to support communication between players in complex projects. The graphical interface has helped to realise that potential.

Many of the uncertainty issues being addressed in this thesis are generic in nature. Inspiration for the new theoretical developments in uncertainty representation have come from outside coastal engineering, originating instead from the realms of artificial intelligence, hydroinformatics, systems and decision theory. The developments in uncertainty inference with IPT are generic and have been recognised as such. There is scope to test and apply the new developments to other domains, starting productively with other infrastructure systems. Application of early versions of the techniques to handling uncertain information in oil exploration attracted considerable attention from industry (CMPT, 1997), but also demonstrated the need to tackle some of the issues addressed in this thesis. Future applications include decision support for condition monitoring and asset management activities for dams, flood defences and engineered and natural slopes. Each of these applications will represent, in Popperian terms, a further and perhaps more ingenious test of the hypothesis that the uncertainty management framework and the specific tools and techniques of uncertainty representation with IPT in process models can be a productive support to decision-making. Each new application will also serve to enrich appreciation of uncertainty in engineering decision-making and provides potential to share best practice across industry sectors.

One of the barriers to wider use of process modelling techniques, in particular with uncertainty representation, is that model construction and maintenance can represent another time-consuming activity for already over-stretched decision-makers. User-friendliness has reduced the overhead associated with model construction. However, the balance between benefits and costs (which are mostly opportunity costs of time) will shift in favour of these approaches as they become more and more closely integrated with the other activities that decision-makers are enacting in a computer-based environment. There is a need to bring the two streams of uncertainty (information content and dependability) together in the same software environment. Most coastal monitoring and management data are now stored and analysed using computer systems so there is scope for intimate integration of knowledge bases with the actual process of assembling evidence and making decisions. The need to reflect information content and process dependability is consistent with the increasing emphasis being paid to meta-data in information systems (Millard, 1999). The aim should be to extract measures of information content automatically from available knowledge bases and integrate it with measure of dependability that emerge from intelligent automatic monitoring computer-based process enactment. This fits well with current developments in the field of machine learning and automatic acquisition of rules from data. For the time being, integrating these two sides of uncertainty being relies on value judgements of the relative importance of information content and dependability. In future there is scope to support these judgements with evidence from case histories. Developments in the field of case-based reasoning are relevant in this respect. Process models capture valuable information about decisions, so there is scope in future for automatic storage of process models as case histories and retrieval to support decision-making in analogous situations (Laing, 1998).

The aim therefore is for decision support systems that can handle a generalised range of uncertainty representations. In this thesis uncertainty representation has focussed on the use of IPT, which has proved to be useful method for representing uncertain evidence about process dependability. The links between IPT and other approaches to uncertainty representation have been demonstrated. However, interval representation is by no means universally appropriate. In Chapter 4 it was argued that the information content in an item of evidence can be uncertain in several different senses and should be expressed in a syntax that reflects that uncertainty. This calls for further research on generalised uncertainty representation. Mass assignment theory and random set theory provide promising pointers in this that direction. The additional challenge is to bring these approaches into the context of decision-making. The contingency approach to choice (Chapter 6) provides an approach to handling some different types of uncertain information at the moment of choice. Yet there is scope for further work to link generalised representations of uncertainty with decision theory.

Integrating knowledge bases with process enactment, generalised uncertainty representation, and issues of human interaction with computer-based uncertainty representation are therefore aspect of uncertainty management in coastal engineering that still need to be resolved. The research described in this thesis has contributed to the development of uncertainty management in coastal engineering in important respects. It has

- identified the needs for uncertainty management in coastal engineering;
- developed a general approach to uncertainty management which combines uncertainty in information content with modelling of process dependability and promotion of resilient coastal defence strategies;
- developed and demonstrated tools for modelling process dependability;
- suggested how evidence about process dependability could be integrated with other types of evidence; and
- developed a contingency approach to choice.

Whilst, inevitably, the task of completing the picture of uncertainty management for coastal defence systems must be left to future research, it has been drawn in outline and important areas have been filled in detail.

8.4 References

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APPENDIX 1

A critique of reliability theory and quantitative risk assessment

Reliability theory and quantitative risk assessment have attracted particular attention in engineering decision-making. These approaches treat model parameters in probabilistic terms. Thus for example in a probabilistic design of a flood defence embankment, a joint distribution of wave heights and water levels is used as the input into an overtopping model rather than one or a handful of precise input conditions. The design thereby takes account of the full range of possible loads and probabilistic information about their relative likelihood. In coastal engineering, where loads are well described by probability distributions, the move towards probabilistic design is welcome and long overdue (Hall and Meadowcroft, 1995, Meadowcroft *et al.*, 1997).

It is clear, however, that reliability theory only addresses the uncertainty in the model parameters which can be expressed in probabilistic terms. It says nothing about the dependability of the input parameters. The parameters are manipulated in deterministic models to generate probabilistic outputs. Other types of relation, be they fuzzy or other general types of mapping, are not recognised. Moreover there is no accepted mechanism for representing the dependability of the model. Therefore, whilst probabilistic methods are an important step they will, at least in their naïve form, be unsatisfying as a means of representing all of the different types of uncertainty encountered in a modelling process. They cannot contain all of the types of uncertainty implicit in the information and relations and cannot include reflections and evidence about dependability. Further more, because fuzziness is not recognised in reliability calculations, the problem is addressed at just one level of precision. This cannot do justice to complex problems where there is usually information from a diverse range of sources with different degrees of fuzziness. There may also be are range of models, from precise probabilistic ones to high level fuzzy models. Reliability theory admits only one model of the problem.

Not all reliability theorist conform to the naïve view that assumes that all of the phenomena of interest can be captured in one model at one level of resolution. However, they argue from a pragmatic point of view that the best engineers can do is use the most dependable model available in a probabilistic calculation and then proceed *as if* it were a perfect model. Having made this leap of faith, decision-making is a mere formality since, as will be explained in Chapter 6, to the rational person who has probabilistic information about the behaviour of decision options (and no other information) there is only one way of making a decision.

It is important to recognise that the numbers generated by reliability calculations are hardly scientific (in the Popperian sense) as they cannot normally be falsified. They are better viewed as deductions from a set of premises. Some of those premises will be better supported by evidence than others. Yet unless a procedure is adopted for recording and auditing the premises used in a reliability calculation the ultimate deduction will be of limited value.

Structural reliability calculations are therefore now widely recognised as only one part of the assessment of structural safety. The results of such calculations are not 'true' probabilities of failure, rather they are 'notional' probabilities of failure. Even in the relatively constrained context of the process industries, calculations of failure probabilities conducted by independent expert teams around Europe have been shown to typically differ by up to three orders of magnitude and in some cases differences were as large a five orders of magnitude (Lemkowitz *et al.*, 1995). It is not unreasonable therefore to have misgivings about the numbers that are generated in reliability calculations in coastal engineering. These calculations do not generally include the behaviour of individuals and almost never include organisational factors such as organisational culture.

Experienced engineers who are proponents of reliability theory will acknowledge that their models are not perfect. The probability numbers they generate are only as good at the models. However, they argue that doing one's best to include all relevant phenomena in the model and then assuming that it is perfect is a pragmatic way to behave. In situations where the relevant models (or in reliability terms, failure mechanisms) are well know and expressible in deterministic or probabilistic terms, the leap of faith is, arguably, justifiable. Probabilistic methods are a good way of dealing with well-known failure mechanisms. In more complex situations where the engineer intuitively feels that there are aspects which are not well represented in the probabilistic model, and may indeed have some (non-probabilistic) evidence to support that intuition, then probabilistic methods will be unsatisfying. Unfortunately many engineering failures are consequences of things which were not even foreseen by the engineering designer. More often than not they are the unforeseen consequences of human actions (Blockley, 1980).

Some reliability theorists have endeavoured to deal with the problem of relevant evidence not being included in the probability calculations by including extra probabilistic parameters in the constituent equations, or by constructing second order probability distributions in the output space. An example of the former approach is provided by one of Burcharth's (Burcharth and Sørensen, 1999) analyses of the reliability of caisson breakwaters, in which an additional parameter is included in the reliability equations. The value of the parameter is increased if site-specific physical model studies have been carried out. Conducting site-specific model studies will increase the dependability of the design but it is difficult to understand how this increase in dependability can be mapped directly onto a probability number, which is used to directly factor the probability of failure. The empirical evidence supporting this approach is questionable and, as Blockley (1999) argues, its use is inadequate because the level of sophistication of handling such a difficult and important part of the total uncertainty is very much less than for the relatively straightforward issue of parameter uncertainty.

The second order distribution approach involves mapping expert belief in the model onto a probability distribution and constructing a joint distribution, with probability of failure on one scale and belief on the other (Burmaster and Wilson, 1996). This approach is closer to the one proposed in this thesis. However, it has already been argued that probability expresses but one type of uncertainty. The evidence relating to model dependability, which is based on diverse items of information and expert interpretations, will tend to be uncertain in respects that are not well represented by probability. Moreover, the dependency relationship between marginal distributions of model output and model dependability is seldom clear. Nor is it necessarily the case that the distributions of model dependability are connected by the same relations as the model parameters.

The objective of including some information about model dependability in the results of probabilistic analysis is an important one that is central to this thesis. However, for the reasons outlined above, the methodologies adopted by reliability theorists and probabilistic risk analysts are unsatisfactory. Ultimately the problem is one of probability not being a rich enough language to represent the different types of information that are of relevance to most complex engineering decisions.

The foregoing arguments should not lead to the rejection of reliability calculations and probabilistic decision theory. These approaches provide important evidence on which to base decisions. However, engineers should recognise the need to broaden their perspective and explore ways of enriching current methods in order to merge qualitative and quantitative perspectives. A systems approach recognises that an infrastructure can be described at a number of different levels of resolution and from a number of different perspectives. Reliability theory can be fitted within this more general framework.

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APPENDIX 2

Subjective expected utility theory

Subjective expected utility (SEU) theory addresses the question of how a decision-maker should choose when confronted with a set of options and a (fully defined) set of uncertain outcomes. Under these circumstances the decision-maker should choose the option which maximises $\sum_{j=1}^{m} v(x_{ij})p(\theta_j)$ which is known as the expected utility of d_i (see Table A2.1). If the value $v(x_{ij})$ is replaced by the symbol u_{ij} the decision-maker chooses $\max_i \overline{u_i}$ where

$$\overline{u_i} = \sum_{j=1}^{n} u_{ij} p(\theta_j).$$

Table A2.1 Decision table showing utilities

States of nature		θ_1	θ_2	...	θ_m
	d_1	u_{11}	u_{12}	...	u_{1m}
	d_2	u_{21}	u_{22}	...	u_{2m}
Options	.	.	.	...	.
	.	.	.	...	.
	.	.	.	...	.
	d_l	u_{l1}	u_{l2}	...	u_{lm}
Probabilities		$p(\theta_1)$	$p(\theta_2)$	...	$p(\theta_m)$

The approach synthesises two important and controversial theoretical developments

i. that subjective probabilities are an appropriate measure of uncertain information;

ii. that the usefulness or satisfaction of a given outcome can be expressed as a utility.

Utility introduces a subjective measure of the decision-maker's preferences, for example their attitude to risk. Nonetheless, the decision-maker's attitudes or preferences have to be coherently ordered. Utility can therefore be thought of as the normative measure of value. The idea of utilities dates at least from Daniel Bernoulli (1738) but it was not until the present century that a set of axioms of coherent preference were developed. Economists and mathematical economists were largely, though not exclusively responsible for these developments (Fishburn, 1970). Above all, John von Neumann and Oscar Morgenstern in their *Theory of Games and Economic Behaviour* (1947) set out the modern approach to including attitudes to risk in preference structures.

SEU theory embodies a decision rule that maximises utility, an approach that is consistent with rational economic theory. According to Simon (1988) the rational man in economics is a maximiser. However, the subjective expected utility theory is not driven by a special desire to maximise expected utility. Rather, making behaviour equivalent to expected utility maximisation is a logical consequence of adopting a few consistency and continuity axioms. Axiomisations of SEU theory are presented by various authors (see for example Savage, 1954 and Luce and Raiffa, 1957) who demonstrate that the decision rule (*i.e.* that the preferred option is the one which maximises expected utility) can be derived from these axioms. The derivation is straightforward and is not reproduced here. The axioms, however, merit attention because they embody the conditions which the decision-maker is accepting when SEU is accepted as a choice mechanism. Table A2.2 summarises the meaning of the assumptions of SEU theory. It is not a formal axiomisation, having been compiled from the axiomisations of Savage (1954), Luce and Raiffa (1957), Shafer (1986) and French (1988), together with commentary by Tversky and Kahneman (1986) and Kleindorfer *et al.* (1993). Typically for subjective expected utility theory, the notation in Table A2.2 is couched in terms of lotteries.

The assumptions in Table A2.2 seem to be reasonable, intelligent and sensible (to paraphrase a dictionary definition of rationality). Upon them elaborate normative theory of decision-making has been constructed. However, there is ample empirical evidence to indicate that intelligent decision-makers systematically violate these axioms (Allais, 1953, Ellsberg, 1961, Tversky, 1969, Tversky and Kahneman, 1974, Tversky and Kahneman, 1981, Kahneman, Slovic and Tversky, 1982). These behavioural studies highlight the difficult of applying SEU and also suggest that human deviations from the normative model of rationality may not be aberrations but instead intelligent means of coping with complexity (March, 1988).

Table A2.2 Assumptions of subjective expected utility theory

Assumption	Explanation
Comparability	$x_1 \gtrsim x_2$ or $x_2 \gtrsim x_1$. The decision-maker must express some preference at least in terms of weak order. There does not exist a pair of objects x_1, x_2 such that the decision-maker holds neither x_1 to be at least as good as x_2 nor x_2 to be at least as good as x_1. This assumption is referred to as "completeness" by Kleindorfer *et al.* (1993).
Transitivity	If $x_1 \gtrsim x_2$ and $x_2 \gtrsim x_3$ then $x_1 \gtrsim x_3$. A common argument for transitivity is that cyclic preferences (*e.g.* $x_1 \gtrsim x_2 \gtrsim x_3 \gtrsim x_1$) can support a 'money pump' or 'Dutch book' in which the intransitive person is induced to pay for a series of exchanges that return to the initial option.
Continuity	If $x_1 > x_2$ and $x_2 > x_3$ then $\exists$ some p on [0,1] such that $px_1 + (1-p)\,x_3 \sim x_2$. There exists some lottery $\{p, x_1;\ 1-p, x_3\}$ a ticket to which the decision-maker will accept with indifference in exchange for a sure prize x_2. This forms the basis of the reference lottery upon which examples of subjective expected utility theory are often constructed.
Monotonicity	If $x_1 > x_n$ then $p_1x_1 + (1-p_1)\,x_n \gtrsim p_2x_1 + (1-p_2)\,x_n$ if and only if $p_1 \gtrsim p_2$. The lottery which gives the greatest probability of winning the more preferable prize x_1 and, therefore, also the least probability of winning the less preferable prize x_n is preferred.
Substitutability	If $l_1 = \{...;p_i, x_i;...\}$, $l_2 = \{...;p_i, x_j;...\}$, and $x_i \sim x_j$ then $l_1 \sim l_2$. Any state of the nature that yields the same outcome regardless of one's choice can be eliminated from the decision model i.e. states of the nature are only of interest inasmuch as they differentiate between options. This assumption is it referred to as "cancellation" by Tversky and Kahneman (1986) and as "independence" by Kleindorfer et. al. (1993) and was derived by Savage from what he referred to as the "sure thing principle".
Invariance	Different representations of the same choice problem should yield the same preference. That is, the preference between options should be independent of their description. This notion captures the "reduction to compound lotteries" axioms of Luce and Raiffa (1957) and French (1988) which refers to the case where the prize to a lottery is a ticket to another lottery. Provided the ultimate prizes and probabilities with which they are obtained the decision-maker should be indifferent to the number of compound lotteries.

Notes:

1. l is a lottery $\{p_1, x_1; p_2, x_2; p_3, x_3;... p_n, x_n\}$ with n different possible outcomes, where $p_i \geq 0$ is the probability of winning x_i and $\sum_{i=1}^{n} p_i = 1$
2. $>$ denotes preference.
3. $\sim$ denotes indifference.
4. $\gtrsim$ denotes weak preference; so if $x_1 \gtrsim x_2$ then x_1 is at least as good as x_2.

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APPENDIX 3

Value of information theory

Value of information theory relates to decisions about collecting evidence. Is it economical to buy another item of data which will help to refine the probability distribution $p(\theta_1)$, $p(\theta_2)$... $p(\theta_m)$ across the future states of nature? Normative decision theory provides an obvious means of evaluating the benefit of information about future states of nature. The theory begins by examining the value of perfect information and then goes on to address, using Bayes theorem, the situation where information is only partial. Numerical examples are included in introductory texts on decision theory such as Raiffa (1968) and Lindley (1985). The theoretical results are included here to demonstrate once again the power of normative decision theory to identify, under closed world conditions when the axioms apply, uniquely optimal decision strategies.

Before obtaining perfect information, which is usually only available at a price, the decision-maker only knows probabilities $p(\theta_1)$, $p(\theta_2)$,... $p(\theta_m)$ of occurrence of future states of nature θ_1, θ_2,... θ_m. Given perfect information the decision-maker would be certain about which future state of nature, θ_j, will materialise. The problem would the reduce to one of decision-making under certainty. The decision-maker would merely select the option d_i, with the maximum utility u_{ij}. The decision-maker can therefore make a prior estimate of the benefit of obtaining perfect information by multiplying this maximal utility by the corresponding probability of that uncertain state of nature, θ_j, and summing all the products obtained for the different states of nature. The expected utility with perfect information is therefore

$$\sum_{j=1}^{n} \max_{i} u_{ij} p(\theta_j) .$$

The expected value of perfect information is the difference between this quantity and the utility which would be expected in the absence of perfect information:

$$\sum_{j=1}^{n} \max_{i} u_{ij} p(\theta_j) - \max_{i} \sum_{j=1}^{n} u_{ij} p(\theta_j) .$$

To estimate the value of less than perfect information the various forms which the information could take are considered in turn and the expected outcome of the decision in the light of that information averaged over the possible forms with their appropriate probabilities. According to normative theory, any new item of information, however imperfect, can be used to update the probability distribution $p(\theta_1)$, $p(\theta_2)$,... $p(\theta_m)$ by applying Bayes theorem. If the decision-maker obtains information X these prior probabilities will be modified to $p(\theta_j|X)$:

$$p(\theta_j|X) = p(X|\theta_j)\, p(\theta_j) \,/\, p(X).$$

The decision-maker will therefore obtain

$$\max_i \sum_{j=1}^{m} u_{ij} p(\theta_j | X)$$

instead of

$$\max_i \sum_{j=1}^{m} u_{ij} p(\theta_j).$$

The expected value of partial information is therefore obtained by summing over the various possible forms the data X can take and is given by

$$\sum_X \max_i \sum_{j=1}^{m} u_{ij} p(X|\theta_j) p(\theta_j) - \max_i \sum_{j=1}^{m} u_{ij} p(\theta_j).$$

It can be shown that this quantity will be greater than or equal to zero. Partial information therefore always represents a gain in value, which has to be weighed up against the cost of obtaining it.

By calculating the expected value of partial information it is possible to design optimal data collection strategies. Consider, for example, a 1500m long section of decaying seawall. Suppose that there are two scenarios:

i. the seawall is superficially damaged and can be readily repaired, or

ii. the seawall is seriously decayed with large voids behind it and requires reconstruction.

Under the conditions of contract the client will be expected to pay a premium if the quantity of either type of construction turns out to be significantly different to the quantity specified in the tender document. The estimated rates for the various possible scenarios are summarised in Table A3.1.

Table A3.1 Estimated rates for seawall repair and reconstruction

	Quantities as appearing in the tender document	Quantities additional to those appearing in the tender document.
Repair of superficial damage	£900/m	£1200/m
Seawall reconstruction	£4500/m	£6600/m

Initial investigations have indicated that 200m of the seawall requires reconstruction and the remaining 1300m require repairs. However, the consultant carrying out the investigation has stated that the length of seawall requiring reconstruction could be as much as 500m. Suppose now that the client is offered the option of doing more detailed analysis of the conditions of the seawall, which will locate the length of seawall requiring reconstruction with 70% accuracy. This analysis will cost £26,000. The information given above is enough to evaluate whether the client should indeed invest in the additional information provided by the more detailed analysis.

The quantity of seawall reconstruction is discretised into four future states of nature θ_1, θ_2, θ_3, θ_4 as shown in Table A3.2. Without the information the construction cost is expected to be:

$$0.4 \times 2070 + 0.3 \times 2640 + 0.2 \times 3210 + 0.1 \times 3780 = £2640\text{k}$$

Suppose that X_1 depicts the situation where the survey indicates that the quantity of reconstruction is 200m, X_2 indicates 300m, X_3 indicates 400m and X_4 indicates 500m. The situation can then be expressed as shown in Table A3.3. The expected cost with the survey information is therefore

$$0.4 \times 2088 + 0.3 \times 2460 + 0.2 \times 2856 + 0.1 \times 3276 = £2472\text{k}$$

so the expected value of the information is

$$2640 - 2472 = £168\text{k}$$

which easily justifies the expense of £26k for the investigation. Incidentally, the expected value of perfect information can readily be calculated to be £210k.

Table A3.2 Probabilities and costs associated with future states of nature

	Length of seawall reconstruction (m)	$p(\theta_j)$	u_j (£k)
θ_1	200	0.4	2070
θ_2	300	0.3	2640
θ_3	400	0.2	3210
θ_4	500	0.1	3780

Table A3.3 Costs and probabilities for various information scenarios and states of nature

	θ_1	θ_2	θ_3	θ_4
X_1	2070	2640	3210	3780
X_2	2100	2430	3000	3570
X_3	2130	2460	2790	3360
X_4	2160	2490	2820	3150
$p(X_1 \mid \theta_j)$	0.7	0.1	0.1	0.1
$p(X_2 \mid \theta_j)$	0.1	0.7	0.1	0.1
$p(X_3 \mid \theta_j)$	0.1	0.1	0.7	0.1
$p(X_4 \mid \theta_j)$	0.1	0.1	0.1	0.7
$\sum_{k=1}^{4} u_{jk} p(X_k \mid \theta_j)$	2088	2460	2856	3276

The straightforward application illustrates the strengths as well as the weaknesses of value of information methods. When planning and designing data collection and investigation activities having an estimate of the monetary value is a useful guide. However, to do so requires information about probabilities and consequences of events, which may be very difficult to estimate at the time.

It is also necessary to model the situation as a set of discrete states of nature when many relevant phenomena about which data is collected (for example wave heights and water levels) are continuous. The example given above required:

- estimates of the cost of construction under an exhaustive set of states of nature, including estimates for the cost of 'unforeseen' work;
- prior probability estimates of those future states of nature;
- estimates of the effectiveness of the survey technique.

In practical circumstances it may be difficult to obtain credible estimates of any of these precisely because the situation is very uncertain so the engineer does not know what to expect. In a world governed by good estimates of prior probability it would be possible to design optimal data collection exercises and contractors would be able to submit optimal tenders on the basis of the risk they were exposed to and the information available to them. In an open world there are truly unforeseen conditions, which is why contracts do not always proceed as planned. There is however, scope for more use of value of information techniques provided that it is recognised that they provide only partial evidence about the benefits of data collection. Indeed it can be argued that being based on a closed world model value of information techniques can tend to undervalue the benefits of information. Monitoring is one of the activities that helps to cope with the incompleteness in an open world because it gives early warning of unforeseen conditions and events.

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APPENDIX 4

Multi-attribute value theory and multi-attribute utility theory

Multi-attribute value theory

Provided decision-makers are prepared to express their preferences in appropriate terms and obey some rationality axioms then problems of multi-attribute choice can be reduced to the familiar format of mathematical programming i.e. to a problem of maximising some function subject to a set of constraints. Multi-attribute value theory addresses the question of how a decision-maker's preferences can be structured and if necessary simplified so that a complete and computationally tractable optimisation model can be constructed.

Multi-attribute value theory is based on the existence of an ordinal value function, which expresses the decision-maker's preferences in numerical terms. Consider two decision options d_1 and d_2 with sets of attributes $(a_{11}, a_{12},..., a_{1n})$ and $(a_{21}, a_{22},..., a_{2n})$ respectively. Given that the attributes of d_1 are preferred to d_2 i.e.

$$(a_{11}, a_{12},..., a_{1n}) \succsim (a_{21}, a_{22},..., a_{2n})$$

then it is assumed that there is some ordinal value function $v(.)$ agreeing with the preference:

$$(a_{11}, a_{12},..., a_{1n}) \succsim (a_{21}, a_{22},..., a_{2n}) \Leftrightarrow v(a_{11}, a_{12},..., a_{1n}) \geq v(a_{21}, a_{22},..., a_{2n}).$$

The ordinal value function $v(.)$ associates a real number $v(a_{11}, a_{12},..., a_{1n})$ with each set of attributes. Now if

$$v(a_{11}, a_{12},..., a_{1n}) = v(a_{21}, a_{22},..., a_{2n})$$

then the decision-maker is indifferent between d_1 and d_2 so $(a_{11}, a_{12},..., a_{1n})$ and $(a_{21}, a_{22},..., a_{2n})$ are said to lie on an indifference surface.

The situation in two dimensions, where the indifference surface reduces to an indifference curve, is illustrated in Figure A4.1. To simplify the notation the two attributes A_1 and A_2 have been labelled X and Y and the sets of attributes associate with options d_1, d_2 and d_3 have been labelled (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively.

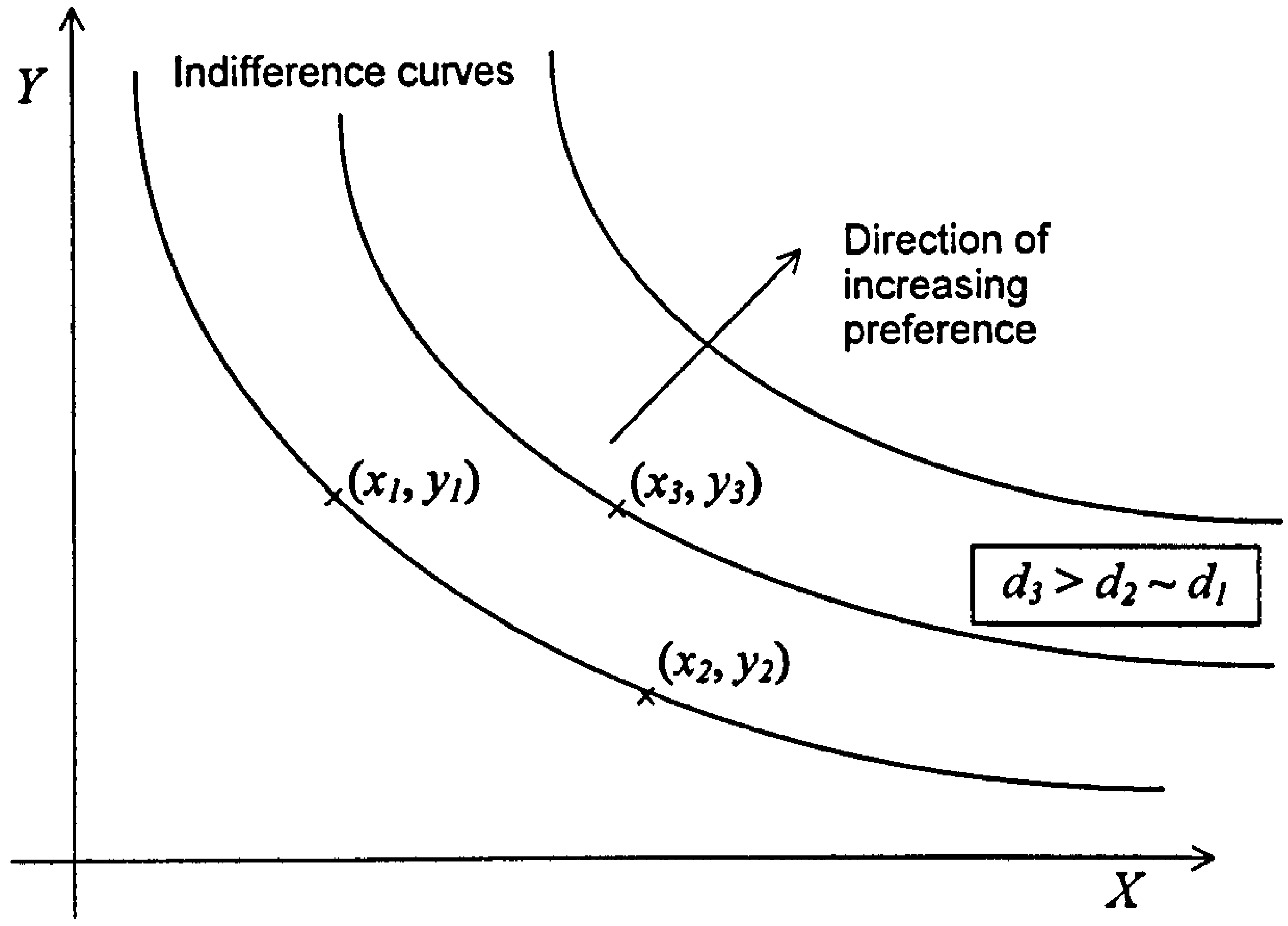

Figure A4.1 Indifference curves (after Keeney and Raiffa, 1976)

The *marginal rate of substitution* λ is the negative reciprocal of the slope of the indifference curve at any given point. It represents the amount of X a decision-maker is willing to 'pay' for a unit of Y. In many instances the marginal rate of substitution will vary depending on the levels of X and Y which are held. In the special case when the local substitution rate equals the global substitution rate then the indifference curves are of the form

$$x + \lambda y = \text{constant}$$

and can be represented by a linear value function

$$v(x, y) = x + \lambda y.$$

In more dimensions a linear value function can be represented as

$$v(a_{i1}, a_{i2},..., a_{in}) = w_1 a_{i1} + w_2 a_{i2} + ... + w_n a_{in}$$

The coefficients $w_1, w_2,..., w_n$ are known as *weighting factors*. If the decision-maker's preferences are modelled as being linear the multi-attribute problem is straightforward to solve using techniques of linear programming, by maximising the linear value function subject to the constraints. Convenient though a linear value function may be it implies a rather special preference structure. Linear value functions are, however, commonly assumed in many areas of economics, commerce and operational research. A specific example of linear value functions is the conventional approach to discounting over time, which is examined presently.

A rather more general case is when preference over X and Y can be represented by an ordinal value function of the form

$$v(x, y) = v_1(x) + v_2(y)$$

i.e. by an *additive* value function. In more general terms

$$v(a_{i1}, a_{i2}, \ldots, a_{in}) = \sum_{k=1}^{n} v_k(a_{ik}).$$

The existence of an additive value function limits the complexity of the multi-dimensional indifference surface and is commonly assumed. In the absence of an additive value function (or some comparable simplification), constructing an indifference surface that represents the decision-makers preferences would become unmanageable, particularly in high-dimensional situations. French (1988) presents five axioms that must be satisfied for an additive value function to exist. Besides some technical assumptions the substantive axiom is that $A_1, A_2, \dots, A_n$ must be *mutually preferentially independent*. Attribute X is preferentially independent of attribute Y if for all $x, x' \in X$

$$(x, \alpha) \gtrsim (x', \alpha) \text{ for some } \alpha \in Y$$
$$\Rightarrow (x, \beta) \gtrsim (x', \beta) \text{ for all } \beta \in Y.$$

Similarly, attribute Y is preferentially independent of attribute X for all $y, y' \in Y$

$$(\alpha, y) \gtrsim (\alpha, y') \text{ for some } \alpha \in X$$
$$\Rightarrow (\beta, y) \gtrsim (\beta, y') \text{ for all } \beta \in X.$$

If both of the above conditions hold then X and Y are mutually preferentially independent.

Preferential independence means that some attribute x will be preferred to x' regardless of the value which is taken by the other attribute. For example, suppose two attributes on which a flood defence scheme is being judged are capital cost and duration of construction. These two attributes are preferentially independent if lower cost is preferred to higher cost given that the construction duration remains the same, irrespective of what the construction duration actually is. Similarly shorter durations are preferred to longer durations given that the cost remains the same, regardless of what the cost actually is. This seems to be reasonable. A preferential independence condition will always be satisfied in cases where more of a given attribute is preferred to less regardless of the value of the other attributes. Preferential independence will not always apply when the attributes are distributed through time, a case that is examined below. However, in circumstances other than temporal streams of attributes preferential independence is a fairly weak condition to satisfy. Keeney and Raiffa (1976) describe some practical methods that can be used to test for preferential independence. Where there is a large number of attributes it is possible to exploit the hierarchical structure of objectives to efficiently test for preferential independence.

Preferential independence does not imply that the attributes are probabilistically independent. So, for example construction cost and construction duration are attributes which under most circumstances will be preferentially independent but which also tend to show a probabilistic dependency i.e. low cost is more often than not associated with rapid construction.

Together with the existence of an additive value function, it is convenient if the indifference curves can be shown to be convex. Figure A4.2 illustrates how in a constrained two dimensional space if there is a convex indifference curve a unique solution point will be located either

- at a tangent between the boundary of the permissible space and a unique indifference curve; or
- at one of the corners of the permissible space.

If both attributes obey the qualitative economic assumption of decreasing marginal worth of a commodity, and the value function is additive, then the indifference curve will be convex (French, 1988).

Multi-attribute utility theory

Multi-attribute utility (MAU) theory extends multi-attribute value theory to the situation of decision-making under risk. It enables the decision-maker's attitude to risk to be included in the construction of a preference function. The most straightforward case is when the decision-maker is

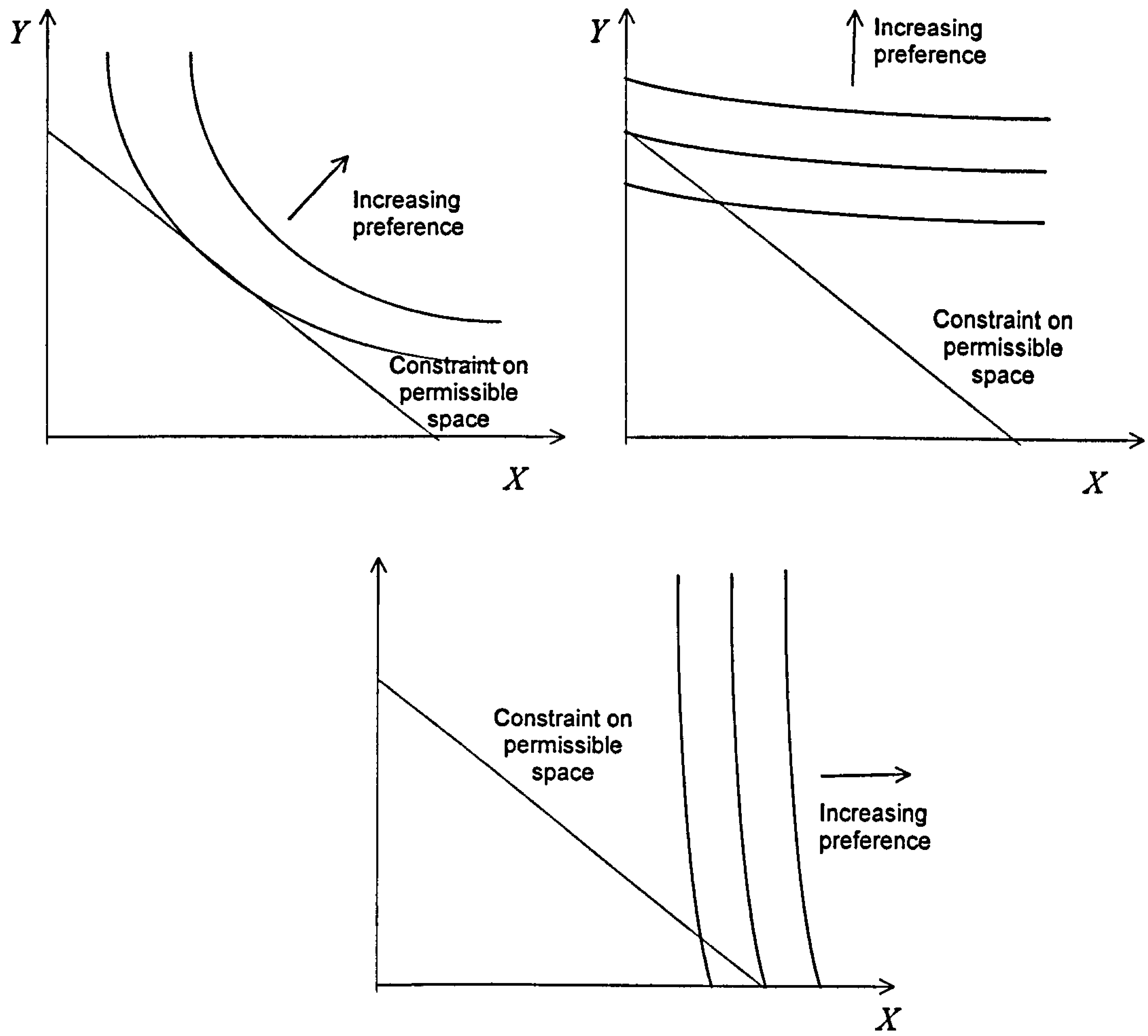

Figure A4.2 Location of the unique solution to an optimisation problem subject to constraints with convex indifference curves (after French, 1988)

risk neutral, in which case the values of the various attributes under consideration can be replaced by their expected values. To do so involves integrating the joint distribution of attributes across all of the future states of nature. This integration will be assisted if the attributes can be assumed to be probabilistically independent but they do not necessarily need to be.

The situation is less straightforward when the decision-maker is not risk neutral *i.e.* does not have a straight-line utility function for some or all of the attributes. In this case according to normative theory it is necessary to construct a multi-attribute utility function. Preference then is based on the integral of the utility function over the distribution of consequences. If a_k represents a specific level of A_k then the utility function $u(\mathbf{a}) = u(a_1, a_2,..., a_n)$ represents the decision-maker's preferences between lotteries involving prises in $A_1, A_2,..., A_n$, so the options should be ranked according to their expected utility:

$$E(u(\mathbf{a})|p) = \sum_{j=1}^{m} p(\theta_j)u(\mathbf{a}_j).$$

This is only usually practical if it is possible to obtain a representation of the utility function such that

$$u(a_1, a_2,..., a_n) = f[f_1(a_1), f_2(a_2),..., f_n(a_n)]$$

where f_k is a function of attribute A_k only and where f has a simple form, an additive or multiplicative form for example. Constructing a multi-attribute utility function is assisted if it can be shown that the utility functions over each attribute are *utility independent.* Utility independence is to some extent analogous to preferential independence in multi-attribute value theory though it is considered to be a somewhat stronger condition (Keeney and Raiffa, 1976). Utility independence means that the utility function, which expresses a decision-maker's attitude to risk on one attribute, is independent of the value taken by the other attributes. Keeney and Raiffa (1976) begin to explore situations where utility independence does not apply, a topic which has subsequently been developed to some sophistication by Farquhar and Fishburn (1981). Meyer (1976) tackles dependencies in the context of discounting over time, the topic that is now addressed.

Discounting over time

A very important class of multi-attribute value problems is situations involving attributes materialising at different moments in time. How should a given attribute, such as a cash payment, which materialises in five years time be compared with one that materialises immediately after the choice? The usual analysis is of a discrete stream of payments a set of annual payments. A project d_i giving rise to cash-flows a_{ik}, $k = 1, 2, ..., n$, over n years may thereby be considered a multi-attribute option. Conventional discounting procedures to a net present value (NPV) assume a linear value function

$$v(\mathbf{a}_i) = \sum_{k=1}^{n} w_k a_{ik}.$$

where

$\mathbf{a}_i = (a_{i1}, a_{i2}, ..., a_{in}),$

$w_k = 1/(1+r)^{k-1}$

and r is the discount rate. Because of its linear value function, discounting to net present value using a constant discount rate implies preferential independence and a constant marginal rate a substitution. The assumption of a constant discount rate is a very strong one. There is no *a priori* reason to suppose that the discount rate should be constant. Indeed it is possible to construct credible counter examples showing that cash flows over time do not necessarily even conform to the much weaker assumption of preferential independence required to justify an additive value function (Meyer, 1976). To apply a constant discount rate under conditions of uncertainty also requires an additive value function for which, as has been discussed, the condition of utility independence needs to be satisfied. Once again, counter-examples can be constructed to demonstrate that there are rational reasons why preferences over time are not necessarily utility independent. A decision-maker's attitudes to risk will tend to vary with time, particularly if her assets are changing with time. Time preferences are affected by the timing of the resolution of uncertainties and the horizon to be used (Meyer, 1976).

In common with other approaches to choice addressed in this chapter, the validity of methods of discounting over time is contingent on context. For government funded projects a constant discount rate and risk-neutral attitude is stipulated (HM Treasury, 1997). MAFF (1993) and Meadowcroft *et al.*, (1997) provide examples of risk discounting for flood and coast defence schemes in which a risk-neutral attitude has been assumed. Even so, annual-spending constraints may mean that particular phasings of cost are in practice more preferable than their net present value would suggest. Decision-makers and organisations within the public sector are not necessarily risk neutral and may tend to super-impose satisficing rules on the normative procedure to reflect their attitude to risk. The controversies raised by attempts to value time streams of non-traded goods (such as environmental benefits) are typical of the value conflicts introduced in Chapter 2 and evidence in Chapter3.

In the private sector it cannot be assumed that:

- the decision-maker can borrow or invest any sum of money for any period of time; and
- the rate of interest paid for borrowing equals the rate of interest earned by investing; and
- the rate of interest is independent of the sum involved and the period for which it is borrowed or leant.

In this case the assumptions of discounting at a fixed rate are questionable. The subtleties of the decision-maker's time-dependent preferences (including attitude to risk) and the vagaries of the financial markets will need to be examined in more detail before an appropriate multi-attribute value function can be constructed.

Problems of discounting over time represent the hardest test of conventional normative multi-attribute theory. French (1988) reaches the following general conclusion.

> *In decisions in which the consequences do not involve any time streams; i.e. in which all the attributes are received simultaneously, utility and additive independence are often (but not invariably) found to be reasonable assumptions. However, as soon as time enters the picture and some attributes are 'banked' before others arrive, independence conditions become very questionable.*

Summary

This appendix has involved a rather rapid tour through the fundamentals of multi-attribute decision theory. It has been mostly expository in nature, drawing on now well-established work by several previous researchers. The aim has been to develop of few key concepts of normative theory and demonstrate the importance of the assumptions on which they are based. Discounting over time is a particularly important topic whose axiomatic implications are rarely addressed by the coastal engineers who routinely use it in decision-making.

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APPENDIX 5

Fuzzy decision theory

Whilst probability theory has been used to model choice under conditions of risk, fuzzy decision theories attempt to deal with the vagueness or fuzziness inherent in subjective or imprecise objectives, options and evidence concerning future states of nature. Fuzzy decision theory is a theory of choice. As is the case with normative decision theory, little or no attention is paid to the process of obtaining options and objectives, save for the statement that it may be easier for human subjects to express options and objectives and people's beliefs about future sates of nature in fuzzy terms.

Fuzzy theories of choice are still in their early age yet there is already a proliferation of alternative formulations of choice problems in fuzzy terms though for the most part they have consisted of 'fuzzifications' of normative theories of choice. The intention here is to present the most established formulation, outline some alternative formulations and make some observations regarding the applicability of fuzzy theories of choice.

In fuzzy decision theory goals and constraints are treated in the same way. According to Bellman and Zadeh (1970) they are "symmetrical". This is a welcome insight, which was introduced in Chapter 6 in the discussion of satisficing and optimising, when it was suggested that objectives and constraints are isomorphic. The normative distinction between objectives and constraints is a consequence of insisting that maximisation (or minimisation) is the only rational basis for decision-making and is therefore the only admissible form of objective. If this is the case then satisficing criteria have to be redefined as constraints rather than objectives. The fuzzy decision theory approach of treating objectives and constraints in the same way is more general, recognising that both optimisation and satisficing are legitimate ways of achieving one's goals.

In the most established formulation of fuzzy choice (Bellman and Zadeh, 1970), constraints and objectives are defined as fuzzy sets, whilst the expected outcomes are assumed to remain deterministic or probabilistic. Suppose that D is the discrete set of options $\{d_1, d_2, \ldots, d_l\}$. The goals G and constraints C are characterised by membership functions

$\mu_G : D \rightarrow [0, 1]$

and

$\mu_C : D \rightarrow [0, 1]$.

The membership function of the fuzzy goal in this case serves much the same purpose as a utility or objective function that orders the outcomes according to preferability in normative theory. The term goal is customarily used to indicate that it can be satisfied (albeit in fuzzy terms) rather than maximised (or minimised). A fuzzy decision F may then be defined as the choice that satisfies both the goals G and constraints C, *i.e.* as the intersection of the fuzzy sets G and C, and so

$\mu_F(d) = min[\mu_G(d), \mu_C(d)]$

It is usual to choose the option $d_j \in D$ that attains the maximum membership grade in F.

Use of the intersection as given above to find the membership grade in D does not allow for any interdependence, interaction or trade-off between goals and constraints. The fuzzy union, *i.e.* the *max* operator, can be used to determine the membership grade in F when positive compensation exists between the goals and constraints, but the conventional *min/max* operators do not provide for the intermediate situation of partial dependence. Fuzzy choice therefore suffers from the same normative criticism as satisficing approaches to multi-attribute choice inasmuch as it just takes account of performance against the worst (or best) criterion and does not reflect the overall performance of each option. Several intermediate confluence functions have now been proposed. Dubois and Prade (1984) describe how an appropriate function can be selected by interrogating the decision-maker to establish trade-offs amongst goals and constraints, but this introduces another layer of judgements into the choice situation.

The choice situation explained above can be illustrated by an example from coastal engineering. Suppose that there are four options d_1, d_2, d_3, d_4 for a coastal defence project. Options will be chosen on the basis of benefit-cost ratio (BCR) subject to constraints of being environmentally and technically sound. Options d_1, d_2, d_3, d_4 have benefit-cost rations of 2.4, 1.8, 1.6 and 1.1 respectively. The first constraint of environmental soundness is represented by the fuzzy set C_1 defined on the set of coastal defence options as follows:

$C_1 = 0.4/ d_1 + 0.6/ d_2 + 0.8/ d_3 + 0.6/ d_4$.

The second constraint C_2 of technical soundness is defined by the fuzzy set C_2 such that

$C_2 = 0.2/ d_1 + 0.7/ d_2 + 0.9/ d_3 + 1.0/ d_4$.

The numerical values of BCR have to be fuzzified to define the fuzzy goal G of efficiency. Suppose that a BCR less than 1.0 is considered to be unacceptable then a suitable fuzzification function might be

$$\mu_G(d) = \begin{cases} 1 & for \quad d > 3 \\ 0.5(d-1) & for \quad d \le x \le 3 \\ 0 & for \quad d < 1 \end{cases}$$

so the corresponding goal G induced on the set of coast defence functions is given by

$G = 0.7/ d_1 + 0.4/ d_2 + 0.3/ d_1 + 0.05/ d_1$.

The fuzzy decision F, which is a fuzzy set, is obtained by taking the standard fuzzy set intersection of C_1, C_2 and G, so

$F = G \cap C_1 \cap C_2 = 0.2/d_1 + 0.4/d_2 + 0.3/d_3 + 0.05/d_4$.

The situation is illustrated in Table A5.1.

No particular option is prescribed, though Bellman and Zadeh (1970) suggested that the option with maximum membership in the decision set should be selected. In that case the coastal defence option that seems best to satisfy the goals and constraints is d_2. The choice is sensitive to the membership function that is defined for the goal of maximising BCR. The membership function effectively implies a value comparison between BCR and the constraints.

Table A5.1 Example of fuzzy choice

	μ_{C_1}	μ_{C_2}	μ_G	μ_F
d_1	0.4	0.2	0.7	0.2
d_2	0.6	0.7	0.4	0.4
d_3	0.8	0.9	0.3	0.3
d_4	0.8	1.0	0.05	0.05

The principle variants of the method of fuzzy choice outlined above and other extensions are summarised as follows.

- Multi-stage decision-making was introduced by Bellman and Zadeh (1970).
- Situations with objectives of unequal importance can be addressed by using weighting coefficients (Yager, 1978).
- The fuzzy theory of choice described above was based on only one state of nature. It was effectively a fuzzification of choice under certainty. Dubois and Prade (1980) review formulations of fuzzy choice under risk and examine how fuzziness can be introduced in the knowledge about future states of nature.
- Group decision-making can be included by developing a function that identifies the most acceptable group preference order from possibly conflicting individual orderings (Dubois and Prade, 1980).
- Several variants of fuzzy mathematical programming have been proposed (Zimmermann, 1987, Lai and Hwang, 1992).

- Choice based merely on the maximum grade in *F* can be criticised because it ignores information concerning any of the other alternatives. Methods that calculate the mean or centre of gravity of the fuzzy set *F* may therefore be used instead (Klir and Folger, 1988).

Fuzzy theories of choice enable information that is germane to a choice situation to be expressed in fuzzy rather than precise deterministic or probabilistic terms. In situations where decision-makers are reluctant to articulate their preferences or the trade-offs between objectives in precise terms, fuzzy choice mechanisms may be appropriate. Using fuzzy measures avoids spurious precision and can reflect a decision-makers vague state of preference or knowledge. The need to make trade-offs is not, however, eliminated. It is merely superseded by the problem of agreeing fuzzy membership functions.

Those normative theorists who have addressed fuzzy decision theory have, perhaps unsurprisingly, found that it does not meet their own criteria for a coherent theory of choice. Dyson (1980) has shown that under some conditions the Bellman and Zadeh approach to choice corresponds to the maximin approach to decision-making under uncertainty, which has been rejected by normative theorists as being incoherent. Lindley (1982) has shown that several theories of fuzzy choice allow Dutch books to be made against the decision-maker.

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